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Gap Study for the Impact of Braced Deep Excavation on the Behavior of Excavation Bed

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A B S T R A C T

Settlement and stress distribution beneath the raft of multi-story building depends on both value and distribution of the modulus of subgrade reaction. On other hand, deep excavation releases the overburden pressure and allows the excavation bed to uneven heave. This phenomena affects both value and distribution of modulus of subgrade reaction of soil at excavation bed, accordingly affects the expected settlement and stress distribution beneath foundation. Two geotechnical topics were investigated in this research; shored deep excavation and modulus of subgrade reaction. Both topics were extensively studied by many earlier researchers. The aim of this study is to answer the following question, "Is deep excavation effects on excavation bed behavior of sufficiently investigated? Or there still un-studied gaps should be fulfilled?" Literatures were collected, reviewed and classified for both topics especially in the interaction zone between the two topics, and it is concluded that deep excavation effect on the modulus of subgrade reaction values and distribution at excavation bed level is not sufficiently addressed especially the effect of excavation bed heave and stiffness of shoring system.

1. INTRODUCTION

Interaction between foundation and the soil beneath is one of the extensively addressed points in geotechnical engineering. It is common practices to use the concept of subgrade reaction to simulate the soil behavior in simple and commercial structural models because it is much simpler than sophisticated soil-structure interaction models. Modulus of subgrade reaction's value and distribution are affected by many parameters including stress path and boundary conditions (which is the case in deep excavation). Braced deep excavation is usually used to construct multi-basement buildings. One of the major concerns of deep excavation is the associated deformations which are lateral displacement of the wall, soil settlement behind the wall, and heave at the bottom of the excavation. These deformations are controlled by the bracing system stiffness which affected by many factors such as flexural stiffness of wall, axial stiffness and spacing of struts and the embedded depth of wall. Several researches were carried out to correlate the stiffness of the shoring system with its geometrical characteristics and element's properties. On other hand, many researchers tried to predict the associated deformations of deep excavation using subsurface site conditions and the stiffness of the shoring system. The aim of this research is to study all previous work regarding to the braced deep excavation and the effect of the stiffness of the bracing system on both of associated deformations and behavior of the excavation bed to figure out the un-studied gaps in this issue. The deep excavation affects the behavior of the excavation bed due to removing the overturned pressure which causes the bed heave. Accordingly, it affects the value of modulus of subgrade reaction at excavation bed. This research is divided into four parts; the first one includes the earlier researchers that concerned in defining and estimating the stiffness of the shoring system, the second part discusses the previous

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work regarding to the prediction of heave at the excavation bed, while the third part is concerned in the proposed methods to estimate the value of the modulus of sub-grade reaction. Finally, the fourth part concludes the research outcomes and discusses the un-studied gaps regarding to these issues.

2. STIFFNESS OF BRACED EXCAVATION SYSTEM

The definition of deep excavation bracing system stiffness is a complex parameter that includes both element properties (flexural stiffness of the wall, axial stiffness of the struts) and geometrical characteristics of excavation (excavation depth, spacing between struts and wall penetration depth).Several formulas were developed to describe this parameter starting with Rowe (1952), who proposed a formula for the bracing system flexibility (1/bracing system stiffness) contains only the flexural stiffness of the wall (EI) and the excavation depth (H) as shown in Eq. (1).

$$\text{Flexibility number} = \frac{H^4}{EI} \qquad (1)$$

Clough (1989) suggested a dimensionless system stiffness parameter includes flexural stiffness of the wall (EI) and the vertical spacing between struts (h) as shown in Eq. (2). He developed a design chart to predict the maximum lateral wall deformation based on his system stiffness parameter besides the basal safety factor after Terzaghi, (1943) for wide deep excavation (excavation depth / excavation width <1.0) and after Bjerrum and Eide (1956) for narrow deep excavation (excavation depth / excavation width >1.0) as shown in Fig. (1)

$$\text{System stiffness (S)} = \frac{\mathrm{EI}}{\gamma_w h^4} \qquad (2)$$

Addenbrooke, (1994) defined a displacement flexibility number as per Eq. (3) by integrating the simple beam equation to find the displacement over the span distance, his results showed a wide scatter in the measured displacements for each flexibility number

$$\text{Displacement flexibility number} = \log\left(\frac{h^5}{\mathrm{EI}}\right) \qquad (3)$$

Long, (2001) compared the accuracy of the predicted maximum lateral wall displacement using earlier stuffiness and flexibility factors. He concluded that for stiff clay Clough's stiffness factor (S) has no significant effect on lateral deformation of wall, while in soft soil with low basal safety factor, Clough's stiffness factor has a significant effect on lateral deformation of the wall.

