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The laminar boundary layer over a rotating paraboloid

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Abstract: This work is concerned with the derivation of the steady boundary layer equations that gives the laminar flow profile over the outer surface of a paraboloid rotating in an otherwise still fluid. Also, the series solution formulation for the laminar flow equations for a rotating paraboloid is given. The series solution were numerically calculated and the laminar flow profiles are visualized in detail. Further, we showed that the formulation of the laminar flow equations for paraboloid has a mathematical flaw and this mistake led to the work of P. D. Verma [1].

Keywords: Laminar flow in 3-D boundary layers; Navier-Stokes equations; Mathematical method.

1 Introduction

The history of three dimensional (3-D) boundary-layer flow transition studies over the rotating disk is rather old and has a huge associated literature (see, for example, [2, 3, 4, 5, 6, 7, 8, 9, 10]). These studies served as the foremost model problem for the subsequent investigations of the 3-D boundary-layer flows over axisymmetric bodies of revolution. Although, both theoretically and experimentally, the case of a flow field structure of the laminar boundary-layer flow over rotating sphere had been greatly clarified in the investigations of [11, 12, 13, 14, 15, 16, 17]. The flow visualization studies led by Kohama and Kobayashi [18, 19, 20, 21, 22, 23, 24] were related to the transition of the laminar boundary-layer flow over rotating sphere and cone.

The theoretical studies of in refs [25, 26, 27, 28, 29, 30, 31, 32] are related to the transition phenomena of the laminar boundary-layer flow over various rotating geometries like disk, sphere and cone were carried out in such a way that the governing laminar flow equations were first derived using some appropriate coordinate system for each geometry. These laminar flow equations are actually a set of simultaneous 3-D nonlinear partial differential equations. These equations were solved using advanced numerical methods. Subsequently the perturbation equations that govern the transition of the laminar boundary-layer, were derived for each body. The solutions of the laminar flow equations are then used in solving the related perturbation equations for each body. Recently, in Refs [33, 34, 35], the authors used the techniques in the aforementioned investigations and successfully derived the laminar flow equations for the general family of rotating prolate spheroids and oblate spheroids. The solutions were then used in the transition analysis of the laminar boundary-layer flow over the general families of each type of spheroid.

2 The laminar boundary layer over a rotating paraboloid

In this section, we give the derivation of the steady boundary layer equations that gives the laminar flow profile over the outer surface of a paraboloid rotating in an otherwise still fluid. We can not fix an appropriate coordinate system to model the laminar flow equations within the boundary layer over rotating paraboloid in a consistent way to the work related to other geometries, for example, sphere, spheroids and cones. However, to our knowledge the only existing work for the boundary layer equations of rotating paraboloid is due to [1]. We reproduce these equations in this section using the same coordinate system with the intention to solve these by the series solution method. However, we found major inconsistency in the formulation of [1] and we will elaborate this in detail. In §3, we derive the laminar flow equations over rotating paraboloid in a similar way to P.

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D. Verma. In §4, we derive the series solution of the laminar flow equations of paraboloid in a similar way to that of [17,33]. In §5, we discuss the discrepancy in the formulation. We also show in detail how the numerical results of the series solution for the laminar flow equations of paraboloid in the coordinates of P. D. Verma [1] does not converge and that the results are not consistent with other geometries.

3 Formulation of laminar boundary layer flow over rotating paraboloid

A cartesian frame of reference is used, the paraboloid rotates with constant angular velocity $\Omega^\star$ about the z-axis. We use the following transformation of the coordinates system for paraboloid of revolution.

$$x^\star = l^\star \alpha\beta \cos\theta,\quad y^\star = l^\star \alpha\beta \sin\theta,\quad z^\star = \tfrac{1}{2} l^\star (\alpha^2 - \beta^2),$$

where α , β are non-dimensional variables, $l^\star$ is a dimensional quantity and θ is the variable through which the paraboloid rotates. Further, $0 \le \theta \le \pi$ and $\alpha, \beta \ge 0$. Note that this coordinate system (α, β, θ) is a simplified form of that used by [1]. Further, we note that the new coordinate system (α, β, θ) as introduced above is orthogonal. We used the continuity and full Navier Stokes equations derived by [33] in general orthogonal curvilinear coordinates and transform them into the aforementioned paraboloidal coordinate system. These equations are shown in Appendix 6 as equations (15)–(18).

