Journal of Engineering Research

Volume 1 | Issue 1 Article 1

2015

MULTI-OBJECTIVE STRATEGY FOR OPTIMIZING REPETITIVE CONSTRUCTION PROJECTS USING LINEAR PROGRAMMING MODELS

Sherif M. Hafez
sherif.hafez@alexu.edu.eg

Enas S. Morgan

Mohamed A. Hamad

The Journal of Engineering Research Volume 1 No.1 Faculty of Engineering-Tanta University

MULTI-OBJECTIVE STRATEGY FOR OPTIMIZING REPETITIVE CONSTRUCTION PROJECTS USING LINEAR PROGRAMMING MODELS

Sherif M. Hafez[1], Enas S. Morgan[2] , Mohamed A. Hamad[2]
[1] Professor of Construction Engineering, Department of Civil Engineering, Alexandria University, Egypt.
E-mail: sherif.hafez@alexu.edu.eg
[2] M.Sc. Candidates, Department of Civil Engineering, Alexandria University, Egypt.
E-mail: enas_morgan@yahoo.com , m_fadeel@hotmail.com

1. ABSTRACT

Decision making has become much more complicated than in the past due to increased decision alternatives, uncertainty, and cost of making errors. As a result, it is very difficult to rely on a trial and error approach in decision making. Nowadays business managers are dealing with different types of projects ranging from implementing a large scale manufacturing plant to a simple sales campaign. While dealing with projects, to become competitive, sometimes it is required to complete a project within the predetermined deadline to keep cost at lowest possible level. Failure to do so ultimately leads to increase in total cost. This would direct managers to encounter a decision situation: which activities of the project will be crashed to minimize the total cost of crashing project. In this paper, we provide a hypothetical example to clarify the framework of how to convert from LOB to CPM and then how to create a model to crash a project time to reach an optimum time-cost solution. Microsoft Excel custom made sheets used to the conversion, also Solver add-in used to solve the model while it implements Linear Programming. As a check, results from Solver and LiPS software are compared.

Keywords: **Line-Of-Balance, LOB, Linear Programming, LP, Project Crashing, Time-Cost Trade-Off, CPM**

2. INTRODUCTION

Most of construction managers are continually facing a situation in which they must take a decision whether to complete the project sooner than originally specified in the contract because of the clients request and /or to optimize the cost of expediting.
The planned duration is decreased by crashing all critical activities either by authorizing overtime work or applying additional resources.

3. EXISTING PROJECT-PLANNING TECHNIQUES

The technique used for project scheduling will vary depending on the project's size, complexity, duration, personnel, and owner's requirements.
The project manager must choose a scheduling technique that is simple to be used and is easily interpreted by all project participants, *Oberlender, G.* (2008).

3.1. Bar Chart

3.2. Network-Based Methods.

There are two widely known network based techniques:

3.2.1. **C**ritical **P**ath **M**ethod, "**CPM**"

3.2.2. **P**rogram **E**valuation and **R**eview **T**echnique, "**PERT**"

3.3. Linear Projects

Several techniques were developed for projects with discrete units, such as floors, houses, offices, etc. The names used have included the following:

1. **L**ine **O**f **B**alance, "**LOB**" [*O'Brien, J.* (1969)];
2. Construction planning techniques [*Peer, S.* (1974)];
3. **V**ertical **P**roduction **M**ethod, "**VPM**" [*O'Brien, J.* (1975)];
4. Time space scheduling method [*Stradal, O.* (1982)];
5. Time-location matrix model [*Carr, R.* (1993)].

Line–**O**f-**B**alance, "**LOB**" is one production scheduling and controlling technique, which tries to surpass the CPM difficulties for the multi-story building scheduling. It was developed into manufacturing environment by the US Navy, and had its origins at the time of the World War II, *Burke, R.* (1996).

Hafez, S. (1997a), developed a tool for time and resource scheduling for repetitive construction projects, this has been done in three stages. This model called ***Modified Repetitive Project Model***, "**MRPM**", which depends on the integration between the principles of line of balance method and critical path method. Crashing the critical activity can alter the direction of the critical path. Therefore, it is necessary to re-examine the critical path after each cycle of applying the developed system, to ensure that the overall project duration is shortened to the required limit. Otherwise, the current critical path will be targeted for further crashing.

3.4. Non-Linear Projects

Russell, A. (1993) commented that the majority of construction projects with repetitive activities are non-linear.

Hafez, S. (2005), surveyed the different issues, which related to schedule repetitive construction process. It can be used in the development of a computerized scheduling system. Firstly, applying resource-driven scheduling methods, visual presentation of line of balance diagram, optimize project cost, and resource utilization were discussed. Finally, it studied the acceleration routine, and integration scheduling methods.

4. LINEAR PROGRAMMING TECHNIQUE

Linear programming is a tool for decision making under certain situation. So, the basic assumption of this approach is that we have to know some relevant data with certainty. The basic data requirements are as follows:

1. The project network with activity time, which can be achieved from PERT and CPM.
2. To what extent an activity can be crashed.
3. The crashing cost associated with per unit of time for all activities.
4. The same data mentioned in 2 and 3 for each activity-option if available.

To reduce the time to complete the activity, more resources are applied in the form of additional personnel and overtime. As more resources are applied, the duration is shortened, but the cost rises. The maximum effort is applied so that the activity can be completed in the shortest possible time.