Moormann, (2004) tried to validate Clough's stiffness factor (S). He concluded that neglecting the embedment depth of wall, soil properties, groundwater condition, surrounding buildings effect, irregularity of excavation geometry, excavation sequence and pre-stressing of struts are the main reasons behind the lack of accuracy of predicted lateral deformations.

Zapata, (2012) proposed a new factor called relative stiffness factor (R) to overcome all the shortcomings of Clough's stiffness factor (S). The new factor considered the flexural stiffness of wall (EI), horizontal and vertical struts spacing (S_H, h), Excavation depth (H) and wall embedment depth (D=H-He) besides three soil proprieties which are unit weight (γ_s), undrained cohesion (c_u) and elastic modulus (E_s). His formula is shown in Eq. (4). He also developed a design chart to estimate the maximum lateral wall deformations relative stiffness factor (R) and the system safety factor (FS) as shown in Fig. (2).

$$R = \frac{E_s}{E}\,\frac{S_H.h.H}{I}\,\frac{\gamma_s H_e}{C_u} \qquad (4)$$

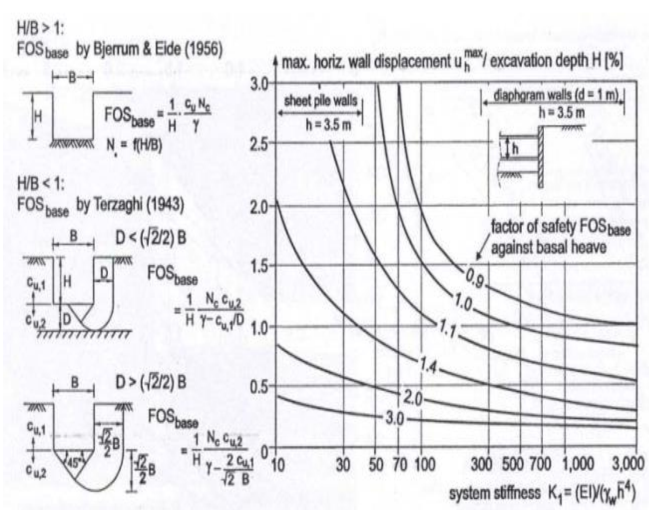


Fig. 1 Clough Design Chart

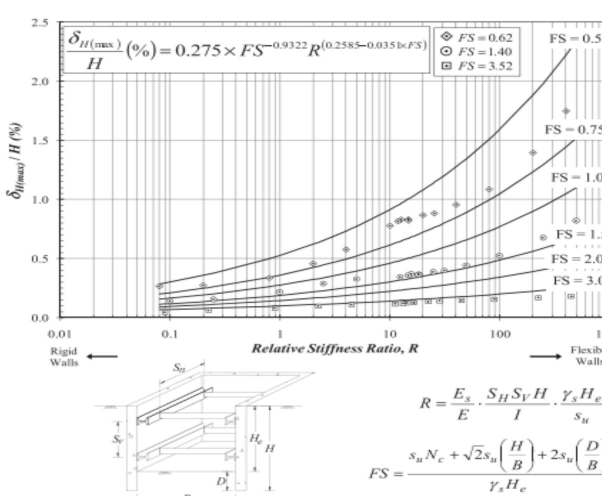


Fig. 2 Zapata 2012 Design Chart

Ramadan (2018) proposed a modified relative stiffness factor (R') for cantilever sheet pile wall based on Zapata's formula as shown in Eq. (5),he also provided a formula for wall lateral deflection based on the modified relative stiffness factor (R') and the basal safety factor (FS_{base2}) as shown in Eq, (6).

$$R' = \frac{E_s}{E_p}\,\frac{L_P.S_h.L'}{I_p}\,\frac{\gamma_s.H_q}{C_u} \qquad (5)$$

$$\frac{U_x}{H_e} = 0.0197\ FS_{base2}{}^{-0.6+\frac{0.376}{\ln R'}} \quad (6)$$

Harry (2019) collected a database of 389 case studies including the 235 case studies used by (Long, 2001). He applied "data mining techniques" on the database to figure out the effect of each parameter on the lateral deformations of the wall. He summarized his results in two points; wall flexibility number after (Addenbrooke, 1994) is best choice for stiff clay and the lateral deformation of deep excavation wall is soft clay is controlled mainly by undrained clay strength and excavation depth.

