Future Computing and Informatics Journal

Volume 2
Issue 2 *2017, Volume 2, Issue 2*

Article 6

2017

Pentagonal fuzzy number, its properties and application in fuzzy equation

Sankar Mondal
Department of Mathematics, Midnapore College (Autonomous), Midnapore, West Midnapore 721101, West Bengal, India, sankar.res07@gmail.com

FUTURE UNIVERSITY IN EGYPT
FACULTY OF COMPUTERS AND INFORMATION TECHNOLOGY

Available online at www.sciencedirect.com

ScienceDirect

Future Computing and Informatics Journal 2 (2017) 110–117
http://www.journals.elsevier.com/future-computing-and-informatics-journal/

Pentagonal fuzzy number, its properties and application in fuzzy equation

Sankar Prasad Mondal*, Manimohan Mandal

Department of Mathematics, Midnapore College (Autonomous), Midnapore, West Midnapore 721101, West Bengal, India

Received 1 July 2017; accepted 14 September 2017
Available online 20 October 2017

Abstract

The paper presents an adaptation of pentagonal fuzzy number. Different type of pentagonal fuzzy number is formed. The arithmetic operation of a particular type of pentagonal fuzzy number is addressed here. The difference between two pentagonal valued functions is also addressed here. Demonstration of pentagonal fuzzy solutions of fuzzy equation is carried out with the said numbers. Additionally, an illustrative example is also taken with the useful graph and table for usefulness for attained to the proposed concept.

Keywords: Pentagonal fuzzy number; Fuzzy equation

1. Introduction

1.1. Fuzzy sets and number

In 1965, Lotfi A. Zadeh [1], delivered new concept namely Fuzzy Sets theory. The theory of unsharp amounts has been applied with great success in many various fields. Chang and Zadeh [2] introduced the concept of fuzzy numbers. Different mathematicians have been studying the theory (one-dimension or n-dimension fuzzy numbers, see for example Refs. [3,4]). With the various improvement of theories and applications of fuzzy sets theory the topic become a topic of great interest.

1.2. Pentagonal fuzzy number

Many researcher take pentagonal fuzzy number with different types of membership function. In this subsection we study on some published work which is associated with pentagonal fuzzy number:

From the above literature survey we see that linear membership function with symmetry is only taken most of the cases. But what happen if we take non linear membership function or asymmetry on both ends or generalized case or their combinations? Obviously the results are different. This article we propose to show all type of possibility.

1.3. Motivation

Fuzzy sets theory play an important role in uncertainty modeling. Now the question is if we wish to take a fuzzy number then how its geometrical representations are. What is its membership functions? So if decision maker take a fuzzy number which can be graphically looks like a pentagon then how its membership function can be defined. From this point of view we try to define different type of pentagonal fuzzy number which can be a better choice of a decision maker in different situation.

* Corresponding author.
E-mail address: sankar.res07@gmail.com (S.P. Mondal).
Peer review under responsibility of the Faculty of Computers and Information, Future University in Egypt

https://doi.org/10.1016/j.fcij.2017.09.001

Authors Information	Types of membership function	Main contribution	Application area
Panda and Pal [5]	Linear membership function of with symmetry	Define arithmetic operation and a exponent operation	Fuzzy matrix theory
Anitha and Parvathi [6]	Linear membership function	Find expected crisp value	Inventory control problem
Helen and Uma [7]	Linear membership function	Find the parametric form of pentagonal fuzzy number	Proof of all arithmetic operation using parametric form concept Find the ranking of pentagonal fuzzy number
Siji and Kumari [8]	Linear membership and non membership function	Define all arithmetic operation Find the ranking of Intuitionistic fuzzy number	Application in network problem
Raj and Karthik [9]	Linear membership function	Define all arithmetic operation	Application in Neural network problem
Dhanamand and Parimaldevi [10]	Linear membership function	Find the ranking of pentagonal fuzzy number using circumcenter of centroids and an index of modality	Apply in multi objective multi item inventory model
Pathinathan and Ponnivalavan [11]	Reverse order linear membership function	Define arithmetic operation	Define different type of reverse order fuzzy number
Ponnivalavan and Pathinathan [12]	Linear membership and non membership function	Define arithmetic operation	Find score and accuracy function
Annie Christi and Kasthuri [13]	Linear membership and non membership function	Define arithmetic operation and ranking	Transportation problem

1.4. Novelties

There is various articles where pentagonal fuzzy sets and number are introduced and apply to different fields. But there are so many scopes to work on that topic. We try to summarize the work done on pentagonal fuzzy number as follows:

(i) Formation of different types of pentagonal fuzzy number in easier manner. i.e., Symmetric linear pentagonal fuzzy number, asymmetric linear pentagonal fuzzy number, symmetric non linear pentagonal fuzzy number, asymmetric nonlinear pentagonal fuzzy number are defined.
(ii) The parametric form of the said above types of number are defined.
(iii) Arithmetic operation of symmetric linear fuzzy numbers is defined and how can we prove it is illustrated.
(iv) The number is considered with equation i.e., pentagonal fuzzy equation are defined and solved.

