Appl. Math. Inf. Sci. **6**, No. 1, 113-116 (2012)

Applied Mathematics & Information Sciences

An International Journal

Numerical Calculation of Rate-Distortion Function of Information Source

Qianzhao Lei

School of Physics and Electrical Engineering, Weinan Normal University, Weinan, China
Email Address:: leiqzh@126.com

Received: Received May 02, 2011; Revised July 25, 2011; Accepted September 12, 2011
Published online: 1 January 2012

Abstract: In order to maximize transmission efficiency, fully use available channels, we need to solve rate-distortion function of information source $R(D)$. For a given source, presented the distortion matrix, and set the accuracy ε, an iterative method can be used to get $R(D)$. Firstly, given the initial value of parameter S, which meaning the slope of the sought graph, matrix of the channel transition probabilities and the average distortion D with parameter S were calculated; then $D \leq D_0$ as fidelity criteria, whether $\left|D_{n+1} - D_n\right| \leq \varepsilon$ being true as iteration sentencing guidelines, iterative computation was done until the parameters of S close enough to 0. The rate-distortion curve simulated by iteration method fits well with the theoretical value, the simulation can provide theoretical guidance for determining the best encoding method.

Keywords: information source; average distortion; rate-distortion function; iterative method; channel transition probabilities

Introduction

Information transmitted in the channel is doubtlessly disturbed by noise, this view, Shannon even described it in his early foundation work about information theory, and Shannon's second theorem also pointed out that when the information transfer rate *R* is greater than the channel capacity, it is impossible for the channel to transfer it without distortion. For an actual communication system, information transfor rate *R* is often greater than the channel capacity; in addition, a certain amount of information will also be lost in carrying out data compression. So in real life, to ensure the reliability of communication can only be some degree of approximation, and too much emphasis on reliability, accuracy often requires paying the unbearable economic and technical cost.

All the solutions of these problems can be converted into the very solution: after setting an allowed distortion value, the main task is to find the best channel to get the smallest bit rate from information source to its destination, the very solution is seeking rate-distortion function R(D) of information source, presently, information rate-distortion theory is the theoretical basis of quantification, digital-to-analogue conversion (DAC), band and data compression, so solving IRDF is very important [1]. Although, by calculating average mutual information, both of rate-distortion function and channel capacity are got, but calculation of the later is much more complex; and its explicit expression is generally difficult to obtain, usually being parametric expression, calculating the parameter expression needs using iterative approximation method [2].

2 Mathematical model of information rate-distortion function

Random variables as distortion array $d(a_i,b_j)$, the value of limited distortion uses its mathematical expectation i.e. average distortion [3]

$$D=\sum_i \sum_j p(a_i)p(b_j \mid a_i)d(a_i,b_j) \quad (1)$$

For any communications system, the demanded is its average distortion degree D no more than the set allowed one [4]

$$D \le D_0 \quad (2)$$

This is fidelity criteria [5], where, D being the average distortion introduced when coding, D_0 for the given average distortion beforehand. When source $p(a_i)$ and the degree of each symbol distortion $d(a_i,b_j)$ are given, selecting different experimental channel $p(b_j \mid a_i)$ is equivalent using different coding method; from which, produced average distortion degree is different.

Given the source probability $p(a_i)$ and distortion function $d(a_i,b_j)$, to solve the rate-distortion function *R(D)* of information source, is to seek the minimum of average mutual-information function $I(X;Y)$, under conditions of the fidelity criteria and the taken experimental channel as the domain of $I(X;Y)$. From the receiving end, that means when satisfying fidelity criteria [6] by looking for some kind of encoding and decoding methods, the necessary lowest average information needs to reappear the news of information source, i.e.

$$R(D)=\min_{P_D} I(X;Y)$$
$$=\min_{p_{ji}\in P_D}\sum_i\sum_j p(a_i)p(b_j \mid a_i)\log\frac{p(b_j \mid a_i)}{p(b_j)} \quad (3)$$

where, P_D is the set of experimental channels that all meet the fidelity criteria, from P_D, a channel can be determined to make its corresponding average mutual-information $I(X;Y)$ reach the minimum. Compared with channel capacity calculation [6,7], we can see that this problem is actually the dual issue of it [8].