Hsieh et al., (2019) studied the effect of cross walls on both Clough's system stiffness (S) and basal factor of safety (FS_{base1}) as it not considered in the previous studies and it has a major effect on the overall system stiffness as shown in Fig. (3). To account for the effect of additional system stiffness due to the junction between cross walls and perimeter walls, they suggested a correction factor (PSR) after (Ou et al. 1996) as shown in Eq. (7) and Fig. (4). this correction factor (also called Plan Strain Ratio) depends on the distance from the cross wall and the aspect ratio of the wall. Another correction factor was suggested to account for the additional adhesion force between the clay and the cross wall. The modified undrained shear strength of clay (C_u*) could be calculated using Eq. (8) and accordingly, the modified basal safety factor could be calculated.

$$Sc = \frac{S}{PSR} \quad (7)$$

$$C_u^* = C_u(1 + \beta\frac{L_{cw}.N_{cw}}{L}) \quad (8)$$

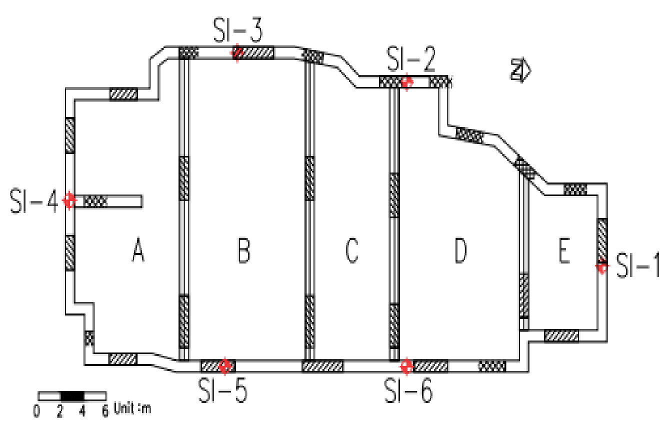


Fig. 3 Layout of Cross Wall

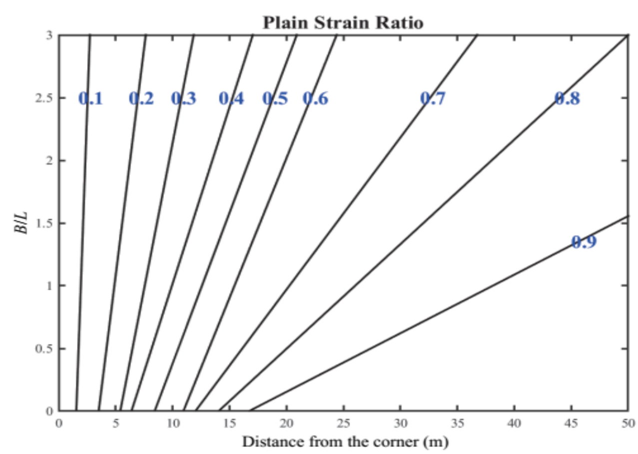


Fig. 4 Plane Strain ratio after (Ou et al. 1996)

3. HEAVE OF BOTTOM OF EXCAVATION

Deep excavation in soft soil causes both short-term and long-term heave as shown in Fig. 5.The long-term heave causes an upward force on the basement's raft. The heave distribution below the basement's raft could be estimated by the intersections between soil curve and structural curves as shown in Fig. 6. Soil curve presents the relation between relaxation ratio (reduced vertical effective stress / initial vertical effective stress) and heave displacement as per continuum theory, while the structure curves presents the stress - settlement curve for each point below of the raft.

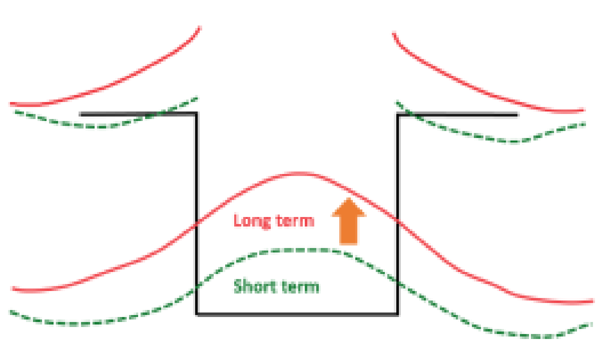


Fig. 5 Schematic Diagram Showed heaves associated soft soil

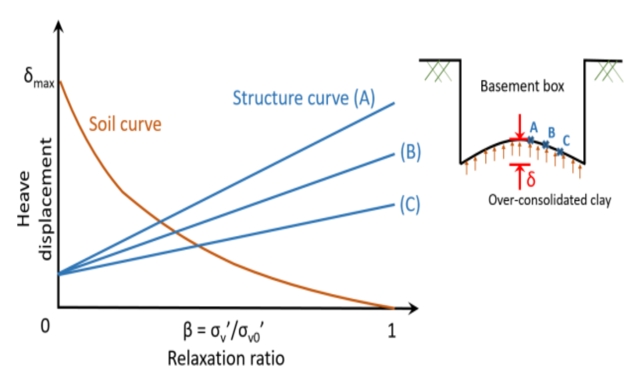


Fig. 6 Schematic Diagram Showed Relaxation ratio methods

Buford, (1988) reported a case study for underground tunnel on London, the measured heave at tunnel's bed started with 20-30 mm and settled at final value of 50 mm. G. Zheng, (2012) presented another case study for braced deep excavation in Tianjin-China as shown in Fig.7. The excavation depth was

25.8m and excavation sides were braced using concrete slabs at three levels, the slabs were supported on columns rested on bored piles with diameter 2.2 m and length of 55 m. The soil profile consists of silt and silty clay underlain by sand layers 80 m below the ground surface.