1.5. Structure of the paper

The paper is organized as follows. In Section 2, the basic concept on fuzzy number and fuzzy difference are discussed. In Section 3, we give a brief description on how we can choose a suitable membership function in different pentagonal form. In Section 4, we addressed some arithmetic operation on Linear pentagonal fuzzy number with symmetry. In Section 5, solution of fuzzy equation with pentagonal fuzzy number with example is discussed. The conclusions are written in Section 6.

2. Preliminaries

Definition 2.1. Fuzzy Number: A fuzzy set $\tilde{A}$, defined on the universal set of real number R, is said to be a fuzzy number if its possess at least the following properties:

(i) $\tilde{A}$ is convex.
(ii) $\tilde{A}$ is normal i.e., $\exists x_0 \in R$ such that $\mu_{\tilde{A}}(x_0) = 1$.
(iii) $\mu_{\tilde{A}}(x)$ is piecewise continuous.
(iv) A_α must be closed interval for every $\alpha[0,1]$.
(v) The support of $\tilde{A}$, i.e., *support* ($\tilde{A}$) must be bounded.

Definition 2.2. Generalized Hukuhara difference: [14] The generalized Hukuhara difference of two fuzzy numbers $a, b \in \Re_{\mathcal{F}}$ is defined as follows

$$a \ominus_{gH} b = k \Leftrightarrow \begin{cases} a = b + k \\ b = a + (-1)k \end{cases}$$

In terms of α-cut set we have

$$\left[a \ominus_{gH} b\right]_\alpha = [\min\{r_1(\alpha), r_2(\alpha)\}, \max\{r_1(\alpha), r_2(\alpha)\}]$$

where, $r_1(\alpha) = a_1(\alpha) - b_2(\alpha), r_2(\alpha) = a_2(\alpha) - b_1(\alpha)$

Let $e = a \ominus_{gH} b$

The conditions for which the existence of $a \ominus_{gH} b$ exists if

$e_1(\alpha) = a_1(\alpha) - b_2(\alpha), e_2(\alpha) = a_2(\alpha) - b_1(\alpha), e_1^{'}(\beta) = a_1^{'}(\beta) - b_2^{'}(\beta)$ and $e_2^{'}(\beta) = a_2^{'}(\beta) - b_1^{'}(\beta)$ with, $e_1(\alpha), e_2^{'}(\beta)$ are increasing and $e_2(\alpha), e_1^{'}(\beta)$ are decreasing function for all $\alpha, \beta \in [0,1]$ and $e_1(\alpha) \le e_2(\alpha), e_2^{'}(\beta) \le e_1^{'}(\beta)$.

Remark 2.1. Throughout the paper, we assume that $a \ominus_{gH} b \in \Re_{\mathcal{F}}$.

3. Pentagonal fuzzy number and its variation

In this section we develop different type of pentagonal fuzzy number in different viewpoint.

Definition 3.1. Pentagonal fuzzy number: A pentagonal fuzzy number $\tilde{A} = (a_1, a_2, a_3, a_4, a_5)$ should satisfy the following condition

(1) $\mu_{\tilde{A}}(x)$ is a continuous function in the interval [0,1]
(2) $\mu_{\tilde{A}}(x)$ is strictly increasing and continuous function on $[a_1, a_2]$ and $[a_2, a_3]$
(3) $\mu_{\tilde{A}}(x)$ is strictly decreasing and continuous function on $[a_3, a_4]$ and $[a_4, a_5]$

Definition 3.2. Equality of two Pentagonal fuzzy number: Two pentagonal fuzzy number $\tilde{A} = (a_1, a_2, a_3, a_4, a_5)$ and $\tilde{B} = (b_1, b_2, b_3, b_4, b_5)$ are equal if

$$a_1 = b_1, a_2 = b_2, a_3 = b_3, a_4 = b_4, a_5 = b_5$$

Now we try to define some new types of pentagonal fuzzy number in their different form.

