Applied Mathematics & Information Sciences
An International Journal

© 2012 NSP
Natural Sciences Publishing Cor.

The Skewed Double Inverted Weibull Distribution

A. Flaih[1], *H. Elsalloukh*[2], *E. Mendi*[3] *and M. Milanova*[3]

[1] Department of Applied Science, University of Arkansas at Little Rock, Arkansas 72204, USA
[2] Department of Mathematics and Statistics, University of Arkansas at Little Rock, Arkansas 72204, USA
[3] Department of Computer Science, University of Arkansas at Little Rock, Arkansas 72204, USA

Received: Jul 8, 2011; Revised Oct. 4, 2011; Accepted Oct. 6, 2011
Published online: 1 May 2012

Abstract: In this paper, we study a class of univariate skewed distributions, obtained by converting the symmetric inverted weibull distribution into a skew one. We discuss basic properties of skewed double inverted weibull distribution $SDIW$, the probability density function, cumulative distribution function, the moments, maximum entropy are derived. Maximum likelihood estimators and Fisher information matrix for the $SDIW$ are provided. We have used two real failure time datasets and compared our work with the results reported from [15], the exponentiated exponential model was the best fitted model in the first dataset, and the $SDIW$ is the better fit in the second datasets.

Keywords: Skewed double inverted weibull distribution; Information matrix; Kurtosis; Maximum likelihood; Skewness.

1. Introduction

Over the past years many researchers have been worked on searching alternative distributions that have accurately described and model data from wide range of application. In real life, for a particular dataset the construction of the symmetric families distribution may not exactly the appropriate distributions to model the data. Given this situation an increased interest to construction skewed distribution which is analytically tractable. A skewed distribution extends the symmetric distributions through adding a new shape parameter that controls the weight of the distribution tail or controls the skewness. Much work has been done with parametric skewed families distributions. "Normality is a myth; there never was; and never will be; a normal distribution" . . . [12].

[6] introduced the univariate skew-normal(SN) distribution,this class of distribution includes the normal distribution as a special case. The skew normal is:

$$h(x/\beta) = 2\phi(x)\Phi(\beta x); \quad x \in \Re \tag{1}$$

here $\phi(x)$ and $\Phi(x)$ denoted the density and the distribution function respectively. The parameter $\beta \in \Re$ controls the skewness in the real line. [8] formalized the skew multivariate normal distribution as an extension of the skew univariate normal distribution. [7] examined further properties of skew multivariate normal distribution. [16] introduced the stochastic representation of the univariate skew normal distribution $SN(\lambda)$ in terms of normal and truncated normal laws. [11] introduced a new method to convert any symmetric distribution into skewed.

$$h(x/\kappa) \tag{2}$$
$$= \frac{2\kappa}{1+\kappa^2}[f(\kappa x)I\{x<0\} + f(x/\kappa)I\{x>0\}]$$

where $\kappa > 0$ is the skewness parameter.

[26] constructed the skewed generalized T (GT) distribution, with skewness parameter $-1 < \epsilon < 1$, depends on the following way:

$$h(x/\epsilon) = f_1(\frac{x}{1-\epsilon})I\{x<0\} + f_2(\frac{x}{1+\epsilon})I\{x \geq 0\} \tag{3}$$

[3] introduced the skew-generalized normal distribution which is a closed related to the skew-normal distribution (SN) introduced by [6]. [4] considered a general class of skewed univariate distributions depending on the skewness parameter. [24] extended a family of skewed distribution generated using a symmetric density which depends on two parameters λ and β. [5] presented the skewed exponential power distribution depending on [11]. [19] considered new class of skew-normal distribution which is the

* Corresponding author: e-mail: anflaih@ualr.edu

© 2012 NSP
Natural Sciences Publishing Cor.