In order to figure out the effect of staged dewatering on the excavation bed heave, he measured the heave after each dewatering stage and carried out different triaxial tests to predict the heave amount analytically. Fig. 8 shows the measured heave after each dewatering stage.

Changjie XU, (2015) studied the long-short pile retaining system effect on the amount of heave, He studied four groups of piles with different ratios between the length of the long pile and short pile to find out the best group that control the associated heave. The layout of his case study and typical comparison between calculated and measured heave are shown in Fig. 9

Deryck Chan, (2017) provided two case studies that monitored long time heave of soft soil, First Case is a 20 m deep underground vault built in British library campus on Euston Road in 1984 and the monitoring of movement continued until 1992 as shown in Fig. 10. The second case was a 10-m deep basement built on Horseferry Road in 1966-67 and the monitoring continued until 1989 the measured heave shown in Fig. 11. He recommended carrying out an advanced finite-element analyses that take long term behavior of soil for such deep excavation projects.

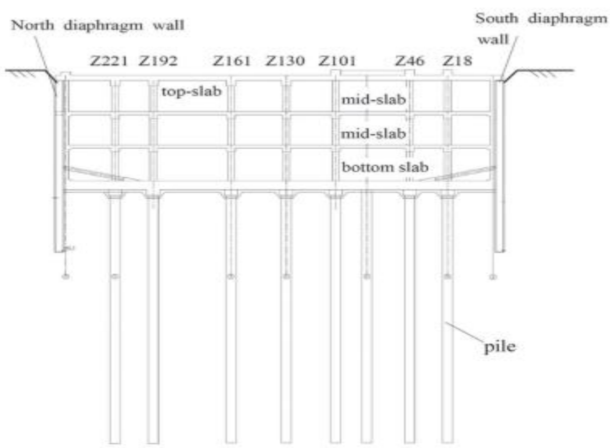


Fig. 7 Case Study used in G. Zheng Study (2012)

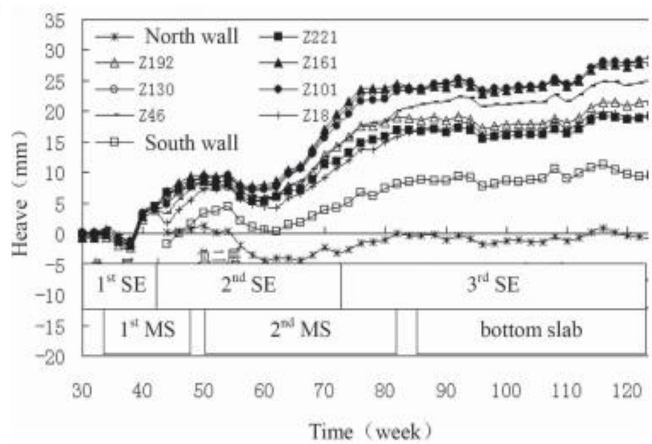


Fig. 8 Measured Heave in Case Study used in G. Zheng Study (2012)