3.1. Linear pentagonal fuzzy number with symmetry

Definition 3.3. Linear pentagonal fuzzy number with symmetry (LPFNS): A linear pentagonal fuzzy number is written as $\tilde{A}_{LS} = (a_1, a_2, a_3, a_4, a_5; r)$ whose membership function is written as

$$\mu_{\tilde{A}_{LS}}(x) = \begin{cases} r\dfrac{x-a_1}{a_2-a_1} \; if \; a_1 \le x \le a_2 \\ 1-(1-r)\dfrac{x-a_2}{a_3-a_2} \; if \; a_2 \le x \le a_3 \\ 1 \;\; if \; x = a_3 \\ 1-(1-r)\dfrac{a_4-x}{a_4-a_3} \; if \; a_3 \le x \le a_4 \\ r\dfrac{a_5-x}{a_5-a_4} \; if \; a_4 \le x \le a_5 \\ 0 \;\; if \; x > a_5 \end{cases}$$

Definition 3.4. α-cut or parametric form of LPFNS: α-cut or parametric form of LPFNS is represented by the formulas

$$A_\alpha = \left\{ x \in X \middle| \mu_{\tilde{A}_{LS}}(x) \ge \alpha \right\}$$

$$= \begin{cases} A_{1L}(\alpha) = a_1 + \dfrac{\alpha}{r}(a_2 - a_1) \; for \; \alpha \in [0, r] \\ A_{2L}(\alpha) = a_2 + \dfrac{1-\alpha}{1-r}(a_3 - a_2) \; for \; \alpha \in [r, 1] \\ A_{2R}(\alpha) = a_4 - \dfrac{1-\alpha}{1-r}(a_4 - a_3) \; for \; \alpha \in [r, 1] \\ A_{1R}(\alpha) = a_5 - \dfrac{\alpha}{r}(a_5 - a_4) \; for \; \alpha \in [0, r] \end{cases}$$

where $A_{1L}(\alpha), A_{2L}(\alpha)$, is increasing function with respect to α and $A_{2R}(\alpha), A_{1R}(\alpha)$, is decreasing function with respect to α.

Key point 3.1. The basic concept of the above number is the left picked point and right picked point are same (See Fig. 1 the picked point is r).

3.2. Linear pentagonal fuzzy number with asymmetry

Definition 3.5. Linear pentagonal fuzzy number with asymmetry: A linear pentagonal fuzzy number is written as $\tilde{A}_{LAS} = (a_1, a_2, a_3, a_4, a_5; r, s)$ whose membership function is written as

$$\mu_{\tilde{A}_{LAS}}(x) = \begin{cases} r\dfrac{x-a_1}{a_2-a_1} \; if \; a_1 \le x \le a_2 \\ 1-(1-r)\dfrac{x-a_2}{a_3-a_2} \; if \; a_2 \le x \le a_3 \\ 1 \;\; if \; x = a_3 \\ 1-(1-s)\dfrac{a_4-x}{a_4-a_3} \; if \; a_3 \le x \le a_4 \\ s\dfrac{a_5-x}{a_5-a_4} \; if \; a_4 \le x \le a_5 \\ 0 \; if \; x > a_5 \end{cases}$$

Note:

(1) If $r = s$ the asymmetry pentagonal fuzzy number becomes symmetry pentagonal fuzzy number.
(2) For asymmetry pentagonal fuzzy number may be $r < s$ or $r > s$

Definition 3.6. α-cut or parametric form of LPFNS: α-cut or parametric form of LPFNS is represented by the formulas

$$A_\alpha = \left\{ x \in X \middle| \mu_{\tilde{A}_{LS}}(x) \ge \alpha \right\}$$

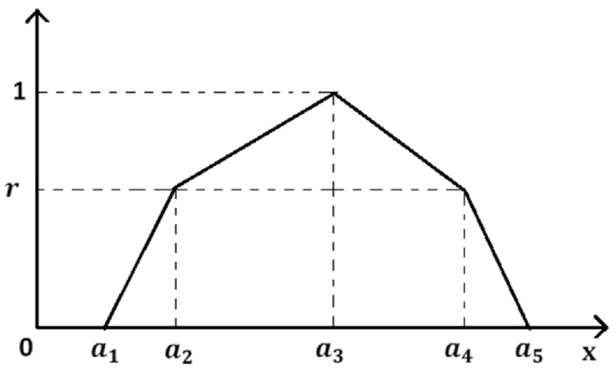


Fig. 1. Linear pentagonal fuzzy number with symmetry.

$$= \begin{cases} A_{1L}(\alpha) = a_1 + \dfrac{\alpha}{r}(a_2 - a_1) \ for\ \alpha \in [0, r] \\ A_{2L}(\alpha) = a_2 + \dfrac{1-\alpha}{1-r}(a_3 - a_2) \ for\ \alpha \in [r, 1] \\ A_{2R}(\alpha) = a_4 - \dfrac{1-\alpha}{1-s}(a_4 - a_3)\ for\ \alpha \in [s, 1] \\ A_{1R}(\alpha) = a_5 - \dfrac{\alpha}{s}(a_5 - a_4)\ for\ \alpha \in [0, s] \end{cases}$$

where $A_{1L}(\alpha), A_{2L}(\alpha),$ is increasing function with respect to α and $A_{2R}(\alpha), A_{1R}(\alpha),$ is decreasing function with respect to α.

Key point 3.2. The basic concept of the above number is the left picked point and right picked point are not same (See Fig. 2 the left picked point is r but right picked point is s).