By applying the Prandtl's boundary-layer assumptions to the continuity and Navier-Stokes equations in the paraboloidal coordinate system for a particular body surface $\beta = \beta_0$, we obtained the following dimensional laminar boundary layer flow equations for the paraboloid rotating in an otherwise still incompressible fluid,

$$\frac{U^\star}{l^\star\sqrt{\alpha^2+\beta_0^2}}\frac{\partial U^\star}{\partial \alpha} + \frac{W^\star}{l^\star\sqrt{\alpha^2+\beta_0^2}}\frac{\partial U^\star}{\partial \beta} - \frac{V^{\star 2}}{l^\star\alpha\sqrt{\alpha^2+\beta_0^2}} = \nu^\star \frac{1}{l^{\star 2}(\alpha^2+\beta_0^2)}\frac{\partial^2 U^\star}{\partial \beta^2} \tag{1}$$

$$\frac{U^\star}{l^\star\sqrt{\alpha^2+\beta_0^2}}\frac{\partial V^\star}{\partial \alpha} + \frac{W^\star}{l^\star\sqrt{\alpha^2+\beta_0^2}}\frac{\partial V^\star}{\partial \beta} + \frac{V^\star U^\star}{l^\star\alpha\sqrt{\alpha^2+\beta_0^2}} = \nu^\star \frac{1}{l^{\star 2}(\alpha^2+\beta_0^2)}\frac{\partial^2 V^\star}{\partial \beta^2} \tag{2}$$

$$\frac{1}{l^\star\sqrt{\alpha^2+\beta_0^2}}\frac{\partial U^\star}{\partial \alpha} + \frac{1}{l^\star\sqrt{\alpha^2+\beta_0^2}}\frac{\partial W^\star}{\partial \beta} + \left(\frac{\alpha}{l^\star(\alpha^2+\beta_0^2)^{\frac{3}{2}}} + \frac{1}{l^\star\alpha\sqrt{\alpha^2+\beta_0^2}}\right)U^\star = 0, \tag{3}$$

where $U^\star$, $V^\star$ & $W^\star$ are the dimensional velocities in the α, θ and β directions respectively. These equations are derived by assuming steady-state incompressible flow with the assumption $\delta^\star/r^\star \ll 1$. Further, we have applied the boundary-layer assumptions that $U^\star \sim O(1)$, $V^\star \sim O(1)$, $W^\star \sim O(\delta^\star)$ & $(\partial/\partial\alpha) \sim O(1)$ where $\delta^\star = (\nu^\star/\Omega^\star)^{1/2}$ is the boundary-layer thickness and $\nu^\star$ is the coefficient of kinematic viscosity. Using these in the continuity equation we can find that $\partial/\partial\beta^\star \sim O(\delta^{\star -1})$, and from the normal component of Navier-Stokes equations we find $P^\star = P^\star(\theta)$. Since, the paraboloid is rotating in an otherwise still fluid, $P^\star$=constant. Note that $r^\star$ is the local radius of the paraboloid and is given by $r^\star = l^\star\alpha\beta_0$. In the fixed frame of reference, the above equations are subject to the following boundary conditions,

$$\begin{aligned} U^\star = W^\star = V^\star - l^\star\alpha\beta_0\Omega^\star = 0 \quad &\text{on } \beta = \beta_0, \\ U^\star = V^\star = 0 \quad &\text{as } \beta \to \infty. \end{aligned} \tag{4}$$

In order to obtain the non-dimensional boundary-layer equations we scale the velocities on the equatorial surface speed of the paraboloid , as in equation below. The dimensionless Similarity variables are defined as

$$\begin{aligned} &U = \frac{U^\star}{l^\star\alpha\beta_0\Omega^\star}, V = \frac{V^\star}{l^\star\alpha\beta_0\Omega^\star}, W = \frac{W^\star}{(\nu^\star\Omega^\star)^{1/2}}, \\ &\eta = (\Omega^\star/\nu^\star)^{1/2} l^\star\beta_0\,(\beta-\beta_0)\,. \end{aligned} \tag{5}$$

In the above non-dimensionalization of the velocity components was indeed carried on the maximum speed at each local radius of the paraboloid. The non-dimensional normal η has been taken as the difference between the radii of the circles formed for $\alpha = \beta$ and $\alpha = \beta_0$. This non-dimensionalization of η is similar to that of [1]. This is indeed transforming the orthogonal coordinates (α, θ, β) into a non-orthogonal coordinate system (α, θ, η). In fact replacing $l^\star\beta$ by the radii differences of the two circles is what the formulation leads to inconsistency with the other related geometries. We will further discuss it in §5. We again note that our formulation is completely consistent with that of P. D. Verma [1].