The Journal of Engineering Research Volume 1 No.1 Faculty of Engineering-Tanta University

4.1. Literature Review

Selinger (1980) developed a dynamic programming model of a linear project. His work ignored to incorporate the cost as decision variable in the optimization process. As extension of the *Selinger's* work, *Russel and Caselton* (1988) formalized a N-stage dynamic programming solution into two state variable to determine the minimum project duration. In the optimization process, the developed model ignored the activities costs as a decision variable. *Reda* (1990) developed a linear programming to identify minimum cost maintaining constant production rates and repetitive projects. This method can only be used for non-typical linear project and not applicable to construction projects. Most of the developed models assume the activities are accomplished serially. In reality, most construction activities are accomplished concurrently while others accomplished serially.

Elmaghraby (1997) considered completion schedules on an arbitrary set of milestone events by developing an efficient algorithm to determine the project schedule, which minimizes the sum of the total cost plus penalties for late completion. Another extension was by *Moore* (1998) by using goal programming to consider multiple objectives, such as completion times, resources leveling and operation within a limited budget. *Senouci* (1996) presented a dynamic programming formulation for the scheduling of non-sequential or non-serial activities to determine the project time-cost profile which determines possible project duration and their minimum project total cost. The formulation considers the effects of interruptions, minimum project direct cost, and minimum project duration.

5. RESEARCH OBJECTIVES

This research study is expected to transform the optimization of repetitive construction resource utilization problems from an intractable problem to a feasible and practical one. The application of these research developments in planning the construction projects holds a strong promise to:

1. Increase the efficiency of resources used for typical-repetitive large-scale construction projects;
2. Reduce construction duration period;
3. Minimize construction cost (sum of direct cost and indirect cost).

5.1. Considered Assumptions

The mathematical formulation of the present model is based on the following assumptions:

1. No idle time is allowed for employed crews,
2. A constant average duration is set for the same activity at all stages to maintain a constant production rate.
3. The learning phenomenon, is neglected;
4. The work on each activity is conducted by one unit at a time.

5.2. Employed Techniques

1. For each activity (k), (where k = 1, 2,…, K) in the typical-repetitive network, **LOB** is used to represent the activity schedule at all stages in project time plan;
2. Transformation from the traditional LOB to modified CPM must be done in the model;

3. Each activity (k), (where k = 1, 2,..., K) has a time buffer ($TB_{k,kk}$), at each stage (s), (where s = 1, 2,…, S) between the completion time of the activity (k)and the start time of each following activity (kk) in the network;
4. Any two sequential activities may have a stage buffer ($SB_{k,kk}$), of a specific number of stages at any time to meet practical and / or technological purposes, this stage buffer has to be identified by the planner for these activities;

6. FORMULATION OF THE PROPOSED MODEL

6.1. Objective function of Normal Duration and Cost (Optimum Alternative):

In this model, project duration will be estimate using a converted LOB to a new modified CPM for scheduling typical-repetitive large-scale construction projects.

6.1.1. Objective Function Of Normal Duration:

$$Z_1^n = min \sum_{k=1}^{k=KK} (NT_K^n + (FS_m)_{K,KK}^n) \quad (1)$$

$$NT_K = S \times ND_K^n \quad (2)$$

$$NT_{KK} = S \times ND_{KK}^n \quad (3)$$

$$No.of\ Alt(Solutions) = n = No.of\ Alt_{k1}\ x\ No.of\ Alt_{k2} x \ldots\ldots\ldots\ldots . No.of\ Alt_K \quad (4)$$

$$TB_{K,KK}^n \geq FS_{K,KK} \quad (5)$$

$$TB_{K,KK}^n \geq SS_{K,KK} - ND_K^n \quad (6)$$

$$TB_{K,KK}^n \geq FF_{K,KK} - ND_{KK}^n \quad (7)$$

$$TB_{K,KK}^n \geq SF_{K,KK} - (ND_K^n + ND_{KK}^n) \quad (8)$$

Take the max of equation from (5 to 8), Then;

6.1.1.1. Case of Critical Stage at First floor:

For any two sequential activities (k) and (kk), if $ND_K^n \leq ND_{KK}^n$ then critical stage is firststage, (see Figure 1). Modified finish to start at a new CPM between these two sequentialactivities (k) and (kk) can be calculated as shown in the next formulas:

$$(FS_{K,KK})_m^n \geq TB_{K,KK} + (1 - S) \times ND_K^n \quad (9)$$

$$(FS_{K,KK})_m^n \geq (SB_{K,KK} - S) \times ND_K^n \quad (10)$$

Take the max of equation from (9 to 10)