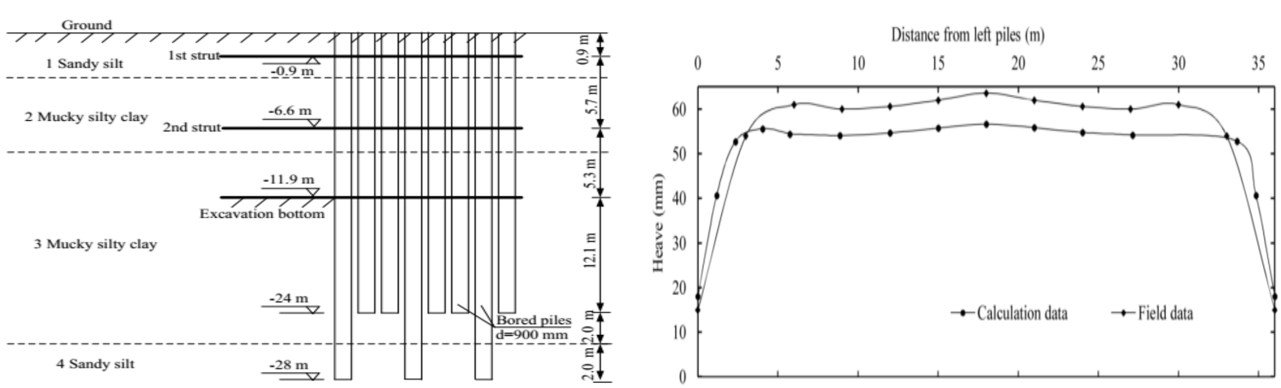
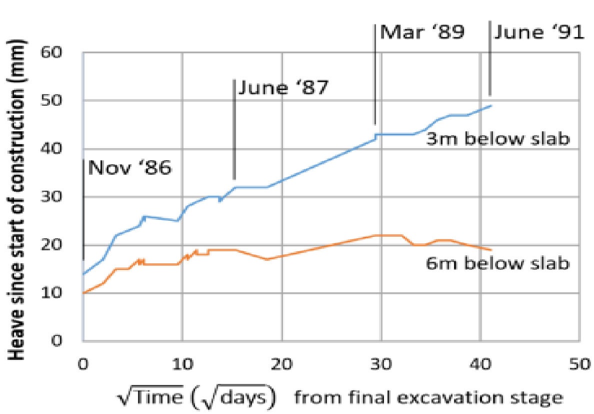


Fig. 9Layout of Case study and comparison between calculated and measured heave. Changjie XU (2013)



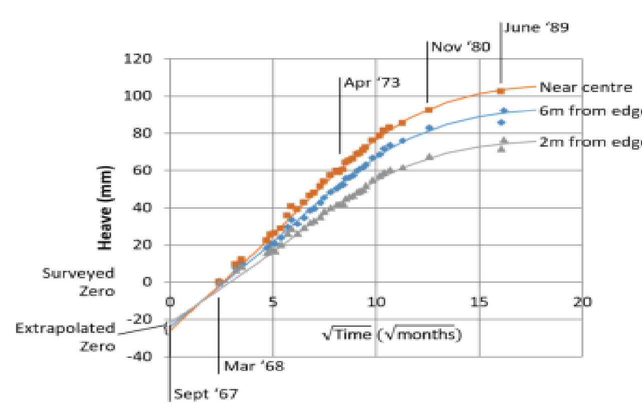


Fig. 10 Measured heave of underground vault in British library

Fig. 11 Measured heave of deep basement on Horseferry Road

4. MODULUS OF SUBGRADE REACTION

Soil is naturally non-linear, anisotropic, heterogeneous, and its deformation depended on the stress level. Accordingly, its behavior under loads is too complicated to be accurately modeled in common structural designs. To simplify the analysis, soil usually modeled as linear springs (or Winker springs) and the stiffness of these springs is called "Modulus subgrade reaction" (Ks). Hence, (Ks) is a ratio between the applied stress on soil and the corresponding deformation in the direction of the stress as shown in Fig. 12. As per previous definition, the modulus of subgrade reaction is not a fundamental soil property. The main shortages in modulus of subgrade reaction concept are the linearity of the spring stiffness and the assumption that each spring can settle independently from other adjacent springs. Most international and local geotechnical codes specified field tests to measure the (Ks) in site and empirical formulas to estimate its value based on soil properties. The following paragraphs will discuss the most used field test and empirical formulas to estimate (Ks) value.

4.1. Estimating the (Ks) value using field tests

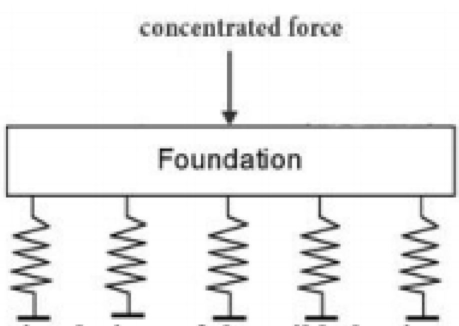
4.1.1. Plate load test

Plate load test is the most used field test to measure the value of the modulus subgrade reaction (Ks), it depends on loading a circular steel plate and measure its settlement. Most international codes and standards such as (DIN 18134, 2012), (BS 1377 Part 9., 1990), (BS EN 1997 Part 2., 2007), (AASHTO T-222, 1981),(ASTM D1196, 1997) and (IS 9214:2007) specified method statement for this test and defined the (Ks) value as the ratio between the vertical stress applied on 762mm diameter steel plate to cause settlement of 1.25 mm. Increasing the plate diameter increases the effective depth (stress bulb) and accordingly increases the settlement and decreases the (Ks) value as shown in Fig. 13. (Terzaghi, 1955) proposed Eq. (9) and (10) to estimate the (Ks) value of full-scale square foot based on the measured (Ks) value using test plate, and Eq. (11) to estimate the (Ks) value for rectangular foot based on the (Ks) value for equivalent square foot.