3.3. Non linear pentagonal fuzzy number with symmetry

Definition 3.7. Non Linear pentagonal fuzzy number with symmetry: A linear pentagonal fuzzy number is written as $\tilde{A}_{LNS} = (a_1, a_2, a_3, a_4, a_5; r)_{(n_1, n_2; m_1, m_2)}$ whose membership function is written as

$$\mu_{\tilde{A}_{NS}}(x) = \begin{cases} r\left(\dfrac{x - a_1}{a_2 - a_1}\right)^{n_1} \ if\ a_1 \le x \le a_2 \\ 1 - (1-r)\left(\dfrac{x - a_2}{a_3 - a_2}\right)^{n_2} \ if\ a_2 \le x \le a_3 \\ 1 \ \ if\ x = a_3 \\ 1 - (1-r)\left(\dfrac{a_4 - x}{a_4 - a_3}\right)^{m_1} \ if\ a_3 \le x \le a_4 \\ r\left(\dfrac{a_5 - x}{a_5 - a_4}\right)^{m_2} \ if\ a_4 \le x \le a_5 \\ 0\ if\ x > a_5 \end{cases}$$

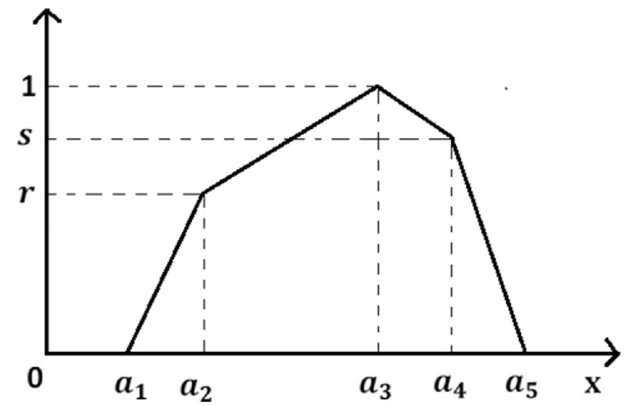


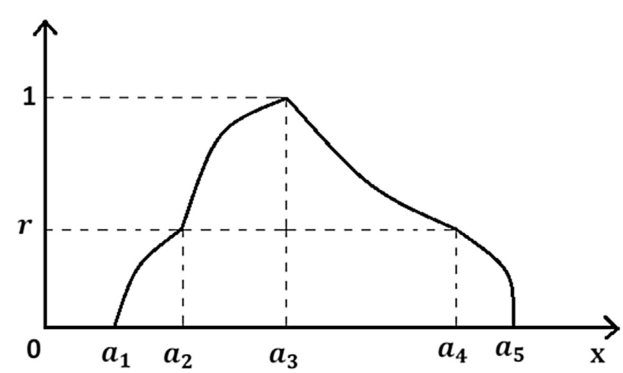
Fig. 2. Linear pentagonal fuzzy number with asymmetry.

Definition 3.8. α-cut or parametric form of LPFNS: α-cut or parametric form of LPFNS is represented by the formulas

$$A_\alpha = \left\{x \in X \middle| \mu_{\tilde{A}_{LS}}(x) \ge \alpha\right\}$$

$$= \begin{cases} A_{1L}(\alpha) = a_1 + \left(\dfrac{\alpha}{r}\right)^{n_1}(a_2 - a_1) \ for\ \alpha \in [0, r] \\ A_{2L}(\alpha) = a_2 + \left(\dfrac{1-\alpha}{1-r}\right)^{n_2}(a_3 - a_2)\ for\ \alpha \in [r, 1] \\ A_{2R}(\alpha) = a_4 - \left(\dfrac{1-\alpha}{1-r}\right)^{m_1}(a_4 - a_3)\ for\ \alpha \in [r, 1] \\ A_{1R}(\alpha) = a_5 - \left(\dfrac{\alpha}{r}\right)^{m_2}(a_5 - a_4)\ for\ \alpha \in [0, r] \end{cases}$$

where $A_{1L}(\alpha), A_{2L}(\alpha),$ is increasing function with respect to α and $A_{2R}(\alpha), A_{1R}(\alpha),$ is decreasing function with respect to α



Non Linear pentagonal fuzzy number with symmetry.

.

Key point 3.3. The basic concept of the above number is the left picked point and right picked point are same but the boundary of the fuzzy area should not be linear always. It can be non linear also. That is the membership function can define as a non linear function. So we can give the non linearity on the membership function (See Fig. 2 the left picked point is r but right picked point is s).

