The internal blood pressure equation involving incomplete $I$-functions

Kamlesh Jangid  
*Department of HEAS (Mathematics), Rajasthan Technical University, Kota, 324010, Rajasthan, India*,  
n.sooppy@psau.edu.sa

S.D. Purohit  
*Department of HEAS (Mathematics), Rajasthan Technical University, Kota, 324010, Rajasthan, India*,  
n.sooppy@psau.edu.sa

Kottakkaran Sooppy Nisar  
*Department of Mathematics, College of Arts and Sciences, Prince Sattam bin Abdulaziz University, Wadi Aldawaser, 11991, Saudi Arabia*,  
n.sooppy@psau.edu.sa

T. Shefeeq  
*Department of Mathematics, Farook College(Autonomous), Kozhikode-673632, Kerala, India*,  
n.sooppy@psau.edu.sa

Follow this and additional works at: [https://digitalcommons.aaru.edu.jo/isl](https://digitalcommons.aaru.edu.jo/isl)

**Recommended Citation**  
Jangid, Kamlesh; Purohit, S.D.; Sooppy Nisar, Kottakkaran; and Shefeeq, T. (2020) "The internal blood pressure equation involving incomplete I-functions," *Information Sciences Letters*: Vol. 9 : Iss. 3 , Article 2. Available at: [https://digitalcommons.aaru.edu.jo/isl/vol9/iss3/2](https://digitalcommons.aaru.edu.jo/isl/vol9/iss3/2)

This Article is brought to you for free and open access by Arab Journals Platform. It has been accepted for inclusion in Information Sciences Letters by an authorized editor. The journal is hosted on Digital Commons, an Elsevier platform. For more information, please contact rakan@aaru.edu.jo, marah@aaru.edu.jo, dr_ahmad@aaru.edu.jo.
The internal blood pressure equation involving incomplete $I$-functions

Kamlesh Jangid, S.D. Purohit, Kottakkaran Sooppy Nisar and T. Shefeeq

1Department of HEAS (Mathematics), Rajasthan Technical University, Kota, 324010, Rajasthan, India
2Department of Mathematics, College of Arts and Sciences, Prince Sattam bin Abdulaziz University, Wadi Aldawaser, 11991, Saudi Arabia
3Department of Mathematics, Farook College(Autonomous), Kozhikode-673632, Kerala, India

Received: 10 May 2020, Revised: 17 Aug. 2020, Accepted: 19 Aug. 2020
Published online: 1 Sep. 2020

Abstract: Mathematical structures are used in the areas of biology, physics, engineering and social sciences. A model will help us interpret the system, analyze the impacts of various components, and make behavioral predictions. In this paper, we developed an internal blood pressure model that involves incomplete $I$-functions. Next, while taking a course in the constraints of incomplete $I$-functions, we give a few special cases of our model, and also mention some known results.

Keywords: Incomplete Gamma Function, Incomplete $I$-functions, $I$-functions, Mellin-Barnes type integrals.

1 Introduction

Mathematical modelling seems to be the expression for physical systems through comprehensible mathematical equations, the theoretical and computational study of which provides constructive interpretation, clarification and direction for the application that occurs. Mathematical modeling is used in diverse fields of biological sciences, engineering, and many more [1,2,3,4,5,6,7,8,9,10,11,12].

Recently, Kumar et al. [13] established the Mathematical modelling of internal blood pressure involving incomplete $H$-functions and then they gave many important special cases of the main results. Motivated from this study, we established the internal blood pressure model involving the incomplete $I$-functions.

We think back to the commonly applied incomplete Gamma functions $I'(s,x)$ and $\gamma(s,x)$ described as:

$$\gamma(s,x) := \int_0^x t^{s-1}e^{-t}dt \quad (\Re(s) > 0; x \geq 0) \quad (1)$$

and

$$\Gamma(s,x) := \int_x^\infty t^{s-1}e^{-t}dt, \quad (x \geq 0; \Re(s) > 0 \text{ when } x = 0), \quad (2)$$

respectively, follow the formula of decomposition presented by:

$$\Gamma(s,x) + \gamma(s,x) = \Gamma(s) \quad (\Re(s) > 0). \quad (3)$$

The incomplete $I$-functions $\gamma I_{\rho,q}^{m,n}(z)$ and $\Gamma I_{\rho,q}^{m,n}(z)$ were presented and analyzed by Jangid et al. [14, Eqs.(1)-(4)] in the following manner:

$$\gamma I_{\rho,q}^{m,n}(z) = \gamma I_{\rho,q}^{m,n} \left[ z \left( \zeta_1, \xi_1; A_1 : x \right), \left( \zeta_j, \xi_j; A_j \right) \right]_{2,p} \quad \left( \eta_j, \Omega_j; B_j \right)_{1,q}$$

$$= \frac{1}{2\pi i} \int_E \phi(s,x) z^s ds \quad (4)$$

and

$$\Gamma I_{\rho,q}^{m,n}(z) = \Gamma I_{\rho,q}^{m,n} \left[ z \left( \zeta_1, \xi_1; A_1 : x \right), \left( \zeta_j, \xi_j; A_j \right) \right]_{2,p} \quad \left( \eta_j, \Omega_j; B_j \right)_{1,q}$$

$$= \frac{1}{2\pi i} \int_E \Phi(s,x) z^s ds \quad (5)$$
for all \( z \neq 0 \), where
\[
\phi(s, x) = \prod_{j=n+1}^{p} \Gamma(\zeta_j - \ell_j s) A_j \prod_{j=1}^{m} \{\Gamma(\eta_j - \zeta_j s)\} B_j
\]
and
\[
\Phi(s, x) = \prod_{j=n+1}^{p} \Gamma(\zeta_j - \ell_j s) A_j \prod_{j=1}^{m} \{\Gamma(\eta_j - \zeta_j s)\} B_j
\]

The incomplete I-functions \( \gamma_{r, q}^{m, n}(z) \) and \( \Gamma_{r, q}^{m, n}(z) \) defined in (4) and (5) exists for all \( x > 0 \) under the same contour and conditions as stated by Rathie [15]. The incomplete I-functions are the generalizations of incomplete \( \Gamma \)-functions, I-function, incomplete \( \Gamma \)-functions, incomplete \( H \)-functions, and \( \Gamma \)-functions etc., which are given under:

(i) Considering that \( x = 0 \) for the (5), incomplete I-function \( \Gamma_{p, q}^{m, n}(z) \) reduce to the I-function [15] as follows:
\[
\gamma_{p, q}^{m, n} (z) = \prod_{j=n+1}^{p} \Gamma(\zeta_j - \ell_j s) A_j \prod_{j=1}^{m} \{\Gamma(\eta_j - \zeta_j s)\} B_j
\]

(ii) Setting \( B_j \) (\( j = 1, \cdots, m \)) = 1, the functions (4) and (5) reduce to the incomplete \( \Gamma \)-functions [16] as follows:
\[
\gamma_{p, q}^{m, n} (z) = \prod_{j=n+1}^{p} \Gamma(\zeta_j - \ell_j s) A_j \prod_{j=1}^{m} \{\Gamma(\eta_j - \zeta_j s)\} B_j
\]

(iii) Setting \( B_j \) (\( j = 1, \cdots, m \)) = 1 and \( A_j \) (\( j = n + 1, \cdots, p \)) = 1, the functions (4) and (5) reduce to the incomplete \( \Gamma \)-functions [17] as follows:
\[
\gamma_{p, q}^{m, n} (z) = \prod_{j=n+1}^{p} \Gamma(\zeta_j - \ell_j s) A_j \prod_{j=1}^{m} \{\Gamma(\eta_j - \zeta_j s)\} B_j
\]

\[
= \prod_{j=n+1}^{p} \Gamma(\zeta_j - \ell_j s) A_j \prod_{j=1}^{m} \{\Gamma(\eta_j - \zeta_j s)\} B_j
\]

and
\[
\Gamma_{p, q}^{m, n} (z) = \prod_{j=n+1}^{p} \Gamma(\zeta_j - \ell_j s) A_j \prod_{j=1}^{m} \{\Gamma(\eta_j - \zeta_j s)\} B_j
\]

\[
= \prod_{j=n+1}^{p} \Gamma(\zeta_j - \ell_j s) A_j \prod_{j=1}^{m} \{\Gamma(\eta_j - \zeta_j s)\} B_j
\]

The present paper is organized as follows:
Section 2 provides the main findings of the study. In Section 3, we provided some particular cases of the main results and finally the concluding remark is given in Section 4.