$$\mathrm{K_{s\ Square\ foot}} = \mathrm{K_{s\ plate}} \frac{B_{plate}}{B_{foot}} \qquad \text{For cohesive soil} \qquad (9)$$

$$\mathrm{K_{s\ Square\ foot}} = \mathrm{K_{s\ plate}} \left(\frac{B_{foot}+B_{plate}}{2\,B_{foot}}\right)^2 \qquad \text{For granular soil} \qquad (10)$$

$$\mathrm{K_{s\ Rectangular\ foot}} = 0.67\ \mathrm{K_{s\ Square\ foot}} \left(1+\frac{B}{L}\right) \qquad (11)$$



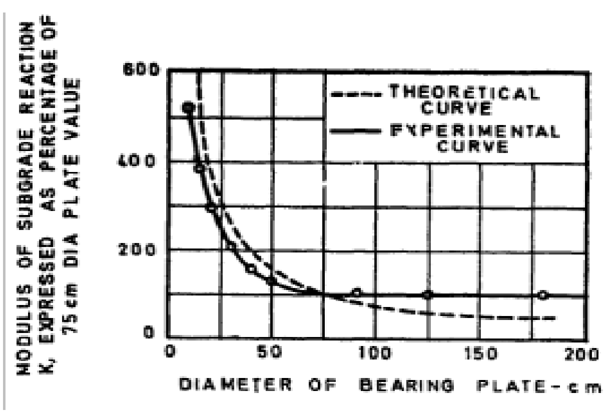


Fig. 12The concept of modulus of subgrade reaction

Fig. 13 The relation between (Ks) value and used plate diameter, IS 9214:2007

Bowles (1997) proposed a simple Eq. (12) to estimate the (Ks) value using the allowable bearing capacity (qa). Where (qa) equals the ultimate bearing capacity extracted from the plate load test (corresponding to settlement of 25mm) divided by safety factor (SF).

$$\mathrm{Ks\ (SI\ units)} = 40\ \mathrm{qa}\,.\,\mathrm{SF} \qquad (12)$$

Abd-Elsamee, (2013), carried out 25 plate load tests on granular soil using different plate shapes and sizes at different foundation depths, test results were analyzed to study the effect of each parameter on the (Ks) value. Finally, they proposed Eq. (13) to estimate the (Ks) value for square footing rested on granular soil based on foundation depth (D), angle if internal friction (φ), allowable bearing capacity from the plate load test (qa = q ult /3) and its corresponding settlement (Δa) all in (KN,m).

$$\mathrm{Ks} = 2529.25\,\mathrm{D} + 290.75\,\emptyset + 53.68\,\mathrm{q_{all}} - 170413\,\Delta_\mathrm{a} - 5881.05 \qquad (13)$$

4.1.2. Standard penetration test (SPT)

Standard penetration test (SPT) is another field test that may be used to estimate the (Ks) value. The proposed empirical formulas to correlate (SPT) results started with Scott R. F (1984), he proposed Eq. (14) for sandy soils. NAVFAC Design Manual 7.02 (1986) suggested an indirect approach by calculating the relative density (Dr) values from the (SPT) results and then uses the calculated (Dr) to estimate the (Ks) values using the chart in Fig. 14.

ZiaieMoayed and Janbaz (2011), proposed Eq. (15) to correlate the (Ks) value of the cemented gravelly deposits in Tehran alluvium to the corresponding (SPT) value. Naeini et al, (2014), correlated (Ks) value to (SPT) values of low plastic clay in Qazvin alluvium using Eq. (16). All (Ks) values from Eq. (12, 13and14) are based on 30cm diameter plate load tests.

$$\text{Ks}\ (\text{MN/m}^3) = 1.8\ \text{N} \qquad (14)$$

$$\text{Ks}\ (\text{MN/m}^3) = 2.8\ \text{N} + 79.6 \qquad (15)$$

$$\text{Ks}\ (\text{MN/m}^3) = 0.96\ \text{N} \qquad (16)$$

4.2. Estimating the (Ks) value using theoretical formulas

4.2.1. Independent Spring Model (uniform Ks)

A lot of researchers (Biot, 1937), (Terzaghi, 1955), (Vesic, 1961), (Meryerhof, 1963), (KloeppelandGlock, 1970), (Selvadurai, 1978), (Horvath, 1983), (Bowles, 1997), and (Sadrekarimi and Akbarzad, 2009) tried to drive formulas to calculate the (Ks) value based on elastic continuum theory. (Biot, 1937) studied an infinite beams resting on 3D continuum under concentrated load; he correlated Winkler model and continuum elastic theory. (Vesic, 1961) used displacements compatibility approach to drive his formula for (Ks). (Horvath., 1983) used Reissner simplified continuum (RSC) to model a raft resting on elastic continuum, then developed his formula for (Ks) considering the soil modulus of elasticity (Es) and the depth of elastic soil (H).Fig. 15 summarized the (Ks) formulas for the previous researchers.

