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Pancharatnam Phase of Two Two-Level Atoms Interacting with a Time-Dependent Cavity Field

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Abstract: The considered system is two independent two-level atoms interacting with a two-mode electromagnetic field (EMF), taking into account the coupling function between atoms and field to be time-dependent. The system's corresponding state function is obtained when the two atoms are initially made in superposition of states and the coherent state of the field is taken. By increasing the value of kerr-like medium parameter χ and time varying parameter ε enhances the number of artificial phase jumps and the chaotic behavior period. We discuss the effect of time-variation coupling and kerr-like medium on atomic inversion, Pancharatnam phase, Husimi Q function and correlation function.

Keywords: Pancharatnam phase ,Husimi Q function, two two-level atoms, correlation function.

1 Introduction

The work in the area of quantum information gained much attention recently, where many of the inventions was reported in the recent years both in the quantum computation and quantum communications [1,2,3,4,5,6,7,8]. Also, several works were have been done to increase the time of the data storage in the quantum memory as in refs [9,10]. The geometric phase one of the important properties which should be investigated for the quantum systems to check the stability of the system [11,12]. The experimental and theoretical studies of the geometric phases have been subjected to extensive research [13,15,14]. The total phase obtained from a corresponding wave function of quantum system which either be noncyclic or cyclic evolution includes two partitions; the geometric phase partition and the dynamical phase partition.the geometric phase relies on the selected trajectory in the space spanned by whole the expected quantum states for the considered system. However, the dynamical phase of The system's corresponding state function is comprehended to be Hamiltonian dependent . The geometric phase was detected by Berry [16,17] for the evolution of cyclic adiabatic , has led to various generalizations. Extension to the case of the non-adiabatic cyclic was explored by Aharonov and Anandan [18]. Pancharatnam ([19], reprinted in [20]) the polarization state of the light beam phase is changed. His remarkable result, when displayed uniformly, the beam which is reinstated to its initial polarized state through two average polarizations. He proved that the acquired phase doesn't reinstate to its initial value but grows by $-\frac{1}{2}\Omega$, wherever Ω represents the area spanned on the Poincaré sphere [21,22]. The influence of Kerr-like medium on Pancharatnam phase during a non-degenerate Raman two-photon process is investigated [23]. The effect of Stark shift and Kerr-like medium on Pancharatnam phase during a non-degenerate Raman two-photon process is investigated [23]. Controlling the atomic dynamics in presence of emfs instead of estimating the dynamical population is studied [24].

The derivation of the time-dependent solution of the master equation for the reduced density operator of a coherent laser driving a damped two level atom is given [25]. The mathematical model of two independent two-level atoms with degenerate two-photon transitions

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interacting with a single-mode of EMF in the presence of parametric amplifiers has been well investigated [26]. A non-linear Tavis-Cummings model, particularly,the interaction between two independent two-level atoms and two modes of EMF with Stark shift effect is analyzed [27, 28, 29, 30]. Sub-Poissonian and population inversion excitation statistics of N two-level atoms in the setting of collective resonance fluorescence is described [31, 32, 33, 34, 35, 36].

In our paper we take a theoretical model of non-linear between two independent two-level atoms-field interaction with emf. By obtaining the corresponding wave function, after adopting a suitable and physically acceptable approximation to the coupled differential equations, obtained from time-dependent Schrödinger equation and study some aspects that describes the dynamical properties of the model.

This paper is prepared: We derive the corresponding wave function in theoretical (mathematical) model in section (2). We investigate the influence of the system parameters on the atomic inversion in section (3) . In section (4), the evolution of Pancharatnam phase is explained. In section (5), the Husimi Q function is studied. In section (6), the second-order correlation is explored. Finally, in section (**??**), a conclusion is presented.