The non-dimensional laminar flow equations of the boundary layer flow over the paraboloid, are shown as,

$$W\frac{\partial U}{\partial \eta} + \alpha U\frac{\partial U}{\partial \alpha} + U^2 - V^2 = \frac{\beta_0}{\sqrt{\alpha^2+\beta_0^2}}\frac{\partial^2 U}{\partial \eta^2} \tag{6}$$

$$W\frac{\partial V}{\partial \eta} + \alpha U\frac{\partial V}{\partial \alpha} + 2VU = \frac{\beta_0}{\sqrt{\alpha^2+\beta_0^2}}\frac{\partial^2 V}{\partial \eta^2} \tag{7}$$

$$\frac{\partial W}{\partial \eta} + \alpha\frac{\partial U}{\partial \alpha} + \left(\frac{\alpha^2}{\alpha^2+\beta_0^2} + 1\right)U = 0. \tag{8}$$

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Subject to the boundary conditions

$$U = W = V - 1 = 0 \text{ on } \eta = 0, \quad (9)$$
$$U = V = 0 \text{ as } \eta \to \infty. \quad (10)$$

These represent the non-slip boundary condition on the paraboloid surface and the quiescent fluid condition at the edge of the boundary layer.

4 Series solution for the laminar flow over rotating paraboloid

In this section, we derive the series solutions for the laminar flow equations (6)–(8) subject to the boundary conditions (9) and (10), using the same techniques in the existing literature.

To solve the above equations at some latitude, a series expansion solution in powers of α is sought of the form,

$$U = F_1 + \alpha^2 F_3 + \alpha^4 F_5 + ..., \quad (11)$$

$$V = G_1 + \alpha^2 G_3 + \alpha^4 G_5 + ..., \quad (12)$$

$$W = H_1 + \alpha^2 H_3 + \alpha^4 H_5 + \dots . \quad (13)$$

Here F_{n+1}, G_{n+1}, H_{n+1} are functions of the non-dimensional variable,
$\eta = (\Omega^\star/\nu^\star)^{1/2} l^\star \beta_0 (\beta - \beta_0)$.
The boundary conditions can be written as,

$$F_{n+1}(0) = H_{n+1}(0) = G_{n+1}(0) - 1 = 0,$$

$$F_{n+1}(\infty) = G_{n+1}(\infty) = 0, \quad (14)$$

for $n = 0, 2, 4, 6, \dots$.

The corresponding series solutions for the laminar flow equations of paraboloid are shown in Appendix 7 as equations (19)–(30). These equations were tried to solve numerically through the well known bvp 4c code in MatLab.

We have observed that the results converge for a very small range of parameters, however not shown here. Further, for most of the parameters involved the results do not converge. For a range of small number of parameters where the solution converges, are such that the value of $\eta \leq 4$ and this seems consistent with that of P. D. Verma results. However, these results are not physically sensible. The no-convergence of the laminar flow profiles for the broad range of parameters and where convergence observed but not sensible, we conclude that it is because of the inconsistency in the formulation of the governing laminar flow equations. We will further discuss this in §5.

5 Discussion

In this section we derived the full Navier-Stokdes equations in the paraboloidal coordinate system which is consistent with the work of [1]. We then showed the non-dimensional laminar flow equations in this coordinate system for the paraboloid. However, we can not fix the η and took it similar to the one used by P. D. Verma. It is showed as a difference of the radii of the local circles of the paraboloid. The local radii of the circles are indeed not normal at each α, θ of the paraboloid. This leads to a non-orthogonal system in η. This is inconsistent with the previous formulations in such geometries.

We suspect that the non-convergence of the laminar flow profiles of the series solutions for a broad range of the values of parameters involved is due to the incorrect use of η. Further, the convergent profiles for a very limited range of values of the parameters the results are not sensible. This lead us to conclude that a different coordinate system to model the laminar flow equations for paraboloid rotating in an otherwise still fluid, should be used and which should be orthogonal till the non-dimensional form of the laminar flow equations.

We at this moment could not sort out such a suitable coordinate system for paraboloid, however, it is observed in detail that the formulation of P. D. Verma is not consistent for the problem of the laminar flow equations for paraboloid.

