

2015

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Recommended Citation

Farag, Reda (2015) "LIMIT EQUILBRIUM SAFETY ANALYSIS OF PILE IN LATERAL SPREAD: THREE-LAYER," *Journal of Engineering Research*: Vol. 1: Iss. 1, Article 8.
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LIMIT EQUILIBRIUM SAFETY ANALYSIS OF PILE IN LATERAL SPREAD: THREE-LAYER

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ABSTRACT

The large amount of uncertainties in liquefaction-induced lateral spreading particularly leads to pile damage after many earthquakes. This paper presents a simplified safety analysis method of the single piles subjected to lateral spread in three layered soils. With the suggested method the analyst can get a quick estimate of the pile safety using few simulations of the pile-soil structural model. The method integrates a limit equilibrium-based finite element model with the response surface method as well as the first or the second order reliability method (FORM/SORM). In the finite element model, the assumptions of limit equilibrium method are simply applied. The soil is represented by 3D solid elastoplastic (Drucker-Prager failure criterion) while the pile is represented by an elastic 3D beam element. The simplified procedure not only estimates the safety but also determines the most sensitive basic pile and soil parameters affecting the response. Both serviceability and ultimate limit states are applicable. The accuracy of the proposed analytical method is evaluated against the Monte Carlo Simulation Method. At the beginning, two simple examples are used to verify, validate and further illustrate the method. After that, a more complex example is used to substantiate the superiority of the method. The effect of numerous uncertainties associated with the system is taken into account.

Keywords: Lateral spread, three layered soil system, limit equilibrium, safety

1 Introduction

Pile sited on poorly consolidated natural deposits or fills that are particularly susceptible to liquefaction and lateral spread, are particularly vulnerable to ground displacement. The safety against the potential lateral ground displacement is surrounded by a lot of uncertainties in the design and environment variables. Moreover, there is a considerable difficulty in obtaining undisturbed samples of loose granular (liquefiable) sediment for laboratory evaluation of constitutive soil properties. As a result, the uses of analytical methods, which rely on accurate measurements of constitutive properties, are usually limited to critical projects or to research. This complexity increases the value of the information those can be provided by the advanced analysis methods. Moreover, it had encouraged many simplified methods of analysis to be sought such as Beam-on-Winkler-Springs, Bradley, et al (2011), pseudo-static method, Tabesh and Poulos (2001) and the limit equilibrium method, Dobry, et al (2003).

Bradley, et al (2011), have proposed a probabilistic framework for Pseudostatic analysis of pile foundations in liquefied and lateral spreading soils. A pseudo-static method involves applying static displacements and forces to a typical beam-spring model, has been used in Monte Carlo simulation. It has been observed that the significant uncertainties in the Pseudostatic analysis result in significant uncertainty in both pile-head displacement and pile bending moment for a given level of input ground motion. Consequently it has been reported that decision making based on a single reference model is potentially erroneous.

In the present paper, the safety of pile under lateral spread in three layer soil profile is estimated using a proposed method. (Pile in two layer soil profile is manipulated in an ongoing paper). The method consists of a limit equilibrium-based finite element model coupled with the response surface method (RSM) as well as the first or second order reliability method (FORM/SORM). The limit state

functions are formulated in terms of the random variables that affect the structural design of pile under lateral spread. Using serviceability and ultimate limit state functions, first or second order reliability methods, FORM/SORM have been used for safety/reliability calculations. The results are verified using Monte Carlo Simulation Method. Furthermore, the most important random variables are determined.

2 Limit Equilibrium Approach

As the main objective of the present paper is to introduce an approach to compute a quick estimation of the safety information using not so many simulations of the structural model, a simple limit equilibrium approach (LE) that was introduced by Dobry, et al (2003), is used. The basic principle of this method is that the lateral forces applied by the soil against a given deep foundation are limited by the maximum pressure that can be applied by the liquefied soil (case of two-layer soil profile) or by the passive resistance of the non-liquefied layers (case of three-layer-soil profile). This method provides upper bound for the bending moments, displacements and rotations of the pile foundation, and is especially useful as an engineering tool for design and retrofitting decision. As reported by Dobry, et al (2003), the limit equilibrium, LE analysis has been used to explain the excellent performance of a bridge foundation to lateral spreading induced by the 1987 Edgecumbe earthquake in New Zealand; and by Japan Road Association Association (1996) and Yokoyama, et al (1997), to evaluate the response of bridge foundations to lateral soil deformation during the Kobe earthquake.