2 Main Results

Throughout this paper, letting \( p \) denote the internal blood pressure in blood vessels with volume \( v \). Also, we suppose on any time \( p_1 \) and \( v_1 \) represents the partial changes in internal pressure and volume, respectively.
\textbf{Theorem 1.} If $v > v_1, p > p_1$ and $x \geq 0$ then the subsequent rule shall be observed:

$$
\gamma^{m,n+1}_{p+1,q+1} \left[ (\zeta, \ell_1; A_1 : x), (-v, v_1; 1), (\zeta, \ell_2; A_2) \right]_{(\eta_1, \eta_2; B_1)} \left( 1 - v, v_1 ; 1 \right)
$$

We shall report a few important limiting cases of key outcomes in this section. If $B_j (j = 1, \ldots, m) = 1$ is set, the incomplete \(T\)-functions (4) and (5) will reduce the incomplete \(T\)-functions defined by (9) and (10), then Theorems 1 and 2 will give the following corollaries:

\textbf{Corollary 1.} If $v > v_1, p > p_1$ and $x \geq 0$ then the following law is hold:

$$
\gamma^{m,n+1}_{p+1,q+1} \left[ (\zeta, \ell_1; A_1 : x), (-v, v_1; 1), (\eta_1, \eta_2; B_1) \right] \left( 1 - v, v_1 ; 1 \right)
$$

provided $\tau$ is a proportional constant and $s$ is a constant of integration.

\textbf{Proof.} Saxena [19, p.77] presented the mathematical concept of an internal blood pressure in the blood vessel as follows:

$$
v \propto p.
$$

Which results in the next differential equation

$$
\frac{dp}{dx} = \tau; \quad v \rightarrow 0, \quad p \rightarrow 0
$$

where $\tau$ is a proportional constant. With respect to the integration among both ends, we acquire

$$
v = \tau p + s
$$

where $s$ is a constant of integration. The above equation may be described to

$$
\frac{\Gamma(v+1)}{\Gamma(v)} = \tau \frac{\Gamma(p+1)}{\Gamma(p)} + s.
$$

If, $p$ is replaced by $p + p_1$ and $v$ is replaced by $v + v_1$ and multiplying both sides by $\frac{1}{2\pi i} \Phi(s, x) e^{s}$, then integrating with respect to $s$ in the direction of contour $\mathcal{L}$ (defined in 5) and we get the needed result by taking account of (5).

\textbf{Theorem 2.} If $v > v_1, p > p_1$ and $x \geq 0$ then the following law is hold:

$$
\gamma^{m,n+1}_{p+1,q+1} \left[ (\zeta, \ell_1; A_1 : x), (-v, v_1; 1), (\zeta, \ell_2; A_2) \right]_{(\eta_1, \eta_2; B_1)} \left( 1 - v, v_1 ; 1 \right)
$$

We shall report a few important limiting cases of key outcomes in this section. If $B_j (j = 1, \ldots, m) = 1$ is set, the incomplete \(T\)-functions (4) and (5) will reduce the incomplete \(T\)-functions defined by (9) and (10), then Theorems 1 and 2 will give the following corollaries:

\textbf{Corollary 1.} If $v > v_1, p > p_1$ and $x \geq 0$ then the following law is hold:

$$
\gamma^{m,n+1}_{p+1,q+1} \left[ (\zeta, \ell_1; A_1 : x), (-v, v_1; 1), (\eta_1, \eta_2; B_1) \right] \left( 1 - v, v_1 ; 1 \right)
$$

provided $\tau$ is a proportional constant and $s$ is a constant of integration.

\textbf{Proof.} Saxena [19, p.77] presented the mathematical concept of an internal blood pressure in the blood vessel as follows:

$$
v \propto p.
$$

Which results in the next differential equation

$$
\frac{dp}{dx} = \tau; \quad v \rightarrow 0, \quad p \rightarrow 0
$$

where $\tau$ is a proportional constant. With respect to the integration among both ends, we acquire

$$
v = \tau p + s
$$

where $s$ is a constant of integration. The above equation may be described to

$$
\frac{\Gamma(v+1)}{\Gamma(v)} = \tau \frac{\Gamma(p+1)}{\Gamma(p)} + s.
$$

If, $p$ is replaced by $p + p_1$ and $v$ is replaced by $v + v_1$ and multiplying both sides by $\frac{1}{2\pi i} \Phi(s, x) e^{s}$, then integrating with respect to $s$ in the direction of contour $\mathcal{L}$ (defined in 5) and we get the needed result by taking account of (5).