2 Theoretical model

The considered system is two independent two-level atoms interacting with a two-mode EMF where the coupling is time-dependent. The two-level atom is expressed as excited, and ground levels by $|g\rangle$ and $|e\rangle$ with energies ω_1, and ω_2. Also, we consider that before the interaction the EMF passes through a Kerr-like medium and the atoms is effected by Stark shift contributions by applying an external electric field. Under these postulates, the effective Hamiltonian in the rotating wave approximation (RWA) can be written as ($\hbar = 1$):

$$\hat{H}_{eff} = \hat{H}_{A+F} + \hat{H}_{kerr} + \hat{H}_{stark} + \hat{H}_I, \tag{1}$$

where

$$\hat{H}_{A+F} = \sum_{j=1}^{2} \omega_j \hat{a}_j^{\dagger} \hat{a}_j + \frac{1}{2}\sum_{j=1}^{2} \Omega_j \hat{\sigma}_z^j, \tag{2}$$

$$\hat{H}_{kerr} = \sum_{j=1}^{2} \chi_j \hat{a}_j^{\dagger 2} \hat{a}_j^2, \tag{3}$$

$$\hat{H}_{stark} = \sum_{j=1}^{2} \hat{a}_j^{\dagger} \hat{a}_j \left(\beta_j |e_j\rangle\langle e_j| + \gamma_j |g_j\rangle\langle g_j|\right), \tag{4}$$

and

$$\hat{H}_I = \sum_{j=1}^{2} \lambda_j(t)(\hat{a}_j^{\dagger 2}\hat{\sigma}_-^j + \hat{a}_j^2 \hat{\sigma}_+^j). \tag{5}$$

wherever operators $\hat{a}_j \left(\hat{a}_j^{\dagger}\right)$ is the annihilation (creation) operator where ω_j is the frequency of field, they follow the relation $[\hat{a}_\ell, \hat{a}_s^{\dagger}] = \delta_{\ell s}$. χ_j is the dispersive parts of the non-linearity of the Kerr-like medium. $\lambda_j(t) = \bar{\lambda}_j \cos(\varepsilon t)$ is the coupling function between the atom and field-mode with $\bar{\lambda}_j$ is coupling constant and ε is coupling time-modulation parameter. β_j and γ_j are the effective Stark shift parameters. So, the state vector is proposed to take the following formula:

$$\begin{aligned}|\psi(t)\rangle = \sum_{m_1=0}^{+\infty}\sum_{m_2=0}^{+\infty} q_{m_1} q_{m_2} \Big[& A(m_1,m_2,t)e^{-i\bar{\alpha}_1 t}|e_1,e_2,m_1,m_2\rangle \\ & + B(m_1,m_2,t)e^{-i\bar{\alpha}_2 t}|e_1,g_2,m_1,m_2+2\rangle \\ & + C(m_1,m_2,t)e^{-i\bar{\alpha}_3 t}|g_1,e_2,m_1+2,m_2\rangle \\ & + D(m_1,m_2,t)e^{-i\bar{\alpha}_4 t}|g_1,g_2,m_1+2,m_2+2\rangle \Big].\end{aligned} \tag{6}$$

where q_{m_j} describes the initial amplitude of the state of the number $|m_j\rangle$, the functions $A(m_1,m_2,t)$, $B(m_1,m_2,t)$, $C(m_1,m_2,t)$ and $D(m_1,m_2,t)$ are the amplitudes of the probability. $\bar{\alpha}_\ell$ ($\ell = 1,2,3,4.$), and the detuning parameter are given as:

$$\bar{\alpha}_1 = \omega_1 m_1 + \omega_2 m_2 + \frac{\Omega_1}{2} + \frac{\Omega_2}{2}, \tag{7}$$

$$\bar{\alpha}_2 = \omega_1 m_1 + \omega_2 (m_2+2) + \frac{\Omega_1}{2} - \frac{\Omega_2}{2}, \tag{8}$$

$$\bar{\alpha}_3 = \omega_1 (m_1+2) + \omega_2 m_2 - \frac{\Omega_1}{2} + \frac{\Omega_2}{2}, \tag{9}$$

$$\bar{\alpha}_4 = \omega_1 (m_1+2) + \omega_2 (m_2+2) - \frac{\Omega_1}{2} - \frac{\Omega_2}{2}, \tag{10}$$

$$\Delta = \Omega_1 - 2\omega_1 = \Omega_2 - 2\omega_2. \tag{11}$$

In this study, the coherent state, $|\alpha_j\rangle$ of the laser field is counted, where

$$q_{m_j} = \exp\left(-\frac{|\alpha_j|^2}{2}\right)\frac{\alpha_j^{m_j}}{\sqrt{m_j!}}, \tag{12}$$