logarithmic skew-normal distribution (LSN) depending on the formula of [6] with (lnz). [9] introduced a generalization of the univariate skew Balakrishnan normal distribution with two parametrer. [13] discussed the epsilon skew student t distribution. Special case of this distribution is the Epsilon-skew Cauchy, Epsilon skew-normal distribution. [1] defined some skewed-symmetric distributions with properties, skew double gamma distribution, skew double weibull distribution, skew double beta-prime distribution depending on [6]. [2] defined some skew double inverted distributions depending on [6], skew double inverted gamma, skew double inverted weibull, skew double inverted pareto. [21] investigated the goodness of fit of the univariate skew-normal distribution depending on [6]. [14] presented the table of univariate skew-normal distribution with different values of skewness parameter β. [25] introduced a generalize to $SN(\lambda)$ in another way. [20] proposed a flexible class of skew-symmetric distribution which is combined of symmetric density and a skewed density function. [22] proposed two methods to estimate the parameters of the skew-normal distribution and skew exponential power distribution in small samples. The basic idea is to construct a distribution by joining at $x = \mu$ two half double inverted weibull with different shapes parameters $(1-\epsilon)$ for the negative orthant and $(1+\epsilon)$ for the positive orthant. The same principle applies to construction the distribution given in [26] and [11]. The contents of this paper are organized as follows: Section 2 discusses the basic steps to construct the skewed parent distribution from non-symmetric parent, especially the case of inverted weibull distribution. Section 3 focuses on the skewness, kurtosis, entropy, and median. Maximum likelihood estimation for the $SDIW$ model and Fisher information matrix are presented in Section 4.

2. Double inverted weibull distribution

In this section, we describe the basic steps to extend the inverted weibull distribution on the negative part of x-axis to construct the double inverted weibull distribution.

Step 1. Let X have the density $h(x)I\{X \geq 0\}$, since $h(x)$ integrate to one on $(0, \infty)$, we can redefine X on the domain $(-\infty, \infty)$ with density,

$$f(x) = \frac{1}{2}h(|x|) \tag{4}$$

which integrates to one on $(-\infty, \infty)$, therefore $f(x)$ is a symmetric density function.

Step 2. The inverted weibull distribution function is defined as follows:

$$f(x) = \frac{\beta}{\alpha}(\frac{1}{x})^{(\beta+1)}e^{-\frac{1}{\alpha}(\frac{1}{x})^{\beta}} \tag{5}$$

where $\beta, \alpha, x > 0$ and the distribution function is:

$$F(x) = e^{-\frac{1}{\alpha}(\frac{1}{x})^{-\beta}}; \quad x > 0$$

Step 3. The symmetrization procedure in step 1 leads to a double inverted weibull distribution with density function:

$$f(x) = \frac{\beta}{2\alpha}(|x|)^{-(\beta+1)}e^{-\frac{1}{\alpha}(|x|)^{-\beta}} \tag{6}$$

where $\beta, \alpha > 0, -\infty < x < \infty$. The distribution function is:

$$F(x) = \begin{cases} 1-\frac{1}{2}[1-e^{-\frac{1}{\alpha}(\frac{1}{x})^{\beta}}]; & x \geq 0 \\ \frac{1}{2}[1-e^{-\frac{1}{\alpha}(\frac{1}{x})^{\beta}}]; & x < 0 \end{cases} \tag{7}$$

3. Skewed double inverted weibull distribution

In this section, we disccus Skewed double inverted weibull $(SDIW)$ distribution and study their properties, from tracing the approach of [26] and [11] for describing the $(SDIW)$ distribution, we can obtain the $(SDIW)$ probability density function:

$$f(x; \mu, \alpha, \beta, \epsilon) \tag{8}$$

$$= \begin{cases} \frac{\beta}{2\alpha}(\frac{x-\mu}{1+\epsilon})^{-(\beta+1)}e^{-\frac{1}{\alpha}(\frac{x-\mu}{1+\epsilon})^{-\beta}}; & x \geq \mu \\ \frac{\beta}{2\alpha}(\frac{\mu-x}{1-\epsilon})^{-(\beta+1)}e^{-\frac{1}{\alpha}(\frac{\mu-x}{1-\epsilon})^{-\beta}}; & x < \mu \end{cases}$$

where $\alpha, \beta > 0$ and $-1 < \epsilon < 1$.