\textbf{Theorem 2.} If $v > v_1, p > p_1$ and $x \geq 0$ then the following law is hold:

$$
\gamma^{m,n+1}_{p+1,q+1} \left[ (\zeta, \ell_1; A_1 : x), (-v, v_1; 1), (\zeta, \ell_2; A_2) \right]_{(\eta_1, \eta_2; B_1)} \left( 1 - v, v_1 ; 1 \right)
$$

3 Special Cases

We shall report a few important limiting cases of key outcomes in this section. If $B_j (j = 1, \ldots, m) = 1$ is set, the incomplete \(I\)-functions (4) and (5) will reduce the incomplete \(I\)-functions defined by (9) and (10), then Theorems 1 and 2 will give the following corollaries:

\textbf{Corollary 1.} If $v > v_1, p > p_1$ and $x \geq 0$ then the following law is hold:

$$
\gamma^{m,n+1}_{p+1,q+1} \left[ (\zeta, \ell_1; A_1 : x), (-v, v_1; 1), (\eta_1, \eta_2; B_1) \right] \left( 1 - v, v_1 ; 1 \right)
$$

provided $\tau$ is a proportional constant and $s$ is a constant of integration.

\textbf{Proof.} Saxena [19, p.77] presented the mathematical concept of an internal blood pressure in the blood vessel as follows:

$$
v \propto p.
$$

Which results in the next differential equation

$$
\frac{dp}{dx} = \tau; \quad v \rightarrow 0, \quad p \rightarrow 0
$$

where $\tau$ is a proportional constant. With respect to the integration among both ends, we acquire

$$
v = \tau p + s
$$

where $s$ is a constant of integration. The above equation may be described to

$$
\frac{\Gamma(v+1)}{\Gamma(v)} = \tau \frac{\Gamma(p+1)}{\Gamma(p)} + s.
$$

If, $p$ is replaced by $p + p_1$ and $v$ is replaced by $v + v_1$ and multiplying both sides by $\frac{1}{2\pi i} \Phi(s, x) e^{s}$, then integrating with respect to $s$ in the direction of contour $\mathcal{L}$ (defined in 5) and we get the needed result by taking account of (5).

\textbf{Theorem 2.} If $v > v_1, p > p_1$ and $x \geq 0$ then the following law is hold:

$$
\gamma^{m,n+1}_{p+1,q+1} \left[ (\zeta, \ell_1; A_1 : x), (-v, v_1; 1), (\zeta, \ell_2; A_2) \right]_{(\eta_1, \eta_2; B_1)} \left( 1 - v, v_1 ; 1 \right)
$$

In addition, specializing the parameters in (4) and (5), we may obtain internal blood pressure equations as special cases for functions expressed in section 1 of our main results as follows:

(i)Using the relationship (8), we may obtain an internal blood pressure equation involving \(I\)-function.

(ii)Using the relationships (11) and (12), the equations of internal blood pressure involving incomplete \(I\)-functions defined by Kumar et al. [13] may be obtained.

(iii)Using the relationships (13) and (14), we may obtain the internal blood pressure equations involving incomplete \(I\)-functions.

\(2020\) NSP

Natural Sciences Publishing Cor.
(iv) Setting $x = 0$, $B_j (j = 1, \cdots, n) = 1$ and $A_j (j = n + 1, \cdots, p) = 1$ in (16), then incomplete $I$-function $I_{p,q}^\gamma (z)$ reduces to $H$-function (see relationship 15), we get a known result from Chaurasia [20].

4 Conclusions

In this paper, we reproduced internal blood pressure equations involving incomplete $I$-functions. In addition, special cases of our main results have been provided in Section 3. The results of this study would be very useful in the biology study.

Acknowledgment

The research of the author Shefeeq. T is supported under the faculty research Project under RUSA 2.0, MHRD, Govt. of India, Project Ref No: G3/RUSA/FRP(2)/2019.

Conflict of Interest

The authors declare that there is no conflict of interest regarding the publication of this article.

References