Relying typically on laboratory test result, Dobry, et al (2003) had implemented a calibration of the LE method for a wide range of single pile conditions in two- and three-layer soil profiles using centrifuge tests. A limit equilibrium method is proposed to evaluate the bending response of floating and end-bearing pile foundation subjected to actual lateral spreads in the field. Two backfigured values; soil pressure ($p = 10.3 \pm 1.5(15\%)$ kPa) and the rotational flexibility ($k_r = 5738$ kN m/rad) were used to drive analytical solutions for elastic beams (pile). Figure 1, shows a typically pile under lateral spread in two and three layer soil profile, while Figure 2-a shows the LE model of three-layered soils.

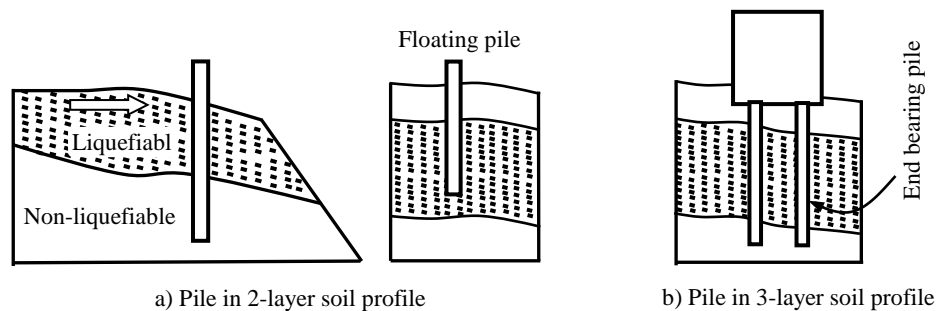


Figure 1. Pile foundation in laterally spreading ground Pile

2.1 Pile in Three-layer Soil Profile

In this system, the liquefiable soil layer is sandwiched between two non-liquefiable layers. As the pile in three-layer is more complicated, the LE assumes that, the displaced pile shape has double curvature and the maximum positive and negative moment occur simultaneously at the top and bottom of the liquefied layer. They occur due to the largest strength of the upper and lower layer. Moreover, the effect of the liquefied layer is negligible compared with the effect of the non-liquefied layers. The loading history involves two stages -in the upper non liquefied layer-; elastic and elastic perfectly plastic. The present work manipulates only the elastic stage.

3.1 Elastic stage

It is assumed in this stage -of low or small lateral displacement ($DH \cong 0.0 - 0.3 \text{ m}$)- that the soil in both top and bottom non-liquefiable layers can be represented by rotational spring, $k_r = 5738 \text{ kN m/rad}$, as shown in Figure 2-a. Moreover, the values of moment at top and bottom boundaries of the liquefied layer are equal, $M_t = M_b$

$$M_t = \frac{D_H}{L_{Li q} (L_{Li q} / 6EI + 1/k_r)} \tag{1}$$

Where L_{liq} , the thickness of the liquefiable soil layer; E, I are the pile elastic modulus and moment of inertia, respectively. The other variables are defined before.