3.4. Non linear pentagonal fuzzy number with asymmetry

Definition 3.9. Non Linear pentagonal fuzzy number with asymmetry: A linear pentagonal fuzzy number is written as $\tilde{A}_{NAS} = (a_1, a_2, a_3, a_4, a_5; r, s)_{(n_1, n_2; m_1, m_2)}$ whose membership function is written as

$$\mu_{\tilde{A}_{NAS}}(x) = \begin{cases} r\left(\frac{x-a_1}{a_2-a_1}\right)^{n_1} \; if\; a_1 \le x \le a_2 \\ 1-(1-r)\left(\frac{x-a_2}{a_3-a_2}\right)^{n_2} \; if\; a_2 \le x \le a_3 \\ 1 \;\; if\; x = a_3 \\ 1-(1-s)\left(\frac{a_4-x}{a_4-a_3}\right)^{m_1} \; if\; a_3 \le x \le a_4 \\ s\left(\frac{a_5-x}{a_5-a_4}\right)^{m_2} \; if\; a_4 \le x \le a_5 \\ 0 \;\; if\; x > a_5 \end{cases}$$

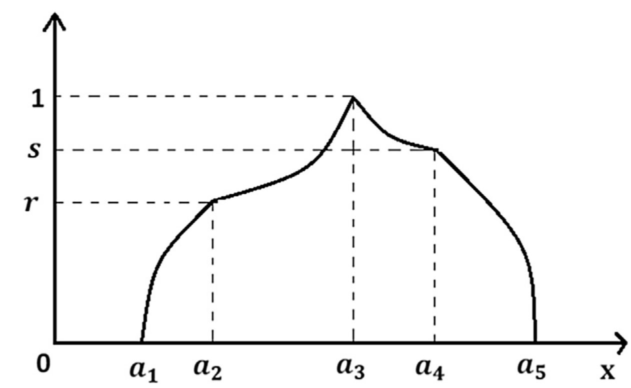
Note: (1) If $n_1 = n_2 = m_1 = m_2 = 1$ then non linear pentagonal fuzzy number becomes linear pentagonal fuzzy number.

Definition 3.10. α-cut or parametric form of LPFNS: α-cut or parametric form of LPFNS is represented by the formulas

$$A_\alpha = \left\{x \in X \,\middle|\, \mu_{\tilde{A}_{LS}}(x) \ge \alpha\right\}$$

$$= \begin{cases} A_{1L}(\alpha) = a_1 + \left(\frac{\alpha}{r}\right)^{n_1}(a_2-a_1)\; for\; \alpha \in [0,r] \\ A_{2L}(\alpha) = a_2 + \left(\frac{1-\alpha}{1-r}\right)^{n_2}(a_3-a_2)\; for\; \alpha \in [r,1] \\ A_{2R}(\alpha) = a_4 - \left(\frac{1-\alpha}{1-s}\right)^{m_1}(a_4-a_3)\; for\; \alpha \in [s,1] \\ A_{1R}(\alpha) = a_5 - \left(\frac{\alpha}{s}\right)^{m_2}(a_5-a_4)\; for\; \alpha \in [0,s] \end{cases}$$

where $A_{1L}(\alpha)$, $A_{2L}(\alpha)$ is increasing function with respect to α and $A_{2R}(\alpha)$, $A_{1R}(\alpha)$ is decreasing function with respect to α.



Non Linear pentagonal fuzzy number with asymmetry.

Key point 3.4. The previous concept is apply here only the basic concept of the number is the left picked point and right picked point are not same.

4. Arithmetic operation on Linear pentagonal fuzzy number with symmetry i.e., $\tilde{A}_{LS} = (a_1, a_2, a_3, a_4, a_5; r)$

(1) *Multiplication by crisp number*

If k is a positive crisp number then $k\tilde{A}_{LS} = (ka_1, ka_2, ka_3, ka_4, ka_5; r)$ and k is a negative crisp number then $k\tilde{A}_{LS} = (ka_5, ka_4, ka_3, ka_2, ka_1; r)$.

Proof: We can proof by interval arithmetic on the parametric method

$A_\alpha = [A_{1\alpha}(\alpha), A_{2\alpha}(\alpha)] = [A_{1L}(\alpha), A_{2L}(\alpha); A_{2R}(\alpha), A_{1R}(\alpha)]$

We can make the above four component $A_{1L}(\alpha), A_{2L}(\alpha), A_{2R}(\alpha), A_{1R}(\alpha)$ into two interval as $A_{1\alpha}(\alpha) = [A_{1L}(\alpha), A_{2L}(\alpha)] = \left[a_1 + \frac{\alpha}{r}(a_2-a_1), a_2 + \frac{1-\alpha}{1-r}(a_3-a_2)\right]$ and $A_{2\alpha}(\alpha) = [A_{2R}(\alpha), A_{1R}(\alpha)] = \left[a_4 - \frac{1-\alpha}{1-s}(a_4-a_3),\; a_5 - \frac{\alpha}{s}(a_5-a_4)\right]$

Note 4.1: The concept on parametric form of a normal fuzzy number is that it behave like a interval number for fixed value of the parameter. For the pentagonal fuzzy number we have to take two interval together.