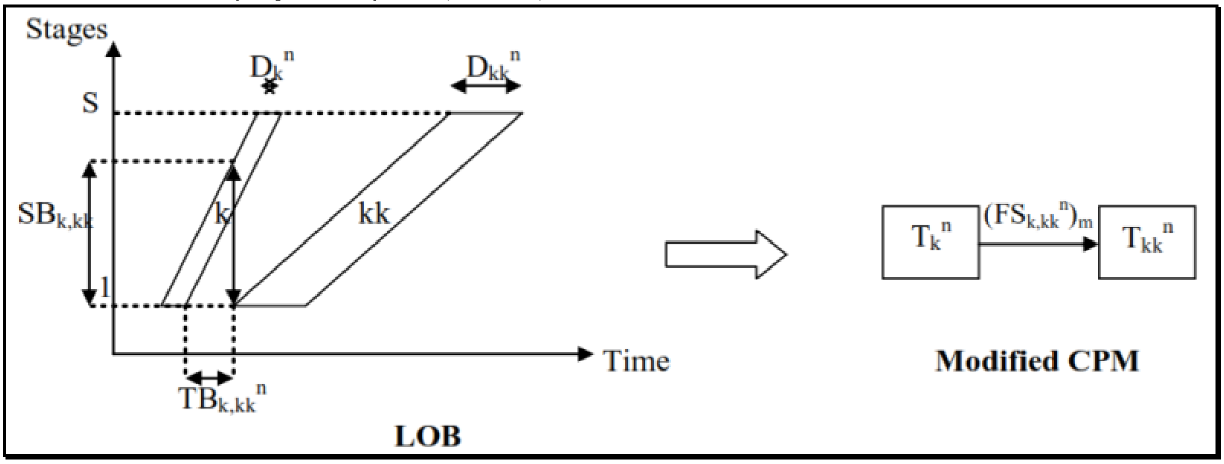

Figure 1: Modified CPM Integrated with LOB when Critical Stage is First Stage

The Journal of Engineering Research Volume 1 No.1 Faculty of Engineering-Tanta University

6.1.1.2. Case of Critical Stage at Last floor:

For any two sequential activities (k) and (kk), if $ND_K^n \geq ND_{KK}^n$, then critical stage is last stage, (see Figure 2). Modified finish to start at a new CPM between these two sequential activities (k) and (kk) can be calculated as shown in the next formulas:

$$(FS_{K,KK})_m^n \geq TB_{K,KK} + (1 - S) \times ND_{KK}^n \quad (11)$$

$$(FS_{K,KK})_m^n \geq (1 + SB_{K,KK} - S) \times ND_{KK}^n - ND_K^n \quad (12)$$

Take the max of equation from (11 to 12)

$$\boldsymbol{PND_{Opt} = Z_{1_{min}}^n} \quad (13)$$

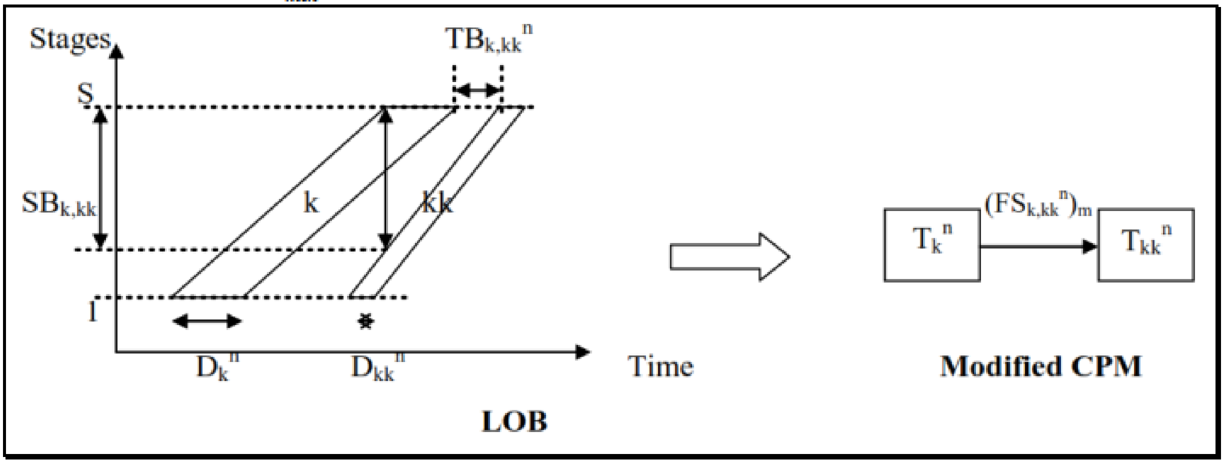

Figure 2: Modified CPM Integrated with LOB when Critical Stage is Last Stage

Where:

- Z_1^n = Minimum Normal Project Duration in (days) using resource utilization (n).
- NT_K^n= Normal Total durations in (days) at all stages of an activity (k) on the critical path at new CPM using resource utilization (n).
- $(FS_m)_{K,KK}^n$ = Modified finish to start at a new CPM between two sequential activities using resource utilization (n).
- S = Number of stages in the project.
- ND_K^n = Normal Total durations in (days) at all stages of a predecessor activity (k) on the critical path at a new CPM using resource utilization (n)
- ND_{kk}^n = Normal Total durations in (days) at all stages of a successor activity (kk) on the critical path at a new CPM using resource utilization (n).
- $SB_{K,KK}^n$ = Stage buffer between the starts of two sequential activities (k) and (kk) at LOB.
- $TB_{K,KK}^n$ = Time buffer in (days) between two sequential activities finish (k) and start (kk) at LOB using resource utilization (n).
- ST_k^n= Start time of a predecessor activity (k) at first stage at LOB using resource utilization (n).
- $SS_{k,kk}$= Start to Start in (days) between two sequential activities (k) and (kk) at LOB.
- $FF_{k,kk}$= Finish to Finish in (days) between two sequential activities (k) and (kk) at LOB.
- $SF_{k,kk}$= Start to Finish in (days) between two sequential activities (k) and (kk) at LOB.