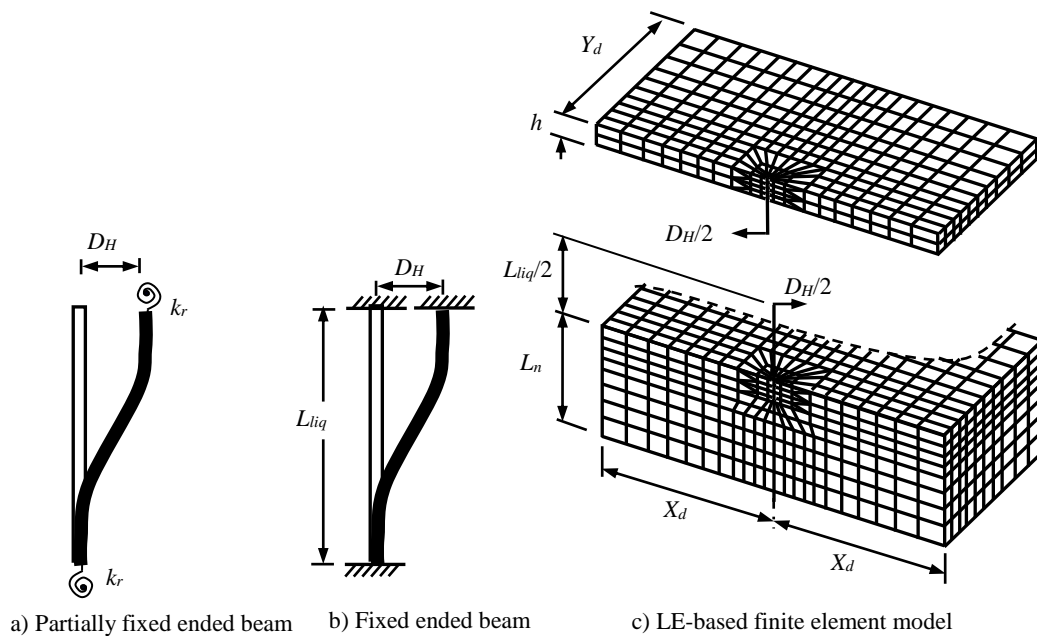


Figure 2. Models of 3-Layer

3.1.1 Simplified elastic beam relation

If the top and bottom layers are assumed to be rock and the pile is completely fixed in these two layers, as shown in Figure 2-b, then, the above equation is reduced to

$$M_t = M_b = 6EID_H / L_{Li q}^2 \tag{2}$$

4 . Proposed LE-Based FE Model

As mentioned above, two backfigured variables; pressure and the rotational flexibility are used to derive analytical solutions for elastic beams. In the present paper, a limit equilibrium -based finite element model (LEFE) is proposed. In this model, the non-liquefied layer is represented by a three dimensional nonlinear elasto-plastic (Drucker-Prager) elements, while the pile is represented by elastic 3D beam elements. As, the system is assumed to be symmetrical about the mid thickness of the liquefied layer, one half is used in the analysis. Instead of displacement of the top non-liquefied soil layer by displacement D_H , $D_H/2$ is applied to the beam element of each half, as shown in Figure

2-c. It should be mentioned that, the Drucker-Prager element is represented by the angle of internal friction (ϕ); soil cohesion (c); elastic modulus (E_s) and the soil density (γ_s). The standalone FE code, COSMOS/M Structural (2000) is used.

5 . Response Surface Method

Coupling the MCS Method with pile under lateral spread is often prohibited by too long simulations. The response surface method (RSM) is commonly used to approximately generate expressions for the performance functions. Then, FORM/SORM integrated with the RSM to evaluate the safety. The RSM can be found in the literature, Haldar and Mahadevan (2000a). First order polynomial is often used to perform preliminary analysis, Eq. (3). While Second-order polynomial is generally used to represent a response surface, Eq. (4). Mathematically, it can be expressed as:

$$\hat{g}(\mathbf{X}) = b_0 + \sum_{i=1}^n b_i X_i \quad (3)$$

$$\hat{g}(\mathbf{X}) = b_0 + \sum_{i=1}^n b_i X_i + \sum_{i=1}^n b_{ii} X_i^2 \quad (4)$$

where X_i ($i = 1, 2, \dots, k$) is the i^{th} random variable, and b_0 , b_i , b_{ii} , and b_{ij} are unknown coefficients to be determined from the deterministic analyses of the problem at specific data points, commonly known as experimental sampling points. The number of unknown parameters are $pp = (k+1)(k+2)/2$, for Eq. (3). The sampling points are generally selected using some multiple of the standard deviation of the random variables according as:

$$X_i = X_i^C \pm h_i \sigma_{x_i} \quad i = 1, 2, \dots, k \quad (5)$$

where X_i is the coded i^{th} variable, X_i^C and σ_{x_i} are the coordinates of the centre point and the standard deviation of a random variable X_i , respectively; h_i is an arbitrary factor that defines the experimental region, and k is number of random variables in the formulation.

Selection of the center point around which the sampling points are selected is the next task in RSM. The initial center point x_{c_1} is selected to be the mean values of the random variable X_i 's. Then, using the values of $g(\mathbf{X})$ obtained from the deterministic FEM evaluations for all the experimental sampling points around the center point, the response surface $\hat{g}_1(\mathbf{X})$ can be generated explicitly in terms of the random variables \mathbf{X} . Once a closed form of the limit state function, $\hat{g}_1(\mathbf{X})$, is obtained, the coordinates of the checking point x_{D_1} can be estimated using FORM/SORM. The actual response can be evaluated again at the checking point x_{D_1} , i.e., $g(x_{D_1})$ and a new center point x_{c_2} can be selected using linear interpolation from the center point x_{c_1} to x_{D_1} such that $g(\mathbf{X}) = 0$. This iterative strategy can be repeated until convergence is met.