in which $\bar{m}_j = |\alpha_j|^2$ the number of the mean photon of the mode j. By applying $i\frac{\partial}{\partial t}|\psi(t)\rangle = \mathscr{H}_{eff}|\psi(t)\rangle$ which is referring to the Schrödinger equation in time-dependent form, the system of the non-linear differential equations is obtained as follow:

$$\begin{aligned} i\dot{A}(t) &= \Gamma_1 A(t) + \tilde{g}_1 e^{i\Delta t} C(t) + \tilde{g}_2 e^{i\Delta t} B(t), \\ i\dot{B}(t) &= \Gamma_2 B(t) + \tilde{g}_1 e^{i\Delta t} D(t) + \tilde{g}_2 e^{-i\Delta t} A(t), \\ i\dot{C}(t) &= \Gamma_3 C(t) + \tilde{g}_1 e^{-i\Delta t} A(t) + \tilde{g}_2 e^{i\Delta t} D(t), \\ i\dot{D}(t) &= \Gamma_4 D(t) + \tilde{g}_1 e^{-i\Delta t} B(t) + \tilde{g}_2 e^{-i\Delta t} C(t). \end{aligned}$$

where

$$k_1 = \Gamma_1 + \Delta - \varepsilon, \tag{13}$$

$$k_2 = \Gamma_4 - \Delta + \varepsilon, \tag{14}$$

$$\Gamma_1 = \chi_1 m_1(m_1-1)+\chi_2 m_2(m_2-1)+\beta_1 m_1+\beta_2 m_2, \quad (15)$$
$$\Gamma_2 = \chi_1 m_1(m_1-1)+\chi_2(m_2+1)(m_2+2)+\beta_1 m_1+\gamma_2(m_2+2), \quad (16)$$
$$\Gamma_3 = \chi_1(m_1+1)(m_1+2)+\chi_2 m_2(m_2-1)+\gamma_1(m_1+2)+\beta_2 m_2, \quad (17)$$
$$\Gamma_4 = \chi_1(m_1+1)(m_1+2)+\chi_2(m_2+1)(m_2+2)+\gamma_1(m_1+2)+\gamma_2(m_2+2), \quad (18)$$

$$v_1 = \frac{\bar{\lambda}_1}{2}\sqrt{\frac{(m_1+2)!}{n_1!}}, \quad (19)$$
$$v_2 = \frac{\bar{\lambda}_2}{2}\sqrt{\frac{(m_2+2)!}{m_2!}}, \quad (20)$$
$$\tilde{g}_\ell = 2v_\ell\cos(\varepsilon t). \quad (21)$$

There exist exponential terms with two different powers in the system of the non-linear differential equations , $e^{\pm i(\Delta+\varepsilon)t}$ and $e^{\pm i(\Delta-\varepsilon)t}$. Approximately, we can ignore the counter oscillating terms $e^{\pm i(\Delta+\varepsilon)t}$. This approximation appears like the RWA and is accepted physically for numerous models [37]. So, the differential equations can be recomposed to be

$$i\dot{A}(t) = \Gamma_1 A(t)+v_1 e^{i(\Delta-\varepsilon)t}C(t)+v_2 e^{i(\Delta-\varepsilon)t}B(t),$$
$$i\dot{B}(t) = \Gamma_2 B(t)+v_1 e^{i(\Delta-\varepsilon)t}D(t)+v_2 e^{-i(\Delta-\varepsilon)t}A(t),$$
$$i\dot{C}(t) = \Gamma_3 C(t)+v_1 e^{-i(\Delta-\varepsilon)t}A(t)+v_2 e^{i(\Delta-\varepsilon)t}D(t),$$
$$i\dot{D}(t) = \Gamma_4 D(t)+v_1 e^{-i(\Delta-\varepsilon)t}B(t)+v_2 e^{-i(\Delta-\varepsilon)t}C(t).$$

After using the method in [38], we get:

$$i\begin{bmatrix}\dot{\bar{A}}(t)\\ \dot{B}(t)\\ \dot{C}(t)\\ \dot{\bar{D}}(t)\end{bmatrix} = \begin{bmatrix}k_1 & v_2 & v_1 & 0\\ v_2 & \Gamma_2 & 0 & v_1\\ v_1 & 0 & \Gamma_3 & v_2\\ 0 & v_1 & v_2 & k_2\end{bmatrix}\begin{bmatrix}\bar{A}(t)\\ B(t)\\ C(t)\\ \bar{D}(t)\end{bmatrix} \quad (22)$$

where

$$\bar{A}(t) = A(t)e^{i(\Delta-\varepsilon)t},$$
$$\bar{D}(t) = D(t)e^{-i(\Delta-\varepsilon)t}.$$