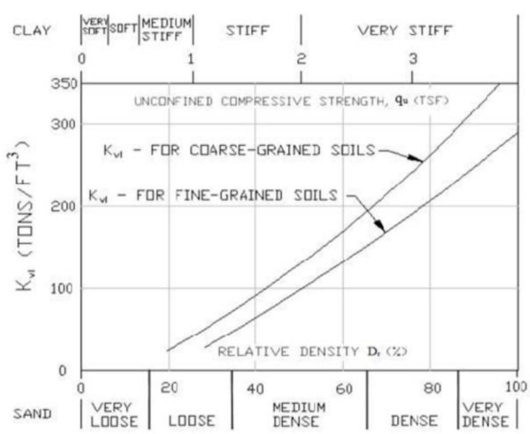


Fig. 14 Modulus of subgrade reaction values after NAVFAC-7.02 (1986)

Investigator	year	Suggested formula
Biot	(1937)	$k_s = \frac{0.95E_s}{B(1-\upsilon_s^2)}\left[\frac{B^4E_s}{(1-\upsilon_s^2)EI}\right]^{0.108}$
Vesic	(1961)	$k_s = \frac{0.65E_s}{B(1-\upsilon_s^2)}\sqrt[12]{\frac{E_sB^4}{EI}}$
Meyerhof and Baike	(1965)	$k_s = \frac{E_s}{B(1-\upsilon_s^2)}$
Selvadurai	(1984)	$k_s = \frac{0.65}{B}\cdot\frac{E_s}{(1-\upsilon_s^2)}$
Bowles	(1998)	$k_s = \frac{E_s}{B\,(1-\upsilon_s^2)mI_sI_F}$

ks = the coefficient of subgrade reaction. *B* = width of footing.
Es = modulus of elasticity. υ_s = poisson's ratio. *EI* = flexural rigidity of foot,
m = takes 1, 2 and 4 for edges, sides nd center of footing, respectively.
I_S and I_F = influence factors depend on the shape of footing

Fig. 15 (Ks) formulas for the previous researchers

Sadrekarimi, (2009), presented a theoretical case study of multistory building rested on a multi-layered soil profile, he investigate the actual behavior of soil from soil-structural interaction FEM model using Plaxis software. Then he carried out a series of Winkler spring models using SAFE software, (Ks) value of each model was calculated using one of the previously mentioned formulas in Fig. 15. He compared the results with the Plaxis output and concluded the error percent in the Winkler spring models outputs is about 35%.

4.2.2. Coupled Models (non-uniform Ks)

Winkler's approach allows each spring to settle independently (un-coupled) which neglect the effect of shear stresses between soil particles and affect the output accuracy. On other hand, the actual soil-structure behavior showed that both settlement and contact stress beneath the footing varied from point to point depending on the location of the considered point and the relative stiffness between soil and footing. Therefore, many researchers developed alternative approaches to overcome the above shortcomings by using different (Ks) value for each point beneath the footing.

Discrete area pseudo-coupled approach depends on an iterative approach. It starts with dividing the footing into zones and analyzing the footing considering uniform (Ks) value for all zones, then the output contact pressure of each zone is used to calculate the settlement of that zone using Boussinesq or Westergaard theories (for uniform and layered soil profiles, respectively). The calculated settlement is used to update the (Ks) value of each zone and the cycles continued till accepted accuracy achieved.

A simpler alternative for the iterative approach is proposed in ACI-336. It depends on dividing the footing into three zones as shown in Fig. 16. The modulus of subgrade reaction of the central zone (Ks1) could be calculated using Eq. (17) where A1, A2 and A3 are the areas of central, middle and edge zones respectively and (Ks) is the conventional modulus of subgrade reaction. The modules of subgrade reaction of the middle and edge zones (Ks2), (Ks3) are equal to 1.5 Ks1 and 2 Ks1 respectively.

$$\text{Ks}_1 = \text{Ks}\frac{\text{A1+A2+A3}}{\text{A1+1.5A2+2A3}} \qquad (17)$$

El-garhy and Osman, (2002), proposed equivalent (Ks) value obtained by dividing the contact pressure by corresponding settlement at each node along the beam length using Eq. (18) where C1, C2 are constants depended on beam width to length ratio and the location of considered point along beam length.