Modified CPM can be constructed at a new CPM with modified finish to start between sequential activities in the network as shown in the previous formulas and then the time objective function with modified CPM can be conducted with Excel Spread sheet ***To Optimize The Minimum Project Normal Duration***.

The Journal of Engineering Research Volume 1 No.1 Faculty of Engineering-Tanta University

6.1.2. Objective Function Of Normal Cost:

$$Z_2^n = Min((S \times \sum_{k=1}^{k=kk} NC_k^n) + IC \times Z_1^n) \quad (14)$$

- Z_2^n =Min. Project Normal Cost {Direct}+{Indirect} in (EGP) using resource utilization (n).
- S = Number of stages in the project
- NC_k^n= Normal cost in (EGP) at one stage of an activity (k) at LOB using resource utilization (n)
- IC = Daily indirect cost rate in (EGP/Day) along project life

The cost objective function as shown in the previous formulation (14) can be conducted with Excel Spread sheet ***To Optimize The Total Cost Of The Project***

.

6.1.3. Optimum Solution (Time -Cost –Trade- Off) for Normal Values

$$Z_1 = \frac{PND_{Opt} - PND_{mean}}{\delta_{ND}} \quad (15)$$

$$Z_2 = \frac{PNC_{Opt} - PNC_{mean}}{\delta_{NC}} \quad (16)$$

$$\delta_{ND} = \frac{\sqrt{n\sum(PND)^2 - (\sum PND)^2}}{n} \quad (17)$$

$$\delta_{NC} = \frac{\sqrt{n\sum(PNC)^2 - (\sum PNC)^2}}{n} \quad (18)$$

$$Z_{min} = w_1 \times Z_1 + w_2 \times Z_2 \quad (19)$$

$$w_1 + w_2 = 1 \quad (20)$$

Where:

- Z = Time-Cost Objective Optimization Function.
- w_1 = Weight of project duration focus.
- w_2 = Weight of project cost focus.
- for optimum duration and corresponding optimum cost use (w_1=w_2= 0.50)

6.2. Crashing Project Duration Model:

In this model, the optimum project normal duration obtained from the first model will be crashed using linear programming in order to solve the inequalities of the model.

6.3. Objective Function:

6.3.1.1. Case of Min Project Total cost:

$$Z = min \sum_{K=1}^{N}(CS_K \times Y_K) + IC \times PD_c \quad (21)$$

$$CS_k = \frac{CC_k - NC_k}{NT_k - CT_k} \quad (22)$$

6.3.1.2. Case of Min Project Duration

$$Z = min\ PD_c \quad (23)$$

6.3.2. Constraints:

6.3.2.1. Non-negative constraints:

All decision variables must be ≥ 0

The Journal of Engineering Research Volume 1 No.1 Faculty of Engineering-Tanta University

(24)

6.3.2.2. Maximum reduction constraints:

$$Y_K \leq NT_k - CT_k$$

(25)

6.3.2.3. Start Time Constraints:

$$S_{KK} \geq S_K + ND_K - Y_K + (FS_m)k, kk$$

(26)

6.3.2.4. Project duration constraint:

$$PD_c \leq PND\ (opt) \quad (27)$$

Where:

- CS_k = Cost Slop of Activity (K)
- CC_k =Crash Cost of Activity (K) in (EGP)
- NC_k = Normal Cost of Activity (K) in (EGP)
- NT_k =Normal Time in (days) of Activity (K) at all stages
- CT_k = Crash Time in (days) of Activity (K) at all stages
- S_k =Start Time of Activity K in (days)
- Y_k = Amount of times in (days) that each activity K will be crashed
- $PND\ (opt)$ = Optimum project duration obtained from formula No. (14)
- PD_c = Crashed Project Duration

7. NUMERICAL EXAMPLE:

A hypothetical 5 Activities network shown in figure (1) will serve as an example for demonstrating the proposed model. Table (1) shows the input data for the model. The studied project has 10 stages and the project Indirect Cost is 300 LE per unit time

Table 1: Activity Data for the Numerical Example

Activity Name	Depend On	Relation Type	Lag Value	Stage Buffer	Resource Options	ND	CD	NC per all stages	CC per all stages
A	--	--	--	--	1	20	17	40000	51000
					2	23	22	50000	58000
B	A	SS	5	2	1	15	11	450000	485000
					2	16	14	350000	371000
C	A	FS	2	1	1	13	12	200000	208000
					2	26	24	150000	154000
D	A	SS	0	3	1	14	12	400000	406000
					2	18	16	320000	325000
E	B	SF	12	1	1	12	10	300000	309000
	C	FS	0	2	2	21	19	240000	264000
	D	FF	3	1					

The Journal of Engineering Research Volume 1 No.1 Faculty of Engineering-Tanta University