We denote the distribution of X by $SDIW_{\beta}(\alpha, \beta, \epsilon, \mu)$; $X \sim SDIW_{\beta}(\alpha, \beta, \epsilon, \mu)$

The parameters α and β correspond to scale, shape respectively, ϵ the skewness parameter and μ is the location parameter. Figure 1 gives a selection of the shapes of the probability distribution functions. The corresponding CDF

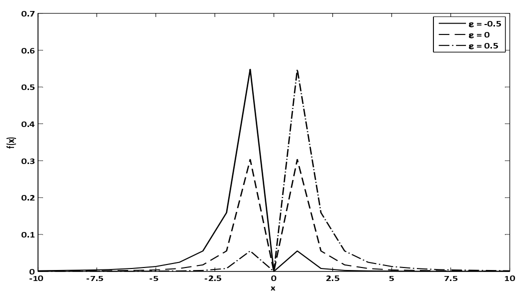

Figure 1 The pdf of the epsilon-skewed double inverted weibull distribution for $\epsilon = -0.5, 0, 0.5$ and $\alpha = 1, \beta = 1$.

© 2012 NSP
Natural Sciences Publishing Cor.

is:

$$F(x) = \begin{cases} 1 - \frac{1+\epsilon}{2}[1 - e^{-\frac{1}{\alpha}(\frac{x-\mu}{1+\epsilon})^{-\beta}}]; & x \geq \mu \\ \frac{1-\epsilon}{2}[1 - e^{-\frac{1}{\alpha}(\frac{\mu-x}{1-\epsilon})^{-\beta}}]; & x < \mu \end{cases} \quad (9)$$

3.1. Special Cases

3.1.1. Case $\beta = 1$

Here the density function (8) simplifies to:

$$f(x;\mu,\alpha,\beta,\epsilon) = \begin{cases} \frac{1}{2\alpha}(\frac{x-\mu}{1+\epsilon})^{-2}e^{-\frac{1}{\alpha}(\frac{x-\mu}{1+\epsilon})^{-1}}; x \geq \mu \\ \frac{1}{2\alpha}(\frac{\mu-x}{1-\epsilon})^{-2}e^{-\frac{1}{\alpha}(\frac{\mu-x}{1-\epsilon})^{-1}}; x < \mu \end{cases} \quad (10)$$

Which is the skewed symmetric double inverted exponential distribution.

3.1.2. Case $\beta = 2$

$$f(x;\mu,\alpha,\beta,\epsilon) = \begin{cases} \frac{1}{\alpha}(\frac{x-\mu}{1+\epsilon})^{-3}e^{-\frac{1}{\alpha}(\frac{x-\mu}{1+\epsilon})^{-2}}; & x \geq \mu \\ \frac{1}{\alpha}(\frac{\mu-x}{1-\epsilon})^{-3}e^{-\frac{1}{\alpha}(\frac{\mu-x}{1-\epsilon})^{-2}}; & x < \mu \end{cases} \quad (11)$$

This is the skewed symmetric double inverted Rayleigh distribution.

3.2. Moments and related parameters

In this subsection, we derive formulas for moments and related parameters of skewed double inverted weibull distribution. For the continuous random variable $X \sim SDIW(\beta,\alpha,\epsilon,\mu)$ the *n*th moment is defined as:

$$E(X-\mu)^n = \frac{\Gamma(1-\frac{n}{\beta})}{2\alpha^{\frac{n}{\beta}}}[(-1)^n(1-\epsilon)^{n+1} + (1+\epsilon)^{n+1}]$$

where n is an integer $n \geq 1$.

The mean of $X \sim SDIW(\beta,\alpha,\epsilon,\mu)$ is defined as:

$$E(X) = \mu + 2\epsilon\alpha^{-\frac{1}{\beta}}\Gamma(1-\frac{1}{\beta})$$

where $\beta > 1$. Its variance is:

$$var(x) = \alpha^{-\frac{2}{\beta}}[\Gamma(1-\frac{2}{\beta})(1+3\epsilon^2) - 4\epsilon^2\Gamma^2(1-\frac{1}{\beta})]$$

where $\beta > 2$. It is easy to find the median:

$$median = \begin{cases} (1+\epsilon)[-\alpha log(1-\frac{1}{1+\epsilon}]^{\frac{1}{\beta}}; & \epsilon < 0 \\ -(1-\epsilon)[-\alpha log(1-\frac{1}{1-\epsilon}]^{\frac{1}{\beta}}; & \epsilon \geq 0 \end{cases}$$