$$\frac{\text{Ks B}\,L^4}{EI} = \text{C1}\left(\frac{16(1-\nu s)^2 EI}{\pi\, Es\, L^4}\right)^{-C2} \qquad (18)$$

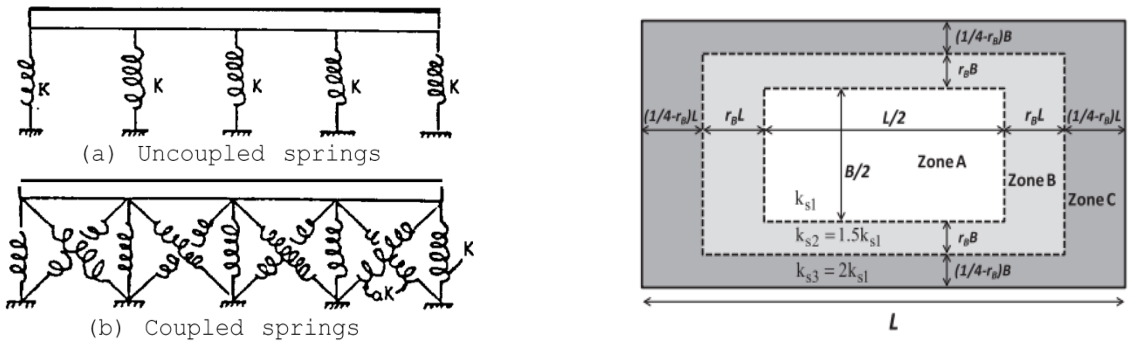


Fig. 16 Simplified pseudo-coupled approach after ACI 336

D.Loukidis and Tamiolokis (2017), used continuum finite element analysis to back-calculated (Ks) distribution under mat foundation using ABAQUS and MATLAB software, the deformations resulting from continuum finite element analysis used as inputs in the mathematical model in the MATLAB.
The proposed equation derived from the back-calculation as follow Eq. (19)

$$Ks = \mathrm{Kr}(0.55 + \mathrm{C_{H1}})\left\{1 + 2C_{H2}\left[\left(\frac{x+0.1ex}{L/2}\right)^6 + \left(\frac{y+0.1ey}{B/2}\right)^6\right] + 4\left[\frac{ex}{L}\left(\frac{x}{L/2}\right) + \frac{ey}{B}\left(\frac{y}{B/2}\right)\right]\right\} \quad (19)$$

Where, x-axis parallel to mat length, ex and ey are the eccentricities of the resultant of column loads along x and y axes. The factors C_{H1} and C_{H2} introduce the influence of an un-deformable layer at depth H below the mat foundation and could be calculated from Eq. (25, 26).

$$\mathrm{C_{H1}} = 0.45 \exp\left(-2.2.\frac{\mathrm{H}}{\mathrm{B}}\right) \quad (25)$$

$$\mathrm{C_{H2}} = \exp\left(-0.4.\frac{\mathrm{B}}{\mathrm{H}}\right) \quad (26)$$

5. DISCUSSION AND CONCLUSIONS

Soil deformations due to braced deep excavation (especially excavation bed heave) were extensively studied by earlier researches. Values of excavation bed heave were observed and recorded in many case studies and for different soil types. Numerical FEM models were developed to predict the heave value distribution at excavation bed considering soil proprieties, excavation depth, bracing system stiffness, ground water condition and many other factors. On other hand, the modulus of subgrade reaction concept still used as a simpler alternative of the complicated soil-structure interaction models. Many empirical formulas were proposed to correlate the (Ks) value to field test results or other soil properties. Besides that, theoretical expressions were derived to calculate the (Ks) value. Coupled models with non-uniform (Ks) distribution were developed to enhance the accuracy of the models based on modulus of subgrade reaction concept.

Few studies considered the foundation depth effect on the (Ks) value (directly as input parameter or indirectly as embedded factor in SPT value), but the range of the studied depths was limited to that of shallow foundations and never considered the effect of excavation bed heave. It is expected that releasing the overburden stress due to deep excavation will decreases the (Ks) values especially for soils sensitive to stress path. On other hand, it is expected that friction between shoring wall and adjacent soil will decreases the amount of heave on the boundary of braced excavation that will affect the distribution of the (Ks) below the foundations.

Based on the previous discussion, it is concluded that no previous studies were sufficiently investigated the effect of excavation bed heave on the value of modulus of subgrade reaction. This gap may by a research point for farther studies to estimate the (Ks) value at deep excavation bed level considering shoring system characteristics, soil proprieties, excavation depth, excavation bed heave and many other factors.

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