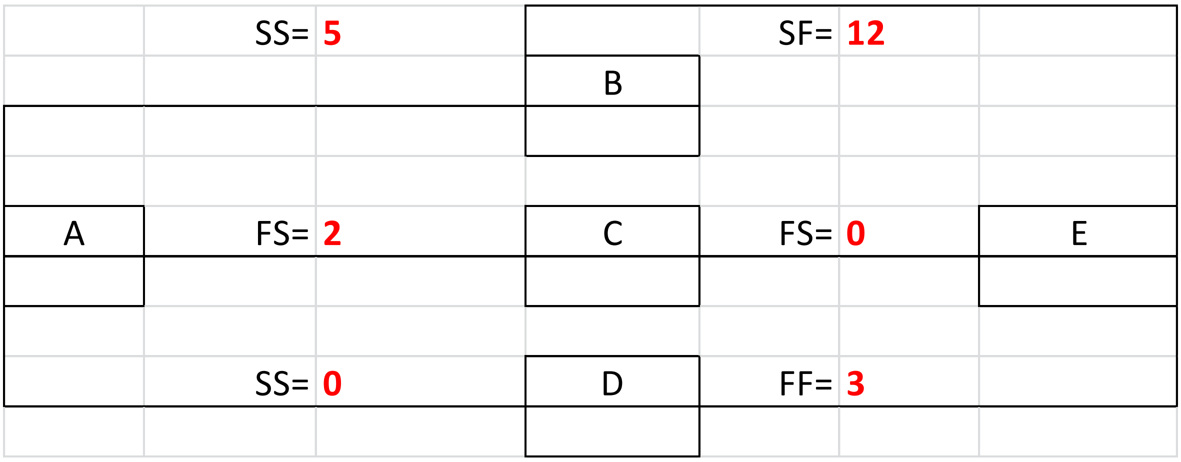

Figure 3: Hypothetical Project Network

7.1.Solution Steps:

1. Number of alternatives (Solutions) = 2x2x2x2x2=**32**
2. Apply equations from (1 to 20) By using **Excel spread sheets macro** to Solve the first model

Table 2: Project Solutions (Alternatives) and corresponding Normal Duration and Cost

Sol #	Solution Address					Corrosponding Duration					Corresponding Cost				
	A	B	C	D	E	A	B	C	D	E	A	B	C	D	E
1	1	1	1	1	1	20	15	13	14	12	40000	450000	200000	400000	300000
2	1	1	1	1	2	20	15	13	14	21	40000	450000	200000	400000	240000
3	1	1	1	2	1	20	15	13	18	12	40000	450000	200000	320000	300000
4	1	1	1	2	2	20	15	13	18	21	40000	450000	200000	320000	240000
5	1	1	2	1	1	20	15	26	14	12	40000	450000	150000	400000	300000
6	1	1	2	1	2	20	15	26	14	21	40000	450000	150000	400000	240000
7	1	1	2	2	1	20	15	26	18	12	40000	450000	150000	320000	300000
8	1	1	2	2	2	20	15	26	18	21	40000	450000	150000	320000	240000
9	1	2	1	1	1	20	16	13	14	12	40000	350000	200000	400000	300000
10	1	2	1	1	2	20	16	13	14	21	40000	350000	200000	400000	240000
11	1	2	1	2	1	20	16	13	18	12	40000	350000	200000	320000	300000
12	1	2	1	2	2	20	16	13	18	21	40000	350000	200000	320000	240000
13	1	2	2	1	1	20	16	26	14	12	40000	350000	150000	400000	300000
14	1	2	2	1	2	20	16	26	14	21	40000	350000	150000	400000	240000
15	1	2	2	2	1	20	16	26	18	12	40000	350000	150000	320000	300000
16	1	2	2	2	2	20	16	26	18	21	40000	350000	150000	320000	240000
17	2	1	1	1	1	23	15	13	14	12	50000	450000	200000	400000	300000
18	2	1	1	1	2	23	15	13	14	21	50000	450000	200000	400000	240000
19	2	1	1	2	1	23	15	13	18	12	50000	450000	200000	320000	300000
20	2	1	1	2	2	23	15	13	18	21	50000	450000	200000	320000	240000
21	2	1	2	1	1	23	15	26	14	12	50000	450000	150000	400000	300000
22	2	1	2	1	2	23	15	26	14	21	50000	450000	150000	400000	240000
23	2	1	2	2	1	23	15	26	18	12	50000	450000	150000	320000	300000
24	2	1	2	2	2	23	15	26	18	21	50000	450000	150000	320000	240000
25	2	2	1	1	1	23	16	13	14	12	50000	350000	200000	400000	300000
26	2	2	1	1	2	23	16	13	14	21	50000	350000	200000	400000	240000
27	2	2	1	2	1	23	16	13	18	12	50000	350000	200000	320000	300000
28	2	2	1	2	2	23	16	13	18	21	50000	350000	200000	320000	240000
29	2	2	2	1	1	23	16	26	14	12	50000	350000	150000	400000	300000
30	2	2	2	1	2	23	16	26	14	21	50000	350000	150000	400000	240000
31	2	2	2	2	1	23	16	26	18	12	50000	350000	150000	320000	300000
32	2	2	2	2	2	23	16	26	18	21	50000	350000	150000	320000	240000

The Journal of Engineering Research Volume 1 No.1 Faculty of Engineering-Tanta University