The skewness , Kourtosis, and Entropy respectively as follows:

$$skewness(X) = \frac{8\epsilon(1+\epsilon^2)\Gamma(1-\frac{3}{\beta})}{[\Gamma(1-\frac{2}{\beta})(1+3\epsilon^2) - 4\epsilon^2\Gamma^2(1-\frac{1}{\beta})]^{\frac{3}{2}}}$$

$$-\frac{6\epsilon\Gamma(1-\frac{1}{\beta})\Gamma(1-\frac{2}{\beta})(1+3\epsilon^2)}{[\Gamma(1-\frac{2}{\beta})(1+3\epsilon^2) - 4\epsilon^2\Gamma^2(1-\frac{1}{\beta})]^{\frac{3}{2}}}$$

$$+\frac{16\epsilon^3\Gamma^3(1-\frac{1}{\beta})}{[\Gamma(1-\frac{2}{\beta})(1+3\epsilon^2) - 4\epsilon^2\Gamma^2(1-\frac{1}{\beta})]^{\frac{3}{2}}}$$

where $\beta > 3$.

$$kurtosis(x) = \frac{\Gamma(1-\frac{4}{\beta})(5\epsilon^4 + 10\epsilon^2 + 1)}{[\Gamma(1-\frac{2}{\beta})(1+3\epsilon^2) - 4\epsilon^2\Gamma^2(1-\frac{1}{\beta})]^{\frac{3}{2}}}$$

$$-\frac{64\epsilon^2\Gamma^2(1-\frac{1}{\beta})\Gamma(1-\frac{3}{\beta})(1+\epsilon^2)}{[\Gamma(1-\frac{2}{\beta})(1+3\epsilon^2) - 4\epsilon^2\Gamma^2(1-\frac{1}{\beta})]^{\frac{3}{2}}}$$

$$+\frac{24\epsilon^2\Gamma^2(1-\frac{1}{\beta})\Gamma(1-\frac{2}{\beta})}{[\Gamma(1-\frac{2}{\beta})(1+3\epsilon^2) - 4\epsilon^2\Gamma^2(1-\frac{1}{\beta})]^{\frac{3}{2}}}$$

$$-\frac{24\epsilon^4\Gamma^4(1-\frac{1}{\beta})}{[\Gamma(1-\frac{2}{\beta})(1+3\epsilon^2) - 4\epsilon^2\Gamma^2(1-\frac{1}{\beta})]^{\frac{3}{2}}}$$

where $\beta > 4$

The Entropy is:

$$H(X) = -log(\frac{\beta}{2}) + \gamma + \frac{\gamma}{2} - \frac{log\alpha}{\beta} + 1$$

where $\gamma = 0.5772$ is the $Euler's$ constant and $\int_0^\infty logze^{-z}dz = -\gamma$

4. Maximum Likelihood Estimation

In this section, we consider ML estimation for the $SDIW$ model, [23] methodology for ML estimation consider. Let $x_{(1)} \leq x_{(2)} \leq ... \leq x_{(n)}$ be the order statistics of a sample from the $SDIW(\beta,\alpha,\epsilon,\mu)$ population, use $x_{(0)} = 0$ and $x_{(n+1)} = \infty$. Let $k = (x_{(1)},...,x_{(n)},\mu)$ be an auxiliary integer such that $x_{(k)} \leq \mu \leq x_{(k+1)}$, and $0 \leq k \leq n$ .In terms of the integer k the loglikelihood $l_k(\beta,\alpha,\epsilon,\mu)$ can be expressed as:

$$l_k(\beta,\alpha,\epsilon,\mu) = \begin{cases} nlog\beta - nlog2 - nlog\alpha - (\beta+1) \\ \sum_{i=1}^{n} log(\frac{x_i-\mu}{2}) - \frac{1}{\alpha}\sum_{i=1}^{n}(\frac{x_i-\mu}{2})^{-\beta}; \\ k = 0 \\ \\ nlog\beta - nlog2 - nlog\alpha \\ -(\beta+1)\sum_{i=1}^{n} log(\frac{\mu-x_i}{2}) - \\ \frac{1}{\alpha}\sum_{i=1}^{n}(\frac{\mu-x_i}{2})^{-\beta}; \quad k = n \\ \\ nlog(\frac{\beta}{2\alpha}) - (\beta+1)\sum_{i=k+1}^{n} log(\frac{x_i-\mu}{1+\epsilon}) - \\ \frac{1}{\alpha}\sum_{i=k+1}^{n}(\frac{x_i-\mu}{1+\epsilon})^{-\beta} - \\ (\beta+1)\sum_{i=1}^{k} log(\frac{\mu-x_i}{1-\epsilon}) - \\ \frac{1}{\alpha}\sum_{i=1}^{k}(\frac{\mu-x_i}{1-\epsilon})^{-\beta}; \quad 1 \leq k \leq n \end{cases} \quad (12)$$

© 2012 NSP
Natural Sciences Publishing Cor.

Lemma 1. If $k = 1$ or $k = 0$ the maximum likelihood estimates $(\hat{\epsilon}, \hat{\mu}, \hat{\alpha}, \hat{\beta})$ is given by:

$$(\hat{\epsilon}, \hat{\mu}, \hat{\alpha}, \hat{\beta}) = \begin{cases} (1, x_{(1)}, \alpha_1, \hat{\beta}_1); & k = 0 \\ (1, x_{(n)}, \alpha_2, \hat{\beta}_2); & k = n \end{cases} \quad (13)$$

where

$$\alpha_1 = \frac{1}{n}\sum_{i=1}^{n}(\frac{x_i - x_1}{2})^{-\beta}$$

and

$$\alpha_1 = \frac{1}{n}\sum_{i=1}^{n}(\frac{x_n - x_i}{2})^{-\beta}$$

$\hat{\beta}_1$ and $\hat{\beta}_2$ can be accomplished by the use of standard iterative procedure (i.e., Newton-Raphson method).

When $1 \leq k \leq n$,the ML estimates are found by solving the following loglikelihood function

$$\frac{\partial l_k(\lambda)}{\partial \lambda_i} = 0, \quad i = 1, 2, 3, 4$$

where $\lambda = (\lambda_1, ..., \lambda_4) = (\hat{\epsilon}, \hat{\mu}, \hat{\alpha}, \hat{\beta})$

In terms of α we can easily solve the likelihood equation $\frac{\partial}{\partial \alpha} l_k(\beta, \alpha, \epsilon, \mu) = 0$ to find:

$$\hat{\alpha} = \frac{1}{n}[\sum_{i=k+1}^{n}(\frac{x_i - \mu}{1+\epsilon})^{-\beta} + \sum_{i=1}^{k}(\frac{\mu - x_i}{1-\epsilon})^{-\beta}] \quad (14)$$

Additionally, numerically methods must be applied to solve the likelihood equations:
$\frac{\partial}{\partial \mu} l_k(\beta, \alpha, \epsilon, \mu) = 0,$
$\frac{\partial}{\partial \epsilon} l_k(\beta, \alpha, \epsilon, \mu) = 0,$
$\frac{\partial}{\partial \beta} l_k(\beta, \alpha, \epsilon, \mu) = 0.$
$\hat{\mu}_k$ will be either $X_{(k)}$ or $X_{(k+1)}$ if $\frac{\partial}{\partial \mu} l_k(\beta, \alpha, \epsilon, \mu) = 0$ has a solution in the interval $(X_{(k)}, X_{(k+1)})$.