3 Numerical calculation of information rate distortion function

The flow chart of iterative operation is shown in Fig. 1, specifically as following: The first step, a group of negatives are set as S values, the negatives' absolute values decreasing gradually and tending to 0 (here setting 0.000005 to represent the approximation of 0). The array's first value is selected as the initial one, and uniform distribution chosen as the transition probabilities of experimental channel $p(b_j \mid a_i)$, and setting $n=1$.

The second step, $p_n(b_j)$ is calculated by using the following

$$p_n(b_j)=\sum_{k=1}^{r} p_n(b_j \mid a_k)p(a_k) \quad (4)$$

The third step is to put $p_n(b_j)$ into the formula $p_{n+1}(b_j \mid a_i)=\dfrac{p_n(b_j)\exp[Sd(a_i,b_j)]}{\sum_j p_n(b_j)\exp[Sd(a_i,b_j)]}$

(5)

from it calculated $p_{n+1}(b_j)$. The fourth step, by using $p_{n+1}(b_j \mid a_i)$ in formula (5), the average distortion D_{n+1} and information rate R_n can be calculated as

$$D_{n+1}=\sum_{j=1}^{s}\sum_{i=1}^{r} p(a_i)p_{n+1}(b_j \mid a_i)d(a_i,b_j), \quad (6)$$

$$R_n=\sum_{i=1}^{r}\sum_{j=1}^{s} p(a_i)p_n(b_j \mid a_i)\log\frac{p_n(b_j \mid a_i)}{p_n(b_j)} \quad (7)$$

The fifth step, if $\left|D_{n+1}-D_n\right| \le \varepsilon$ is satisfied, getting to step 6; otherwise, setting $n=n+1$, and returning to the second step, where ε is pre-determined accuracy.

The sixth step, setting $D(S)=D_n$, $R(S)=R_n$, choosing the next value of parameter S, the initial probability is chosen as uniform distribution; resetting $n=1$, and then jumping to the second step, until the slope is close enough to 0.

For any slope parameter *S*, the corresponding distortion and information rate can be got, the

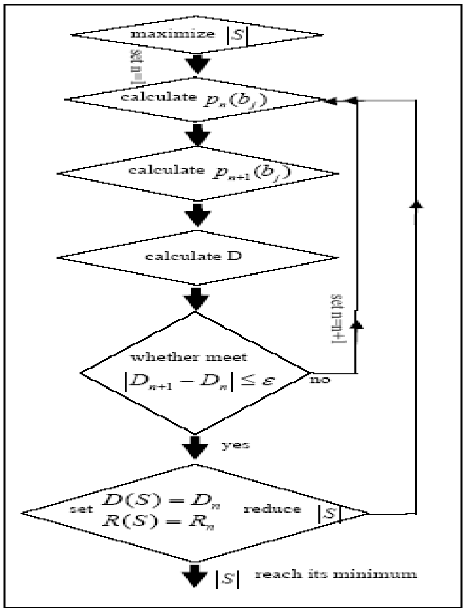

Fig. 1. Flow chart of iterative method used for calculating the rate-distortion function

selected number of S is the needed number of points to scan the *R(D)* curve. Changing probabilities of information source, curves of corresponding distortion function of different sources can be got similarly.