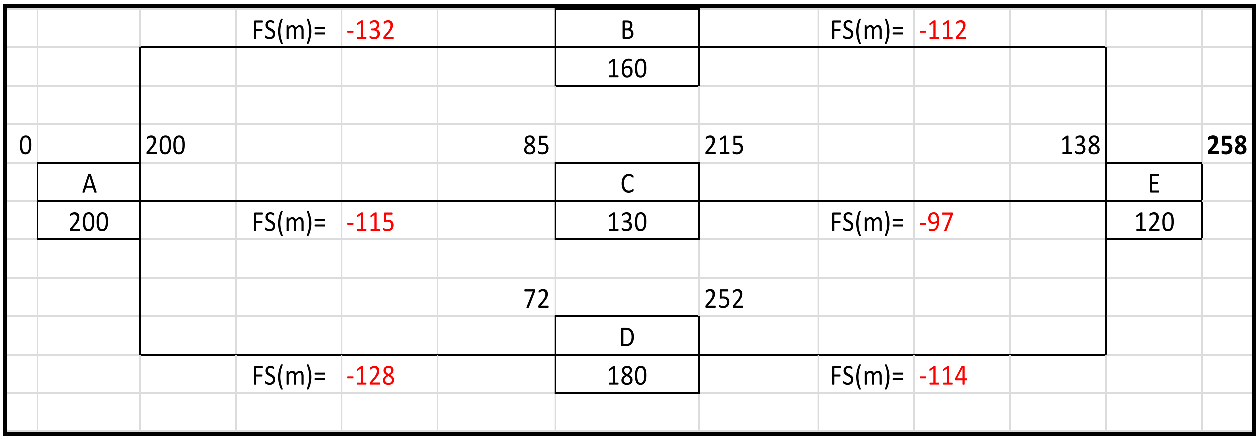

Figure 4: Optimum Project Normal Time & corresponding Total Cost

Table 3: Optimum Project Normal Duration and Cost

Solution	A	B	C	D	E	P.D (days)	P.C (EGP)	Z1	Z2	0.5 Z1 + 0.5 Z2
1	1	1	1	1	1	246	1,463,800	-1.98	-78.87	-40.4244
2	1	1	1	1	2	321	1,426,300	0.48	-79.39	-39.4541
3	1	1	1	2	1	258	1,387,400	-1.58	-79.93	-40.7568
4	1	1	1	2	2	321	1,346,300	0.48	-80.50	-40.0082
5	1	1	2	1	1	294	1,428,200	-0.40	-79.36	-39.8837
6	1	1	2	1	2	320	1,376,000	0.45	-80.09	-39.8189
7	1	1	2	2	1	294	1,348,200	-0.40	-80.47	-40.4379
8	1	1	2	2	2	319	1,295,700	0.42	-81.20	-40.3915
9	1	2	1	1	1	246	1,363,800	-1.98	-80.26	-41.1171
10	1	2	1	1	2	321	1,326,300	0.48	-80.78	-40.1468
11	**1**	**2**	**1**	**2**	**1**	**258**	**1,287,400**	**-1.58**	**-81.31**	**-41.4495**
12	1	2	1	2	2	321	1,246,300	0.48	-81.88	-40.7009
13	1	2	2	1	1	294	1,328,200	-0.40	-80.75	-40.5764
14	1	2	2	1	2	320	1,276,000	0.45	-81.47	-40.5116
15	1	2	2	2	1	294	1,248,200	-0.40	-81.86	-41.1306
16	1	2	2	2	2	319	1,195,700	0.42	-82.58	-41.0842
17	2	1	1	1	1	273	1,481,900	-1.09	-78.62	-39.8562
18	2	1	1	1	2	351	1,445,300	1.47	-79.13	-38.8304
19	2	1	1	2	1	285	1,405,500	-0.70	-79.68	-40.1886
20	2	1	1	2	2	351	1,365,300	1.47	-80.23	-39.3846
21	2	1	2	1	1	297	1,439,100	-0.31	-79.21	-39.7590
22	2	1	2	1	2	347	1,394,100	1.33	-79.84	-39.2507
23	2	1	2	2	1	297	1,359,100	-0.31	-80.32	-40.3132
24	2	1	2	2	2	327	1,308,100	0.68	-81.03	-40.1744
25	2	2	1	1	1	273	1,381,900	-1.09	-80.01	-40.5489
26	2	2	1	1	2	351	1,345,300	1.47	-80.51	-39.5231
27	2	2	1	2	1	285	1,305,500	-0.70	-81.06	-40.8813
28	2	2	1	2	2	351	1,265,300	1.47	-81.62	-40.0773
29	2	2	2	1	1	297	1,339,100	-0.31	-80.60	-40.4517
30	2	2	2	1	2	347	1,294,100	1.33	-81.22	-39.9434
31	2	2	2	2	1	297	1,259,100	-0.31	-81.71	-41.0059
32	2	2	2	2	2	327	1,208,100	0.68	-82.41	-40.8671
					∑	9802	42,940,600			
					AVG	306.313	7,156,767			
					$\sum (X^2)$	3032214	57,788,447,260,000			
					δ	30.4851	72,181			

The Journal of Engineering Research Volume 1 No.1 Faculty of Engineering-Tanta University

The table (3) shows that:

1. **Solution No. (11)** is the optimum for min **Project Normal Time & corresponding Total Cost:**