Applying Proposition 5 proposed in [4], the ML estimates of λ becomes $(\hat{\mu}_k, \hat{\epsilon}_k, \hat{\beta}_k)$, where k is such that $l_k(\hat{\mu}_k, \hat{\epsilon}_k, \hat{\beta}_k) \geq l_i(\hat{\mu}_k, \hat{\epsilon}_k, \hat{\beta}_k)$, $\forall\ i \in 0, ..., n$. The Fisher information matrix is given by the elements:

$$-E[\frac{\partial^2 logf}{\partial \mu^2}] = \frac{2\epsilon^2 \alpha^{\frac{3}{\beta}} \Gamma(2+\frac{3}{\beta})}{(1+\epsilon^2)(1-\epsilon^2)} + \frac{(1+\beta)\alpha^{\frac{2}{\beta}} \Gamma(1+\frac{2}{\beta})}{1-\epsilon^2}$$

$$-E[\frac{\partial^2 logf}{\partial \epsilon^2}] = \frac{1+\beta}{1-\epsilon^2} + \frac{2\beta^2\epsilon}{(1+\epsilon^2)(1-\epsilon^2)}$$

$$-E[\frac{\partial^2 logf}{\partial \beta^2}] = \frac{ln^2\alpha}{\beta^2}$$

$$-E[\frac{\partial^2 logf}{\partial \alpha^2}] = \frac{1}{\alpha^2}$$

$$-E[\frac{\partial^2 logf}{\partial \mu \partial \alpha}] = 0$$

$$-E[\frac{\partial^2 logf}{\partial \beta \partial \mu}] = \frac{\beta \alpha^{\frac{2}{\beta}}}{1-\epsilon}\Gamma(2+\frac{2}{\beta})$$

$$-E[\frac{\partial^2 logf}{\partial \mu \partial \epsilon}] = \frac{-\beta \alpha^{\frac{1}{\beta}}\epsilon}{1-\epsilon^2}\Gamma(2+\frac{1}{\beta})$$

$$-E[\frac{\partial^2 logf}{\partial \beta \partial \alpha}] = \frac{1-\epsilon^2 + ln\alpha(1-\epsilon)}{\alpha\beta}$$

$$-E[\frac{\partial^2 logf}{\partial \alpha \partial \epsilon}] = 0$$

$$-E[\frac{\partial^2 logf}{\partial \beta \partial \epsilon}] = \frac{1}{2} - \frac{\beta}{2(1+\epsilon)}$$

5. An Application

In this section, we illustrate the application of the Skewed Inverted Weibull distribution described in the previous sections using two uncensored datasets:

First dataset: The first dataset, available from [17], consists of the number of million revolutions before failure for each of 23 endurance of deep-groove ball bearings.

17.88, 28.92, 33.00, 41.52, 42.12, 45.60, 48.40, 51.84, 51.96, 54.12, 55.56, 67.80, 68.64, 68.64, 68.88, 84.12, 93.12, 98.64, 105.12, 105.84, 127.92, 128.04, 173.40.

We have compared the MLEs of the unknown parameters and the corresponding Log-Likelihood (LL), the $SDIW$ model with the results that reported from [15], as given in Table 1.

Table 1 The $MLEs$ and the corresponding LL

The model	$MLEs$	LL
Gamma(λ, α)	$\hat{\lambda}$=0.0556, $\hat{\alpha}$=4.0196	-113.0274
Weibull(λ, α)	$\hat{\lambda}$=0.0122, $\hat{\alpha}$=2.10502	-113.6887
EE(λ, α)	$\hat{\lambda}$=0.0314, $\hat{\alpha}$=5.2589	-112.9763
SDIW(β, α, μ, ϵ)	$\hat{\beta}$=1.864, $\hat{\alpha}$=0.5576, $\hat{\mu}$=68, $\hat{\epsilon}$=-0.4494	-124403.610

Second dataset: The second data is set available from [18]. They are the time intervals (hours) between failures of the air conditioning system of an airplane:

23, 261, 87, 7, 120, 14, 62, 47, 225, 71, 246, 21, 42, 20, 5, 12, 120, 11, 3, 14, 71, 11, 14, 11, 16, 90, 1, 16, 52, 95.

The MLEs of the unknown parameters and the corresponding Log-likelihood (LL) for the $SDIW$ model have

© 2012 NSP
Natural Sciences Publishing Cor.