6 . Soil-Pile Statistical Properties

In the present work, piles of reinforced concrete or polyetherimide ULTEM 1000, -used in centrifuge tests- are used in the analysis. The pile modulus of elasticity, (E), the cross sectional area of the pile expressed in the external radius (r) and thickness (t) are considered to be random variables. Besides the pile unit density (γ) and Poisson's ratio (ν) as in (JCSS)Joint (2000). The modelling of

uncertainties in geotechnical soil properties are widely reported in the literature. In the present study, the soil elastic modulus (E_s), the cohesion strength (c), the angle of internal friction (ϕ), the soil unit density (γ_s) and the soil Poisson's ratio (ν_s) are considered to be random variables (JCSS)Joint (2006). The uncertainty in the lateral displacement depends on the uncertainties in both soil properties and the earthquake characteristics including accelerations, time histories, duration, etc. In the present work, the lateral displacement is assumed to follow the probability distribution of extreme value Type 1 (EV-I). The statistical characteristics are gathered from the literature for each example.

7. Numerical Examples

The proposed method is demonstrated through the following simple examples. The examples are; Simplified Elastic Beam Model; Centrifuge Test Model and Limit Equilibrium Finite Element model. The superiority of the suggested methods is illustrated in example 3. The MCS Method is used for the sake of verification as it is seen next.

7.1 Example 1: Simplified Elastic Beam Model

An assumed reinforced concrete pile driven in a liquefiable layer sandwiched between two non-liquefiable layers is considered. The thickness of the liquefiable soil layer $L_{liq}=7.00$ m, and the top layer is subjected to lateral displacement $D_H = 5$ cm. The statistical description of the uncertainties associated with the random variables is listed in Table 1.

Assuming f is the pile flexural strength, the flexural limit state is:

$$g(\mathbf{X}) = f - M_t \times r / I = f_c - 6EID_H / L_{Liq}^2 \times r / I \quad (6)$$

Table 1: Statistical characteristic of random variables - Example 4

	Random variables	Symb ol	Distributi on	Nominal	Mean	COV	Ref.
1	Lateral spread	D_H	EV-I	0.03 m	0.03 m	0.20	*
2	Radius	r	LN	0.15 m	0.15	0.10	Bednar (1986)
3	Length	L_{Liq}	N	7.00 m	7.00	0.04	*
4	E-modulus of R.C.	E	LN	2.0×10^7 kN/m ²	2.01×10^7	0.18	*
5	R.C. strength	f_c	LN	22500 kN/m ²	22500	0.15	Ellingwood (1980)

* Data not available. Assumed parameters are based on engineering judgment.

Table 2: Results of reliability analysis- Example 4

		Variables sensitivities					β	P_f	function calls no.
		f_c	D_H	r	E	L_{Liq}			
1	Monte Carlo						2.212	1.35×10^{-2}	10^5
2	SORM	0.421	- 0.665	- 0.267	- 0.504	0.231	2.209	1.36×10^{-2}	1
3	i) Response surface Quadratic polynomial	0.599	- 0.396	- 0.535	- 0.401	0.197	1.845	3.25×10^{-2}	9

The safety index using Monte Carlo simulation and SORM, β -MCS and β -SORM equal 2.212 and 2.209, as listed in Table 2. Using the response surface method, the safety index, β -index = 1.845, this value is 16.5% less than that of Monte Carlo simulation method. The sensitive random variables are; lateral displacement, pile radius and the pile E-modulus with relative importance; 39.6%, 53.5% and 40.1% respectively, as listed in Table 2.

7.2 Example 2: Simplified Elastic Beam Model

A 10 m soil deposit and pile length is studied in this example. The deposit has 6 m layer liquefiable sand encased between 2 m top and 2 m bottom of non-liquefiable soil. The top layer is subjected to lateral displacement $D_H = 0.15$ m. The pile has a circular section of radius 30 cm and a bending stiffness $EI=8000$ kNm². This example is the actual model of a centrifuge test (model 1) in Dobry, et al (2003). The pile is manufactured of polyetherimide ULTEM 1000. Assuming that the modulus of elasticity and the flexural strength $E= 3300$ and $f = 160$ Mpa, respectively, the pile thickness is found to be $t = 3.4$ cm, (ULTEM ® PEI Resin Product Guide Eng/6/2003 CA).