6 Dimensional equations of rotating paraboloid

Now we derive the continuity and full Navier Stokes equations in the paraboloidal coordinate system as discussed in §3 from the continuity and full Navier Stokes equations in general orthogonal curvilinear coordinates [33]. We get the continuity equation as:

$$\frac{1}{l^\star\sqrt{\alpha^2+\beta^2}}\frac{\partial U^\star}{\partial \alpha} + \frac{1}{l^\star\sqrt{\alpha^2+\beta^2}}\frac{\partial W^\star}{\partial \beta} + \frac{1}{l^\star\alpha\beta}\frac{\partial V^\star}{\partial \theta} + \left(\frac{\alpha}{l^\star(\alpha^2+\beta^2)^{\frac{3}{2}}} + \frac{1}{l^\star\alpha\sqrt{\alpha^2+\beta^2}}\right)U^\star + \left(\frac{\beta}{l^\star(\alpha^2+\beta^2)^{\frac{3}{2}}} + \frac{1}{l^\star\alpha\sqrt{\alpha^2+\beta^2}}\right)W^\star = 0 \quad (15)$$

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The α component of the full Navier Stokes equations in paraboloidal coordinates is written as

$$\frac{\partial U^\star}{\partial t}+\frac{U^\star}{l^\star\sqrt{\alpha^2+\beta^2}}\frac{\partial U^\star}{\partial\alpha}+\frac{V^\star}{l^\star\alpha\beta}\frac{\partial U^\star}{\partial\theta}+\frac{W^\star}{l^\star\sqrt{\alpha^2+\beta^2}}\frac{\partial U^\star}{\partial\beta}$$
$$-\frac{{V^\star}^2}{l^\star\alpha\sqrt{\alpha^2+\beta^2}}+\frac{\beta U^\star W^\star}{l^\star(\alpha^2+\beta^2)^{\frac{3}{2}}}+\frac{\alpha{W^\star}^2}{l^\star(\alpha^2+\beta^2)^{\frac{3}{2}}}=$$
$$-\frac{1}{\rho l^\star\sqrt{\alpha^2+\beta^2}}\frac{\partial P^\star}{\partial\alpha}+\nu^\star\Bigg[\frac{1}{{l^\star}^2(\alpha^2+\beta^2)}\frac{\partial^2U^\star}{\partial\alpha^2}+\frac{1}{{l^\star}^2\alpha(\alpha^2+\beta^2)}\frac{\partial U^\star}{\partial\alpha}$$
$$+\left(\frac{3}{{l^\star}^2(\alpha^2+\beta^2)^{\frac{7}{2}}}-\frac{3(\alpha^2+\beta^2)}{{l^\star}^2(\alpha^2+\beta^2)^3}-\frac{1}{{l^\star}^2(\alpha^2+\beta^2)^2}+\frac{1}{{l^\star}^2\alpha^2(\alpha^2+\beta^2)}\right)U^\star$$
$$-\left(\frac{1}{{l^\star}^2\alpha^2\beta\sqrt{\alpha^2+\beta^2}}+\frac{1}{{l^\star}^2\alpha\beta\sqrt{\alpha^2+\beta^2}}\right)\frac{\partial V^\star}{\partial\theta}+\frac{2\beta}{{l^\star}^2(\alpha^2+\beta^2)^2}\frac{\partial W^\star}{\partial\alpha}$$
$$-\left(\frac{\alpha}{{l^\star}^2\beta(\alpha^2+\beta^2)^3}+\frac{\alpha}{{l^\star}^2\beta(\alpha^2+\beta^2)^{\frac{7}{2}}}\right)W^\star-\frac{2\alpha}{{l^\star}^2(\alpha^2+\beta^2)^2}\frac{\partial W^\star}{\partial\beta}$$
$$+\left(\frac{1}{{l^\star}^2\beta(\alpha^2+\beta^2)}+\frac{\beta}{{l^\star}^2(\alpha^2+\beta^2)^2}\right)\frac{\partial U^\star}{\partial\beta}+\frac{1}{{l^\star}^2\alpha^2\beta^2}\frac{\partial^2U^\star}{\partial\theta^2}+\frac{1}{{l^\star}^2(\alpha^2+\beta^2)}\frac{\partial^2U^\star}{\partial\beta^2}\Bigg] \tag{16}$$