Case 1: When k > 0

Now if we multiply by positive crisp number then

$$kA_{1\alpha}(\alpha) = [kA_{1L}(\alpha), kA_{2L}(\alpha)] \text{ and } kA_{2\alpha}(\alpha) = [kA_{2R}(\alpha), kA_{1R}(\alpha)]$$

So the resultant interval is

$$kA_\alpha = [kA_{1\alpha}(\alpha), kA_{2\alpha}(\alpha)]$$
$$= [kA_{1L}(\alpha), kA_{2L}(\alpha); kA_{2R}(\alpha), kA_{1R}(\alpha)]$$

So $kA_\alpha = \left[ka_1 + \frac{\alpha}{r}(ka_2 - ka_1), ka_2 + \frac{1-\alpha}{1-r}(ka_3 - ka_2); ka_4 - \frac{1-\alpha}{1-s}(ka_4 - ka_3), ka_5 - \frac{\alpha}{s}(ka_5 - ka_4)\right]$.

That means kA_α is the α-cut of $k\tilde{A}_{LS} = (ka_1, ka_2, ka_3, ka_4, ka_5; r)$.

Case 2: When $k < 0$

Now if we multiply by positive crisp number then

$kA_{1\alpha}(\alpha) = [kA_{2L}(\alpha), kA_{1L}(\alpha)]$ and $kA_{2\alpha}(\alpha) = [kA_{1R}(\alpha), kA_{2R}(\alpha)]$

So the resultant interval is $kA_\alpha = [kA_{2\alpha}(\alpha), kA_{1\alpha}(\alpha)] = [kA_{1R}(\alpha), kA_{2R}(\alpha); kA_{2L}(\alpha), kA_{1L}(\alpha)]$.

So $kA_\alpha = \left[ka_5 - \frac{\alpha}{s}(ka_5 - ka_4), ka_4 - \frac{1-\alpha}{1-s}(ka_4 - ka_3); ka_2 + \frac{1-\alpha}{1-r}(ka_3 - ka_2), ka_1 + \frac{\alpha}{r}(ka_2 - ka_1)\right]$.

That means kA_α is the α-cut of $k\tilde{A}_{LS} = (ka_5, ka_4, ka_3, ka_2, ka_1; r)$.

If k is a negative crisp number then $k\tilde{A}_{LS} = (ka_5, ka_4, ka_3, ka_2, ka_1; r)$.

(2) ***Addition of two pentagonal fuzzy numbers*:**

Consider two pentagonal fuzzy numbers $\tilde{A}_{LS} = (a_1, a_2, a_3, a_4, a_5; r_1)$ and $\tilde{B}_{LS} = (b_1, b_2, b_3, b_4, b_5; r_2)$ then the addition of the two numbers is given by

$$\tilde{C}_{LS} = (a_1 + b_1, a_2 + b_2, a_3 + b_3, a_4 + b_4, a_5 + b_5; r)$$

where $r = \min\{r_1, r_2\}$.

(3) ***Subtraction of two pentagonal fuzzy numbers*:**

Consider two pentagonal fuzzy numbers $\tilde{A}_{LS} = (a_1, a_2, a_3, a_4, a_5; r_1)$ and $\tilde{B}_{LS} = (b_1, b_2, b_3, b_4, b_5; r_2)$ then the addition of the two numbers is given by

$$\tilde{D}_{LS} = (a_1 - b_5, a_2 - b_4, a_3 - b_3, a_4 - b_2, a_5 - b_1; r)$$

where $r = \min\{r_1, r_2\}$.

5. Solution of fuzzy equation with pentagonal fuzzy number

Consider the linear equation with linear pentagonal symmetric fuzzy number as

$$\tilde{a}_{LS}x + \tilde{b}_{LS} = \tilde{c}_{LS}$$

Note 5.1: The above equation is a linear pentagonal symmetric fuzzy equation since the coefficients and parameters are all linear pentagonal symmetric fuzzy number. If any one of the parameter are linear pentagonal symmetric fuzzy number then it is also known as linear pentagonal symmetric fuzzy equation.