As this model often simulate a reinforced concrete pile, the value of $f_c = 22500$ kN/m² is used in the limit state formulation and in the subsequent analysis. The values of the random variables as well as their statistical properties are gathered from the literature for both the pile and soil and listed in Table 3.

i) Response surface solution:

The analysis is started performing reliability analysis using the linear polynomial, it is observed

Table 3: Statistical Characteristic of Random Variables - Example 5

	Random variables	Sym.	Dist.	Nominal	Mean	Bias	COV	Ref.
1	Lateral displacement	D_H	EV-I	0.15 m	0.15	1.0	0.20	*
2	Radius	r	LN	0.30 m	0.30	1.0	0.10	Bednar (1986)
3	Thickness	t	LN	3.4 cm	3.4	1.0	0.05	Bednar (1986)
4	Length	H_{Liq}	N	6.00 m	6.00	1.0	0.04	*
5	Elastic modulus of pile	E	LN	3300 Mpa	3300	1.0	0.06*	*
6	Rotational spring	k_r	LN	5738 kN m/rad	5738	1.0	0.21*	Dobry, et al (2003)
7	Flexural strength	f_c	LN	22500 kN/m ²	22500	1.0	0.15*	Joint (2000)

* Data not available. Assumed parameters are based on engineering judgment.

Table 4: Results of Reliability Analysis - Example 5

		Variables sensitivities							β	P_f	no. of calls
		f_c	D_H	r	k_r	L_{Liq}	t	E			
1	i) Response surface										
	First order polynomial	0.424	-0.729	0.433	-0.282	0.115	0.080	-0.051	2.521	5.84×10^{-3}	13
2	Second order	0.433	-0.755	0.396	-0.251	0.119	0.082	-0.051	2.601	4.64×10^{-3}	13
3	Scheme 0	0.501	-0.747	0.219	-0.295	0.194			2.666	3.83×10^{-3}	9
4	ii) Explicit limit state										
	MCS-6vars								2.685	3.63×10^{-3}	10^5
5	SORM-6vars	-0.459	-0.773	0.199	-0.324	0.187	-0.064	0.091	2.689	3.56×10^{-3}	1

that, E and t can be considered as deterministic (their sensitivities are small ($< 8\%$)), reducing the number of variables to four variables as shown in row number 1 of Table 4. Then applying the quadratic polynomial, the safety index, β -index=2.666. The most important variables are found to be; lateral displacement (D_H), pile radius (r), the rotational flexibility (k_r) and the thickness of the liquefied layer with relative importance; 74.7%; 21.9%; 29.5% and 19.4%, respectively.

ii) Explicit Limit state

Using the explicit limit state, it is found that the Monte Carlo simulation safety index and the second order safety index β -MCS=2.685 and β -SORM=2.689, respectively. The safety index of response surface is 0.7% less than that value of Monte Carlo Simulation. The sensitivities of the variables can be also compared. For example, the sensitivity of the lateral displacement is; 0.755 and 0.773 for the response surface methodology and the explicit limit state, respectively. It can be noted that the most important variables are the same in both cases.

7.3 Example 3: Simplified Elastic Beam Model

The pile in the above example (example 2), is subjected to 20 cm lateral displacement. The suggested LE-based FE model, (LEFE), is used in the analysis. Table 5 shows the statistical properties of the concerned variables. The stochastic model involves 11 variables and the concrete flexural strength is assumed to be 3750 kN/m².

The analysis is started by setting up the FE model. In the model $D_H/2$ applies to each half. $M_b = 40.4$ kN.m. This value is 6.2 % less than the value of LE approach, Dobry, et al (2003). Therefore, the uncertainty in the model bending moment is taken into account through a model factor α_b .

Following the same procedure as in the previous example, a preliminary analysis is performed using first order polynomial. The large number of variables is reduced to 3 variables; pile radius, (r), the

lateral displacement, (D_H), and the pile thickness, (t) with sensitivities; 0.065; 0.827 and 0.082; respectively. Then, the reliability analysis is preformed using the quadratic polynomial.