The θ component of the full Navier Stokes equations in paraboloidal coordinates is written as

$$\frac{\partial V^\star}{\partial t}+\frac{U^\star}{l^\star\sqrt{\alpha^2+\beta^2}}\frac{\partial V^\star}{\partial\alpha}+\frac{V^\star}{l^\star\alpha\beta}\frac{\partial V^\star}{\partial\theta}+\frac{W^\star}{l^\star\sqrt{\alpha^2+\beta^2}}\frac{\partial V^\star}{\partial\beta}$$
$$+\frac{V^\star U^\star}{l^\star\alpha\sqrt{\alpha^2+\beta^2}}+\frac{V^\star W^\star}{l^\star\beta\sqrt{\alpha^2+\beta^2}}=-\frac{1}{\rho l^\star\alpha\beta}\frac{\partial P^\star}{\partial\theta}$$
$$+\nu^\star\Bigg[\frac{2}{{l^\star}^2\alpha^2\beta\sqrt{\alpha^2+\beta^2}}\frac{\partial U^\star}{\partial\theta}+\frac{1}{{l^\star}^2\alpha^2\beta^2}\frac{\partial^2V^\star}{\partial\theta^2}+\frac{1}{{l^\star}^2\alpha\beta^2\sqrt{\alpha^2+\beta^2}}\frac{\partial W^\star}{\partial\theta}$$
$$+\frac{1}{{l^\star}^2(\alpha^2+\beta^2)}\frac{\partial^2V^\star}{\partial\beta^2}+\frac{1}{{l^\star}^2\beta(\alpha^2+\beta^2)}\frac{\partial V^\star}{\partial\beta}-\left(\frac{1}{{l^\star}^2\beta^2(\alpha^2+\beta^2)}+\frac{1}{{l^\star}^2(\alpha^2+\beta^2)}\right)V^\star$$
$$+\frac{1}{{l^\star}^2(\alpha^2+\beta^2)}\frac{\partial^2V^\star}{\partial\alpha^2}+\frac{\alpha}{{l^\star}^2(\alpha^2+\beta^2)}\frac{\partial V^\star}{\partial\alpha}\Bigg] \tag{17}$$

The β component of the full Navier Stokes equations in paraboloidal coordinates is written as

$$\frac{\partial W^\star}{\partial t}+\frac{U^\star}{l^\star\sqrt{\alpha^2+\beta^2}}\frac{\partial W^\star}{\partial\alpha}+\frac{V^\star}{l^\star\alpha\beta}\frac{\partial W^\star}{\partial\theta}+\frac{W^\star}{l^\star\sqrt{\alpha^2+\beta^2}}\frac{\partial W^\star}{\partial\beta}$$
$$-\frac{\beta{U^\star}^2}{l^\star(\alpha^2+\beta^2)^{\frac{3}{2}}}+\frac{\alpha U^\star W^\star}{l^\star(\alpha^2+\beta^2)^{\frac{3}{2}}}+\frac{{V^\star}^2}{l^\star\beta\sqrt{\alpha^2+\beta^2}}=-\frac{1}{\rho l^\star\sqrt{\alpha^2+\beta^2}}\frac{\partial P^\star}{\partial\beta}$$
$$+\nu^\star\Bigg[\frac{2\alpha}{{l^\star}^2(\alpha^2+\beta^2)^2}\frac{\partial U^\star}{\partial\beta}-\left(\frac{\beta}{{l^\star}^2\alpha(\alpha^2+\beta^2)^2}+\frac{\beta}{{l^\star}^2\alpha(\alpha^2+\beta^2)^{\frac{7}{2}}}\right)U^\star$$
$$-\frac{2\beta}{{l^\star}^2(\alpha^2+\beta^2)^2}\frac{\partial U^\star}{\partial\alpha}+\frac{1}{{l^\star}^2\beta(\alpha^2+\beta^2)}\frac{\partial W^\star}{\partial\beta}+\Bigg(\frac{1}{{l^\star}^2(\alpha^2+\beta^2)^{\frac{7}{2}}}-\frac{3\alpha^2}{{l^\star}^2\beta(\alpha^2+\beta^2)^3}$$
$$-\frac{3\beta^2}{{l^\star}^2(\alpha^2+\beta^2)^3}-\frac{1}{{l^\star}^2(\alpha^2+\beta^2)^2}-\frac{1}{{l^\star}^2\beta^2(\alpha^2+\beta^2)}\Bigg)W^\star+\frac{1}{{l^\star}^2(\alpha^2+\beta^2)}\frac{\partial^2W^\star}{\partial\beta^2}$$
$$+\frac{1}{{l^\star}^2(\alpha^2+\beta^2)}\frac{\partial^2W^\star}{\partial\alpha^2}+\frac{1}{{l^\star}^2\alpha(\alpha^2+\beta^2)}\frac{\partial W^\star}{\partial\alpha}+\frac{1}{{l^\star}^2\alpha^2\beta^2}\frac{\partial^2W^\star}{\partial\theta^2}\Bigg] \tag{18}$$