Solution: Taking α-cut of the equation we get

$$[a_{1L}(\alpha), a_{2L}(\alpha); a_{2R}(\alpha), a_{1R}(\alpha)].[x_{1L}(\alpha), x_{2L}(\alpha); x_{2R}(\alpha), x_{1R}(\alpha)] + [b_{1L}(\alpha), b_{2L}(\alpha); b_{2R}(\alpha), b_{1R}(\alpha)] = [c_{1L}(\alpha), c_{2L}(\alpha); c_{2R}(\alpha), c_{1R}(\alpha)]$$

Now the problem is converted to some interval equation.
We have now follow the following strategy

$$\min\{[a_{1L}(\alpha), a_{2L}(\alpha)].[x_{1L}(\alpha), x_{2L}(\alpha)]\} + b_{1L}(\alpha) = c_{1L}(\alpha)$$

$$\max\{[a_{1L}(\alpha), a_{2L}(\alpha)].[x_{1L}(\alpha), x_{2L}(\alpha)]\} + b_{2L}(\alpha) = c_{2L}(\alpha)$$

$$\min\{[a_{2R}(\alpha), a_{1R}(\alpha)].[x_{2R}(\alpha), x_{1R}(\alpha)]\} + b_{2R}(\alpha) = c_{2R}(\alpha)$$

$$\max\{[a_{2R}(\alpha), a_{1R}(\alpha)].[x_{2R}(\alpha), x_{1R}(\alpha)]\} + b_{1R}(\alpha) = c_{1R}(\alpha)$$

If we consider $\tilde{a}_{invi}$ is positive interval valued intuitionistic fuzzy number then using the concept of generalized characterization theorem we can write

$$a_{1L}(\alpha)x_{1L}(\alpha) + b_{1L}(\alpha) = c_{1L}(\alpha)$$

$$a_{2L}(\alpha)x_{2L}(\alpha) + b_{2L}(\alpha) = c_{2L}(\alpha)$$

$$a_{2R}(\alpha)x_{2R}(\alpha) + b_{2R}(\alpha) = c_{2R}(\alpha)$$

$$a_{1R}(\alpha)x_{1R}(\alpha) + b_{1R}(\alpha) = c_{1R}(\alpha)$$

Whose solution can be written as

$$x_{1L}(\alpha) = \frac{c_{1L}(\alpha) - b_{1L}(\alpha)}{a_{1L}(\alpha)}$$

$$x_{2L}(\alpha) = \frac{c_{2L}(\alpha) - b_{2L}(\alpha)}{a_{2L}(\alpha)}$$

$$x_{2R}(\alpha) = \frac{c_{2R}(\alpha) - b_{2R}(\alpha)}{a_{2R}(\alpha)}$$

$$x_{1R}(\alpha) = \frac{c_{1R}(\alpha) - b_{1R}(\alpha)}{a_{1R}(\alpha)}$$

Note 5.2: Clearly $[x_{1L}(\alpha), x_{2L}(\alpha); x_{2R}(\alpha), x_{1R}(\alpha)]$ is the α-cut of the problems, but it is necessary to check whether the all component of the solution is maintains the pentagonal fuzzy rules or not.

The solution is strong solution if

$$\frac{dx_{1L}(\alpha)}{d\alpha}, \frac{dx_{2L}(\alpha)}{d\alpha} > 0$$

i.e., $x_{1L}(\alpha), x_{2L}(\alpha),$ are increasing function with respect to α.
and

$$\frac{dx_{2R}(\alpha)}{d\alpha}, \frac{dx_{1R}(\alpha)}{d\alpha} < 0$$

i.e., $x_{1L}(\alpha), x_{2L}(\alpha),$ are decreasing function with respect to α.

Table 1
Solution for different value of α.

α	$x_{1L}(\alpha)$	$x_{2L}(\alpha)$	α	$x_{2R}(\alpha)$	$x_{1R}(\alpha)$
0	6.5000		**0**		18.5000
0.1	7.1667		**0.1**		18.1667
0.2	7.8333		**0.2**		17.8333
0.3	8.5000		**0.3**		17.5000
0.4	9.1667		**0.4**		17.1667
0.5	9.8333		**0.5**		16.8333
0.6	10.5000	10.5000	**0.6**	16.5000	16.5000
0.7		11.0000	**0.7**	15.5000	
0.8		11.5000	**0.8**	14.5000	
0.9		12.0000	**0.9**	13.5000	
1		12.5000	**1**	12.5000	

Numerical example 5.1: Consider the pentagonal fuzzy equation

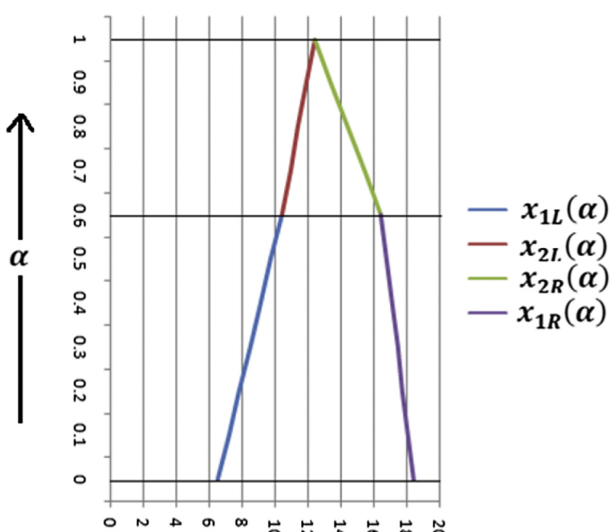
$$ax + \tilde{b}_{LS} = \tilde{c}_{LS}$$

where
$a = 4, \tilde{b}_{LS} = (4, 8, 10, 14, 16; 0.6)$ and $\tilde{c}_{LS} = (30, 50, 60, 80, 90; 0.6)$.