		Random variables	Sym.	Dist.	Nominal	Mean	Bias	COV	Ref.
1	Load	Lateral displacement	D_H	EV-I	0.2 m	0.2	1.0	0.25*	
2	Pile	Pile Flexural modulus	E	LN	3300 Mpa	3300	1.0	0.06*	
3		Poisson's ratio	ν	LN	0.2	0.2	1.0	0.10	*
4		Density	γ	N	16 kN/m ³	16	1.0	0.10	*
5		Radius	r	LN	0.30 m	0.30	1.0	0.10	Bednar (1986)
6		Thickness	t	LN	3.4 cm	3.4	1.0	0.05	Bednar (1986)
7		Length	L_{Liq}	N	6.00 m	6.00	1.0	0.04*	
8	Top layer	Soil E-modulus	E_s	LN	1500 kN/m ²	1725	1.15	0.21*	Joint (2006)
9		Friction angle	ϕ	LN	34.5°	35.535	1.03	0.20	Joint (2006)
10		Cohesion strength	c	LN	5.1 kN/m ²	5.61	1.10	0.37	Joint (2006)
11		Poisson's ratio	ν_s	LN	0.30	0.30	1.0	0.10	Joint (2006)
12		Soil density	γ_s	LN	17 kN/m ²	17	1.0	0.10	Joint (2006)
13		Flexural strength	f_c	LN	3750 kN/m ²	3750	1.0	0.15	Joint (2006)
14	Model factor	Bending moment	α_b	N	1.00	1.00	1.00	0.10	*

The safety index is found to be β -index = 1.618. The relative importance of the above mentioned three important variables are; 48.5%, 29.6% and 9.5%, respectively. These safety results are listed in Table 6.

In this example, more variables related to pile and soil can be taken into account. The number of the considered variables is 14 variables as listed in Table 5.

The proposed method lasts little time. However, it is approximated as it is based on the limit equilibrium method. So, it is appropriate for the preliminary analysis or the not critical projects.

Table 5: Statistical characteristic of random variables - Example 6

*Data not available. Assumed parameters are based on engineering judgment

Table 6: Results of reliability analysis- Example 6

		Variables sensitivities					β	P_f	function calls no.
		f	α_b	r	D_H	t			
1	First order	0.459	-0.284	-0.065	-0.827	0.082	2.970	9.61×10^{-2}	25
		0.424	-0.267	-0.820	-0.262	0.093	2.404	8.11×10^{-3}	7
	Second order								
2	Scheme 0	0.692	-0.434	-0.485	-0.296	0.095	1.618	5.28×10^{-2}	7

8. Conclusion

The present paper introduces a simplified method to get a quick estimate value of the safety of pile under lateral spread. In the suggested method a limit equilibrium method based on static equilibrium and kinematic consideration has been coupled with the classical response surface method and the second order reliability method. The method is applied to single pile in three-layer soil profiles. Furthermore, the variables sensitivities are identified. This approximated method extracts quick estimate of reliability information using tens of limit equilibrium-based finite element model. For the three considered examples, it is found that the most important variables are; the pile radius and the lateral displacement.

1. Nomenclature

$b_0, b_i, b_{ij},$ and b_{ij}	Unknown coefficients of a polynomial to be determined.
C	The soil cohesion strength.
D_H	The maximum liquefaction-induced lateral displacement.
D_p	The pile diameter.
E, E_s	The young's modulus of pile material and soil, respectively.
EI	The flexural rigidity of the pile.
f, f_c	The flexural strength of the pile material and reinforced concrete, respectively.
$g_f(x), g_{ux}(x)$	Explicit expression of flexural and drift limit state function, respectively
$\hat{g}(\mathbf{X})$	Response surface.
$\hat{g}_b(\mathbf{X}), \hat{g}_{ux}(\mathbf{X})$	The response surface function of moment and drift, respectively.
h_i	A chosen factor that defines the experimental/sample region.
L_{liq}	The thickness of the liquefiable soil layer
I	Second moment of inertia of the pile
K	The number of random variables in the formulation.
k_r	The rotational stiffness of the base.
M	The bending moment.
MCS	Monte Carlo Simulation.
P	Soil pressure.
Pp	The numbers of coefficients necessary to define a polynomial.
P_f	The probability of failure.
R	The pile radius.
T	The pile thickness.
u_x	The pile head deflection.

X_{all}	The allowable drift.
\mathbf{x}_{c_2}	Second center point.
\mathbf{x}_{D_i}	The coordinates of the checking point.
$X_i (i = 1, 2, \dots, k)$	The i^{th} random variable
X_i^C	The coordinates of the centre point, i .
β	β -index =Reliability index.
ε	Pre-selected convergence criterion
σ_{x_i}	The standard deviation of a random variable X_i .
ν, ν_s	Concrete and soil Poisson's ratio, respectively.
γ, γ_s	Unit density of reinforced concrete and soil, respectively.
ϕ	The angle of internal friction.

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