Table 2 The $MLEs$ and the corresponding LL

The model	$MLEs$	LL
Gamma(λ, α)	$\hat{\lambda}$=0.0135, $\hat{\alpha}$=0.8134	-152.2312
Weibull(λ, α)	$\hat{\lambda}$=0.0183, $\hat{\alpha}$=0.8554	-152.007
EE(λ, α)	$\hat{\lambda}$=0.0145, $\hat{\alpha}$=0.08130	-152.264
SDIW(β, α, μ, ϵ)	$\hat{\beta}$=1.248, $\hat{\alpha}$=72.067, $\hat{\mu}$=22, $\hat{\epsilon}$=0.5902	-80.9411

been obtained and compared with the results that reported from [15] as given in Table 2.

Our comparison is based on the negative log-likelihood (LL), for the first dataset we observed from the (LL) that $SDIW$ model does not fitted the data well, nevertheless the $SDIW$ model fits the best in the second dataset in terms of the negative Log-likelihood values. So, it is not guaranteed the $SDIW$ will fitted always better than Gamma, Weibull, and Exponentiated exponential (EE), but at least we can say in certain dataset $SDIW$ might work better than Gamma, weibull, or (EE). These four models; $SDIW$, Gamma, Weibull, and (EE) are not nested. However, the Log-likelihood values also can be compared by using Akaike's information criteria (AIC). We observed that the difference, 2 (124403-112.976) is so large, it follows that the exppenentiated exponential model (EE) providing a significantly better fit than Gamma, Weibull, and $SDIW$ model in the first dataset. In the second dataset and according to the AIC value, it is observed that the difference 2 (152.007-80.9411) is the largest, it follows that the $SDIW$ model provides a significantly better fit.

6. Conclusions

In this paper, we considered the $SDIW$ distribution. We derived the basic properties of this new family, PDF, CDF, Mean, variance, Median, Skewness, Kurtosis, and entropy. Also, we provided the maximum likelihood estimators along with the fisher information matrix. We presented two real life datasets: in the first dataset the (EE) model has a better fit model as compared to Gamma, Weibull, and $SDIW$. In the second dataset, we have observed that $SDIW$ has a better fit model as compared to the Gamma, Weibull, and (EE) model. So, it can be concluded that the $SDIW$ model is very sensitive to the type of data dealing with. Obviously, more work needed with $SDIW$ family.