- PND (opt) = 258 Days
- Direct Cost = 1,210,000 LE
- Indirect Cost = 77,400 LE
- Total Project Cost = 1,287,400 LE

2. Apply Equation No. (22, 25) to calculate the Cost Slope and the Max Reduction Y as illustrated in table no. (4)

Table 4: Cost Slope and Max Reduction Units of Project Activities

Act	NT	CT	NC	CC	max Y	CS
A	200	170	50000	58000	30	266.667
B	160	140	350000	371000	20	1050
C	130	120	150000	154000	10	400
D	180	160	320000	325000	20	250
E	120	100	240000	264000	20	1200

7.2. Decision Variables:

X1=Y_A, X2=Y_B, X3=Y_C,X4=Y_D, X5=Y_E, X6=PD, X7=S_B, X8=S_C, X9=S_D, X10=S_E

7.3. Objective Function

$$Zmin = 266.667 \times Y_A + 1050 \times Y_B + 400 \times Y_C + 250 \times Y_D + 1200 \times Y_E + 300 \times PD_c$$

7.4. Constraints:

7.4.1. *Nonnegative constraints:*

$Y_A, Y_B, Y_C, Y_D, Y_E, S_B, S_C, S_D, S_E, PD \geq 0$ (1:10)

7.4.2. *Maximum reduction constraints:*

$Y_A \leq 30$ (11)
$Y_B \leq 20$ (12)
$Y_C \leq 10$ (13)
$Y_D \leq 20$ (14)
$Y_E \leq 20$ (15)

7.4.3. *Start Time Constraints:*

$$S_B \geq S_A + S \times ND_A - Y_A + (FS_m)_{A,B}$$

$$S_A = 0$$

$S_B + Y_A \geq 68$ (16)
$S_C + Y_A \geq 85$ (17)
$S_D + Y_A \geq 72$ (18)

$$S_E \geq S_B - Y_A + 1160 - 112$$

$S_E - S_B + Y_B \geq 48$ (19)
$S_E - S_C + Y_C \geq 33$ (20)
$S_E - S_D + Y_D \geq 66$ (21)

7.4.4. *Project duration constraints:*

$PD_c \geq S_E + ND_E \times S - Y_E$

So; $PD_c - S_E + Y_E \geq 120$ (22)
$PD_c \leq 258$ (23)

- By using Lips Software to solve the model the and Comparing the data with Excel Solver Results following results has been found:

Table 5: Lips Software Results

Variable	Value	Obj. Cost	Reduced Cost
X1	30	266.667	0
X2	0	1050	-1050
X3	0	400	-400
X4	20	250	0
X5	0	1200	-1200
X6	208	300	0
X7	40	0	0
X8	55	0	0
X9	42	0	0
X10	88	0	0

Table 6: Excel Solver Results

SOLVER

	YA	YB	YC	YD	YE	PD	STB	STC	STD	STE	
	266.67	1050	400	250	1200	300	0	0	0	0	
Zmin=	30	0	0	20	0	208	40	55	42	88	75400.0000

Constraints

	YA	YB	YC	YD	YE	PD	STB	STC	STD	STE	LHS		RHS
1	1										30	<=	30
2		1									0	<=	20
3			1								0	<=	10
4				1							20	<=	20
5					1						0	<=	20
6						1					208	<=	258
7	1										30	=>	0
8		1									0	=>	0
9			1								0	=>	0
10				1							20	=>	0
11					1						0	=>	0
12						1					208	=>	0
13	1						1				70	=>	68
14	1							1			85	=>	85
15	1								1		72	=>	72
16		1					-1			1	48	=>	48
17			1					-1		1	33	=>	33
18				1					-1	1	66	=>	66
19					1	1				-1	120	=>	120

Project Crashed to **208** Days with Total Cost = 1210000 + 75400 **1,285,400.00** LE

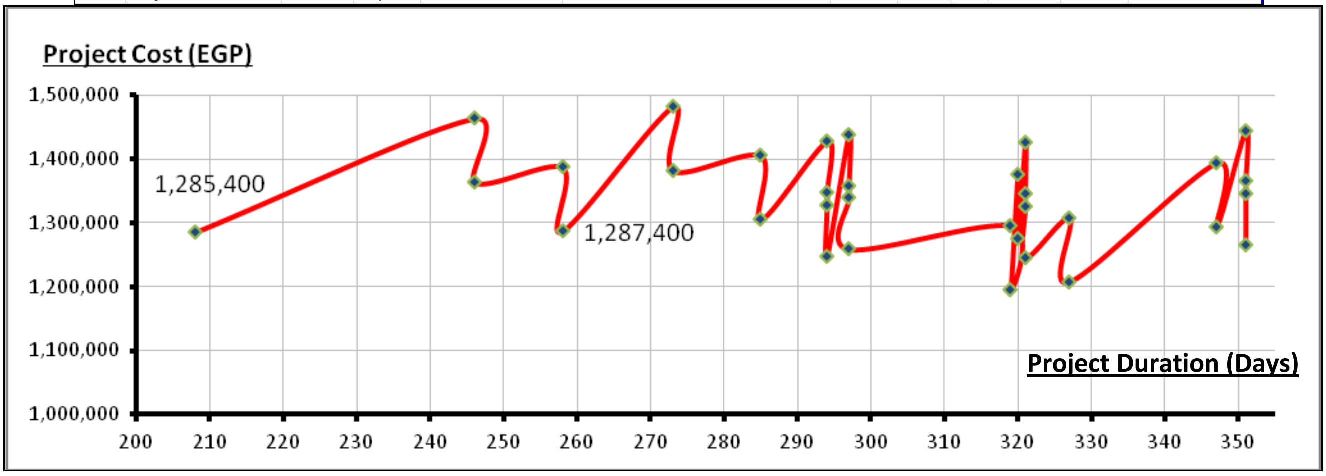

Figure 5: Comparing Model Result (Crashed) with The Normal Values (32 Solutions)

8. Analysis and Results

The linear programming model will not only take into account the activities on the critical path, but will also consider the non-critical activities, which in their turn become critical as the project time decreases.