Solution: The α-cut of the solution is like

$$(\tilde{x}_{LS})_\alpha = [x_{1L}(\alpha), x_{2L}(\alpha); x_{2R}(\alpha), x_{1R}(\alpha)]$$

where, $x_{1L}(\alpha) = \frac{1}{4}\left(26 + \frac{80\alpha}{3}\right), x_{2L}(\alpha) = \frac{1}{4}(62 - 20\alpha)$, $x_{2R}(\alpha) = \frac{1}{4}(26 + 40\alpha)$ and $x_{1R}(\alpha) = \frac{1}{4}\left(74 - \frac{40\alpha}{3}\right)$.



Graph of solution.

Remarks 5.3. From the above graph and Table 1 we see that $x_{1L}(\alpha)$ is increasing for $\alpha \in [0, 0.6], x_{2L}(\alpha)$, is increasing for $\alpha \in [0.6, 1], x_{2R}(\alpha)$, is decreasing for $\alpha \in [0.6, 1]$ and $x_{1R}(\alpha)$ is decreasing for $\alpha \in [0, 0.6]$. Hence the solution is a strong solution.

6. Conclusion

In this paper the concept on different type of pentagonal fuzzy number is defined. The said number valued function is extended to its generalized Hukuhara difference concept, where it is applied to elucidate the pentagonal fuzzy solutions of the equation. Arithmetic operations of a particular pentagonal fuzzy number are also addressed. Further a numerical example is illustrated with pentagonal fuzzy number with fuzzy equation. Mainly the whole work reaches on the following conclusion:

- Demonstrating different type pentagonal fuzzy numbers enabled to meet the imprecise parameters as well, which is approvingly the advantageous for the decision makers to analyze the result in a more precise manner.
- By different situation the decision maker can take different type of pentagonal fuzzy number as per the problem definition.

Thus in future we are interested to use these concepts to find the solution of different problem with different type of pentagonal fuzzy numbers and we can apply this in various fields of engineering and sciences.

Acknowledgement

The first author of the article wishes to convey his heartiest thanks to Miss. Gullu for inspiring him to write the article.

References

[1] Zadeh LA. Fuzzy sets. Inf Control 1965;8:338−53.
[2] Chang SSL, Zadeh LA. On fuzzy mappings and control. IEEE Trans Syst Man Cyberne 1972;2:30−4.
[3] Diamond P, Kloeden P. Metric space of fuzzy sets. Singapore: World Scientific; 1994.
[4] Dubois D, Prade H. Operations on fuzzy numbers. Int J Syst Sci 1978;9:613−26.
[5] Panda Apurba, Pal Madhumangal. A study on pentagonal fuzzy number and its corresponding matrices. Pac Sci Rev B Humanit Soc Sci 2015;1:131−9.
[6] Anitha P, Parvathi P. An inventory model with stock dependent demand, two parameter Weibull distribution deterioration in a fuzzy environment. In: Online International Conference on green engineering and technologies (IC-GET); 2016. p. 1−8.
[7]] Helen R, Uma G. A new operation and ranking on pentagon fuzzy numbers. Int J Math Sci Appl 2015;5(2):341−6.
[8] Siji S, Selva Kumari K. An approach for solving network problem with pentagonal intuitionistic fuzzy numbers using ranking technique. Middle-East J Sci Res 2016;24(9):2977−80.
[9] Vigin Raj A, Karthik S. Application of pentagonal fuzzy number in neural network. Int J Math Appl 2016;4(4):149−54.

[10] Dhanamand K, Parimaldevi M. Cost analysis on a probabilistic multi objective-multi item inventory model using pentagonal fuzzy number. Glob J Appl Math Math Sci 2016;9(2):151−63.
[11] Pathinathan T, Ponnivalavan K. Reverse order triangular, trapezoidal and pentagonal fuzzy numbers. Ann Pure Appl Math 2015;9(1):107−17.
[12] Ponnivalavan K, Pathinathan T. Intuitionistic pentagonal fuzzy number. ARPN J Eng Appl Sci 2015;10(12):5446−50.
[13] Annie Christi MS, Kasthuri B. Transportation problem with pentagonal intuitionistic fuzzy numbers solved using ranking technique and Russell's method. Int J Eng Res Appl 2016;6(2):82−6.
[14] Bede B, Stefanini L. Generalized differentiability of fuzzy-valued functions. Fuzzy Sets Syst, Volume 1 November 2013;Volume 230. Pages 119−141.