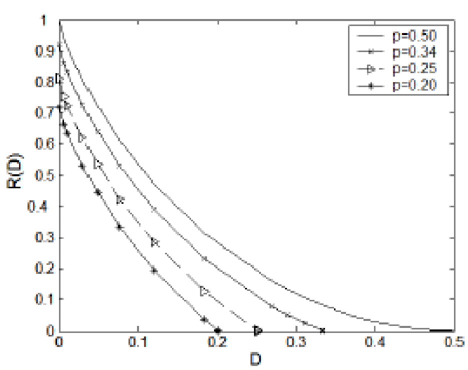

Fig. 2. curves of rate-distortion functions of different information sources

Equipped with binary symbols of information source, distortion function takes Hamming's [9] precision ε as $\varepsilon = 10^{-10}$, S values' range [-500- 5 -4.5 -3.5 -3 -2.5 -2 -1.5 -1 -0.9 -0.8 -0.7 -0.6 - 0.5 -0.4 -0.3 -0.2 -0.1 -0.05 -0.025 -0.01 -0.005 - 0.0025 -0.001 -0.0005 -0.00025 -0.0001 - 0.00005 -0.000025 -0.00001 -0.000005], the simulation results shown in Fig. 2, each curve of rate-distortion function *R(D)* corresponds probability of different information source.

Results analysis: after finding the rate-distortion function *R(D)*, which is no longer relevant to the experimental channel, though, the channel is used just before the finding of extreme value, *R(D)* will be only the parameters of characteristics of information source. The source different, so different is its *R(D)*. Once the probabilities of the source determined, the curve of *R(D* will be strictly monotonically decreasing when the average distortion increases; where the position of zero of average distortion, the maximum of each rate-distortion function.

4 Conclusions

Solving the rate-distortion function of information source is very complex; here used being simple information source, so used the simple distortion measure. Iterative calculation demands that both high precision and computing speed are to be considered. When precision is $\varepsilon \le 10^{-16}$, computing time will be too much longer to stand; however, when $\varepsilon \ge 10^{-10}$, the accuracy's very good, and so acceptable the speed. Selection of the maximum of parameter S (absolute minimum) could also directly affect calculation accuracy and iteration speed; here it takes 0.000005, the requirement basically can be met. By the way, since the selected number of parameter S is limited here, so the got curves are not very smooth.

Acknowledgment

This work is supported by Shaanxi Province Research Foundation of Natural Science (2011JM8020); it is also supported by Weinan Normal University Research Funding (11YKS016).

References

[1] S. F. Jiang, Approach to Estimating and Revising the Value of Information Rate-distortion Funtion R (D). *Journal of Shanghai University of Electric Power*, Volume 22, Issue 2 (2006), 187-191.

[2] R. E. Blahut, Computation of channel capacity and rate-distortion function. *IEEE Trans. Inform. Theory*, Volume 18, Issue 4 (Jul 1972), 460-473.

[3] S. T. Yang, M. J. Zhao, P. L. Qiu, On Wyner-Ziv Problem for general sources with average distortion criterion. *Journal of Zhejiang*

University (Science A: An International Applied Physics & Engineering Journal), Volume 8, Issue 8 (2007), 1263-1270.

[4] Y. L. Li, J. Zhao, The Calculating Method of Discrete Source of the Distortion-Rate Function. *Water Conservancy Science and Technology and Economy*, Volume 12, Issue 8 (2006), 564-566.

[5] A. Wyner, J. Ziv, The Rate-Distortion Function for Source Coding with Side Information at the Decoder. *IEEE TRANSACTIONS ON INFORMATION THEORY*, Volume 22, Issue 1 (JANUARY 1976), 1-10.

[6] C. S. Chen, The Analysis of Methods to Fig. out the Common Channel Capacity. *Computer & Telecommunication*, Issue 1 (2010), 67-70+72.

[7] Y. Guan, Some Alternate Algorithms for Computing the Capacity of Arbitrary Discrete Memoryless Channels. *COMN. ON APPL. MATH.AND COMPUT* . Volume 20, Issue 2 (2006), 19-27.

[8] L. Z. Li, Y. L. Yang, Calculation for Channel Capacity and Information Rate-Distortion Function of Telecontrol. *Automation of Electric Power Systems*, Issue 3 (1991), 9-43+15.

[9] J. X. Deng, Y. H. Kang, *Information theory and coding* (May, 2007) in press.

Q. Z. Lei is an associate professor in School of Physics and Electrical Engineering, Weinan Normal University, China. He received his master degree in radio physics and is working for PhD. His main research fields include signal processing and electromagnetic wave propagation in random medium. For the last 3 years, he has been author of more than 20 articles, of which about 10 articles were published in international journals, others in chinese core journals.