Using LiPS software and Excel Solver; the solution of the model is presented in table(5) and (6),which indicates that:

1. The Optimum Min Project TC = 1,210,000+75,400 = (**1,285,400EGP**) corresponding to (208) Day Project Duration.
2. These results are due to crashing Activity (A) (3 days per stage) and Activity (D) (2 Days per stage).

It requires trial and error method to get the optimal result. It is important for project manager to recognize the flexibility of the system that can be used to explore numerous possible opportunities to the contractor. Moreover, this approach allows the user to easily manipulate different project networks of various difficulties representing real world applications, and to study the effectiveness of the model in the case of large projects.

The implementation of the developed model showed more efficient and reliable results and generated a considerable computational savings along with an increase in robustness.

9. Conclusion

The data needed for crashing project activities by means of linear programming technique are the time and cost for each activity when it is done in the normal way and then when it is fully crashed (expedited). The project manager can investigate the effect on total cost of changing the estimated duration of the project to various alternative values. Using linear programming model, the project manager will be able to determine how much (if any) to crash each activity in order to minimize the total cost of the project.

An algorithmic model based on linear programming incorporated with a minimal time-cost crash in a construction project was introduced. The format of the model lends itself to a wide range of variables and considerations.

The introduced modeling strategy which showed the resources of this interactive approach including a bulk of data to completely analyze the project is easily possible. It allowed a great number of parameters to simulate project conditions and contractor's preference and provided potentially useful tool for decision making on project scheduling.

10.References

1. Hafez S. M. (1997a). "Resource Allocation and Smoothening for Repetitive Projects" M. Sc. Thesis, Structural Eng. Dept., Alex. Univ., Alex., Egypt.
2. Hafez S. M. (2005). "Developing Classical Schedule Technology" Alexandria Engineering Journal, Vol. 44 (2), pp. C267-C277.
3. Selinger S., "Construction planning for linear projects", ASCE Journal of the Construction Division, Vol. 106, 2, (1980), pp.195-205.
4. Russel A.D. and Caselton W.F., "Extensions to linear scheduling optimization", ASCE Journal of Construction Engineering and Management, Vol.114, 1, (1988), pp.36-52.
5. Reda R.M., "RPM: repetitive project modeling", ASCE Journal of Construction Engineering and Management, 116, 2, (1990), pp. 316-330.
6. Elmaghraby S.E. and Pulat P.S., "Optimal Project Compression with Due-Dated Events, "Naval Research Logistics Quarterly, Vol.26, N°2, (1997), pp. 331-348.
7. Moore L.J., Taylor III B.W., Clayton E.R., and Lee S.M., "Analysis of a Multi-Criteria Project Crashing Model," American Institute of Industrial Engineering Trams., Vol. 10, N°2, (1998), pp. 163-169.
8. Senouci A.B. and Eldin N.N., "A time-cost trade-off algorithm for non-serial linear projects", Canadian J. of Engineering, Vol.23, (1996), pp.134-149.
9. Remon Fayek Aziz, 2010, “Time-Cost-Quality-Trade- Off for construction project with repetitive activities”.
10. Smith L.A., "Comparing Commercially Available CPM/PERT Computer Programs", Journal of Industrial Engineering, Vol. 10, N°4, (1997).
11. Schrage L.E., User's manual for LINDO; Scientific Press, Palo Alto, California, (1981).
12. Al Serraj Z.M., "Formal development of line-of balance technique", ASCE Journal of Construction Engineering and Management, Vol.116, 4, (1990), pp. 689-704.
13. Siemens N., "A Simple CPM Time-Cost Trade-off Algorithm," Management Science, Vol.17, No. 6, 1997), pp.B354-363.
14. Abo Elmagd Y. M. (2005). "Time-Cost Trade-Off in Construction Projects Applied on a Single Tower Crane" M. Sc. Thesis, Structural Eng. Dept., Alex. Univ., Alex., Egypt.

15. Adeli H. and Karim A. (1997). "Scheduling/Cost Optimization and Neural Dynamics Model for Construction" J. Constr. Engrg. and Mgmt., ASCE, 123 (4), 450-458.

16. Ahsan K., and Gunawan I. (2010). "Analysis of Cost and Schedule Performance of International Development Projects" International Journal of Project Management, (28), 68–78.

17. Chassiakos A. B. and Sakellaropoulos P.S. (2005). “Time-Cost Optimization of Construction Projects with Generalized Activity Constraints” J. Constr. Eng. And Mgmt., ASCE, 131 (10), 1115-1124.