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7 Details of the series solution for paraboloid

$$F_1^2 + H_1 F_1' - G_1^2 = F_1'' \tag{19}$$

$$4F_1F_3 + H_1F_3' + H_3F_1' - 2G_1G_3 - 1/2\beta_0^2\left(G_1^2 - H_1F_1' - F_1^2\right) = F_3'' \tag{20}$$

$$6F_1F_5 + 3F_3^2 + H_1F_5' + H_3F_3' + H_5F_1' - 2G_1G_5 - G_3^2 - G_1G_3/\beta_0^3 + 1/8\beta_0^4\left(G_1^2 - H_1F_1'\right) + 1/2\beta_0^2\left(H_1F_3' + H_3F_1'\right) + 2F_1F_3/\beta_0^2 - F_1^2/8\beta_0^3 = F_5'' \tag{21}$$

$$8F_1F_7 + 8F_3F_5 + H_1F_7' + H_3F_5' + H_5F_3' + H_7F_1' - 2G_1G_7 - 2G_3G_5 - 1/2\beta_0^2\left(4F_1F_5 + 3F_3^2 + G_3^2 + G_1^2 - H_1F_5' - H_3F_3' - H_5F_1'\right) - F_1F_3/2\beta_0^4 + F_1F_5/2\beta_0^2 + 1/16\beta_0^6\left(F_1^2 + H_1F_1'\right) - G_1G_5/\beta_0^2 + G_1G_3/4\beta_0^4 - 1/8\beta_0^4\left(H_1F_3' + H_3F_1'\right) = F_7'' \tag{22}$$

$$2F_1G_1 + H_1G_1' = G_1'' \tag{23}$$

$$4F_1G_3 + 2F_3G_1 + H_1G_3' + H_3G_1' + H_1G_1'/2\beta_0^2 + F_1G_1/\beta_0^2 = G_3'' \tag{24}$$

$$6F_1G_5 + 4F_3G_3 + 2F_5G_1 + H_1G_5' + H_3G_3' + H_5G_1' + 1/\beta_0^2\left(F_1G_3 + F_3G_1\right) - F_1G_1/4\beta_0^4 + 1/2\beta_0^2\left(H_1G_3' + H_3G_1'\right) - H_1G_1'/8\beta_0^4 = G_5'' \tag{25}$$

$$8F_1G_7 + 6F_3G_5 + 4F_5G_3 + 2F_7G_1 + H_1G_7' + H_3G_5' + H_7G_1' + 1/2\beta_0^2\left(H_1G_5' + H_3G_3' + H_5G_1'\right) - 1/8\beta_0^4\left(H_1G_3' + H_3G_1'\right) + H_1G_1'/16\beta_0^6 + 1/\beta_0^2\left(3F_1G_5 + 2F_3G_3 + F_5G_1\right) - 1/4\beta_0^4\left(2F_1G_3 + F_3G_1\right) + F_1G_1/8\beta_0^6 = G_7'' \tag{26}$$

$$F_1 + H_1' = 0 \tag{27}$$

$$3\beta_0^2F_3 + \beta_0^2H_3' + F_1 = 0 \tag{28}$$

$$5\beta_0^2F_5 + F_1F_5 + H_5' - F_1/\beta_0^2 + F_3 = 0 \tag{29}$$

$$7\beta_0^2F_7 + \beta_0^2H_7' - F_1/\beta_0^4 - F_3/\beta_0^2 + 2F_5 = 0 \tag{30}$$

8 Conclusion

The equations of fluid motion within the boundary layer can be simplified because of the layer's thinness, and exact or approximate solutions can be obtained in many cases. The intent of this manuscript is discuss the laminar flow profile over the outer surface of a paraboloid rotating in an otherwise still fluid which arises from the steady boundary layer equations. Moreover, the laminar flow equations for a rotating paraboloid is derived. Ultimately, we proved that the formulation of the laminar flow equations for paraboloid has a mathematical flaw which leads to the exciting work of P. D. Verma [1].

Conflict of Interest

The authors declare that there is no conflict of interest regarding the publication of this article.

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