References

[1] M. M. Ali and J.Woo, Skew-symmetric reflected distribution. Soochow Journal of Mathematics, **32**, 2, 233-240 (2006).
[2] M. M. Ali, J. Woo and S. Nadarajah, Some skew symmetric inverse reflected distributions, Braz. J. Probab. Stat., **24**, 1-23 (2010).
[3] R. B. Arellano-Valle, H. W. Gómez and F. A. Quintana, A New Class of Skew-Normal Distributions, Communications in Statistics-Theory and Methods,1532-415X, **33**, 7, 1465-1480 (2004).
[4] R. B. Arellano-Valle, H. W. Gómez and F. A. Quintana, Statistical inference for a general class of asymmetric distributions, J. of Statistical Planning and Inference, **128**, 2, 427-443 (2004).
[5] A. Ayebo, Skew exponential power distribution, M.Sc. Thesis, USA (2002).
[6] A. Azzalini, A class of distributions which includes the normal ones, Scand. J. Statist., **12**, 171-8 (1985).
[7] A. Azzalini and A. Capitanio, Statistical applications of the multivariate skew normal distribution, J. R. Statist. Soc. Ser. B., **61**, 579-602 (1999).
[8] A. Azzalini and A. D. Valle, The multivariate skew-normal distribution, Biometrika, **83**, 715-726 (1996).
[9] W. Bahrami, H. Agahi and H. Rangin, A Two-parameter Balakrishnan Skew-normal Distribution, J. Statist. Res. Iran, **6**, 231-242 (2009).
[10] H. Elsalloukh, The epsilon skew exponential power distribution, Ph.D. Thesis, USA (2004).
[11] C. Fernandez and M.F.J.Steel,On Bayesian modeling of fat tails andskewness, Journal of the American Statistical Association, **93**, 359-371 (1998).
[12] R. C.Geary, Testing for normality, Biometrika, **34**, 209-242 (1947).
[13] H. W. Gómez, F. J. Torres and H. Bolfarine, Large-sample inference forthe epsilon-skew-t distribution, Commun.Statist.-Theory Meth., **36**, 73-81 (2007).
[14] A. K. Gupta and T. Chen, Goodness-of-fit tests for the skew-normal distribution, Communications in Statistics-Simulation and Computation, **30**, 907-930 (2001).
[15] A. K. Gupta and D. Kundu, Exponentiated Exponential Family: An Alternative to Gamma and Weibull Distributions, Biometrical Journal, **43**, 117-130 (2001).
[16] N. Henze, A probabilistic representation of the "skew-normal" distribution, Scand. J. Statist., **13**, 271-5 (1986).
[17] J. F. Lawless, Statistical Models and Methods for Lifetime Data, John Wiley and Sons, New York (1982).
[18] H. Linhart and W. Zucchini, Model Selection, Wiley, New York (1986).
[19] G. D.Lin and J. Stoyanov, The logarithmic skew-normal distributions are moment-indeterminate, J. Appl. Prob., **46**, 909-916.(2009).
[20] Y. Ma, and M. G. Genton, Flexible class of skew-symmetric distributions, Scandinavian Journal of Statistics., **31** ,459-468 2004
[21] G. Maute-Figueras, P. Puig and A. Pewsey, Goodness-of-fit tests for the skew-normal distribution when the parameters are estimated from the data, Communications in Statisics-Theory and Methods, **36**, 1735-1755 (2007).
[22] A. C. Monti, A note on the estimation of the skew normal and the skew exponential power distributions, METRON - International Journal of Statistics, **LXI**, 2, 205-219 (2003).
[23] G. S. Mudholkar and A. D. Hutson, The epsilon-skew-normal distribution for analyzing near-normal data, J. Statist.Plan. Infer., **83**, 291-309 (2000).
[24] H. S. Salinas, R. B. Arellano-Valle and H. W. Gómez, The extended skew exponential power distribution and its derivation, Commun. Statist.-Theory & Methods, **36**, 1673-1689 (2007).

© 2012 NSP
Natural Sciences Publishing Cor.

[25] M. Sharafi and J. Behboodian, The Balakrishnan skew-normal density, Stat. Papers, **49**, 769-778 (2008).
[26] P. Theodossiou, Financial data and the skewed generalized T distribution, Management Science, **44**, 1650-1661 (1998).

A. Flaih is Ph.D. student at the University of Arkansas at Little Rock, USA. His research fields are statistical inference, statistical quality control, Bayesian analysis, and regression methods. In June 1995 he obtained a M.Sc. in statistics. He has published more than 15 papers.

E. Mendi is Ph.D. student in Computer Science at the University of Arkansas at Little Rock, USA. He received his M.S. degree in 2006 from Technical University of Munich, Germany and his B.S. degree in 2004 from the Middle East Technical University, Ankara, Turkey. His main research interests are in the areas of image-video processing, computer vision, computer graphics, artificial intelligence, biomedical engineering, statistical inference, and probability distribution theory. He has published and co-authored more than 25 publications, over 8 journal papers, 1 book chapter and numerous conference papers.

M. Milanova is a Professor of Computer Science Department at the University of Arkansas at Little Rock since 2001. She received her M.S. degree in Expert Systems and AI in 1991 and her Ph.D. degree in Computer Science in 1995 from the Technical University, Sofia, Bulgaria. Dr. Milanova did her post-doctoral research in visual perception at the University of Paderborn, Germany. She had grants from the German Research Foundation, the Brazilian Research Foundation, the US National Science Foundation, the European Community, NATO, US Department of Homeland Security and from US Air Force. She serves as a book editor of two books and associate editor of several international journals. Her main research interests are in the areas of artificial intelligence, biomedical signal processing and computational neuroscience, computer vision and communications, machine learning, and privacy and security based on biometric research. She has published and co-authored more than 60 publications, over 33 journal papers, 8 book chapters, numerous conference papers and 2 patents.

© 2012 NSP
Natural Sciences Publishing Cor.