The united system solution in eq.(31) relies on the initial states of the system. Pay attention that the formulas of amplitude probabilities by preparing the atoms initially to be made in superposition of the states

$$|\Psi(0)\rangle_{atom} = \cos(\theta)\,|e_1,e_2\rangle + e^{-i\phi}\sin(\theta)\,|g_1,g_2\rangle,$$

ϕ refers to the corresponding phase of the two states. For $\theta = 0$, both atoms be in excited states. While if we take $\theta = \frac{1}{2}\pi$, the ground states is considered. The solution of this system is given by

$$\mathscr{R}(m_1,m_2,t) = \exp\left[-i\mathscr{M}(m_1,m_2)t\right]\mathscr{R}(m_1,m_2,0). \quad (23)$$

where $\mathscr{M}(m_1,m_2)$ and $\mathscr{R}(m_1,m_2,t)$ take the following forms:

$$\mathscr{M}(m_1,m_2) = \begin{bmatrix}k_1 & v_2 & v_1 & 0\\ v_2 & \Gamma_2 & 0 & v_1\\ v_1 & 0 & \Gamma_3 & v_2\\ 0 & v_1 & v_2 & k_2\end{bmatrix}, \quad (24)$$

$$\mathscr{R}(n_1,n_2,t) = \left[\bar{A}(m_1,m_2,t)\;\; B(m_1,m_2,t)\;\; C(m_1,m_2,t)\;\; \bar{D}(m_1,m_2,t)\right]^T \quad (25)$$

The matrix exponential in eq (23) can be calculated (38) by several analytical methods [?,39].

3 Atomic Population Inversion

The difference between the uppermost states and the lowest states of the probability amplitudes is denoted by the atomic population inversion. the collapse and revival phenomenon referred to the information about the behavior of the interaction between atom and field. For this model, the atomic inversion $W(t)$ is given by

$$W(t) = \rho_{11}(t) - \rho_{33}(t) + \rho_{22}(t) - \rho_{44}(t), \quad (26)$$

where $\rho_{\ell\ell}$ are the diagonal elements of reduced density matrix. We display the evolution of $W(t)$ vs the scaled time $\bar{\lambda}t$ in different cases for the system parameters with constant values of $\chi = 0$ and $\Delta = 0$ in fig (1). Generally, we take notice that the revival and collapse phenomenon is evident in the figures, the inversion, and periodic oscillation is confined between (-1) and $(+1)$. In fig.(1(a)), the atomic inversion shows when $\chi = 0$, $\Delta = 0$, $r = 0$ and $\varepsilon = 0$. In fig.(1(b)), we set $r = 1$, and $\varepsilon = \pi\bar{\lambda}$, the oscillation becomes around (.05) means that the upward shift of the base-line refers to more energy is stored into the atomic system because of the presence of Stark parameters. In fig.(1(c) and 1(d)), we set $\varepsilon = 10\pi\bar{\lambda}$,$\varepsilon = 20\pi\bar{\lambda}$, respectively. We note that the upward shift of the base-line is greater than the previous case and also the oscillation amplitude decreases during the revival period.

4 Pancharatnam phase

Physical sciences include several systems of objects that is particularized to a phase by several parameters. The Pancharatnam phase [41] is of particular interest, when a system undergoes a cyclic evolution, the prepared and targeted states differ by a phase factor. When the system behaves in a non-cyclic evolution, those states are different, and the phase imposition is not trivial. The Pancharatnam phase is influential in the propagation of an EMF where the state of polarization is alternating periodically [42]. Most experimental presentations of Pancharatnam phase [23] include laser beam splitting, the variance in the phase difference of the two beams due to the polarization of one or both of the split beams along different paths. The Pancharatnam phase $\Phi(t)$ defined by

$$\Phi(t) = \arg(\langle\Psi(0)|\Psi(t)\rangle) \quad (27)$$

It is crucial to note that Pancharatnam phase does not rely on the laser phase but slightly on the phase shift between

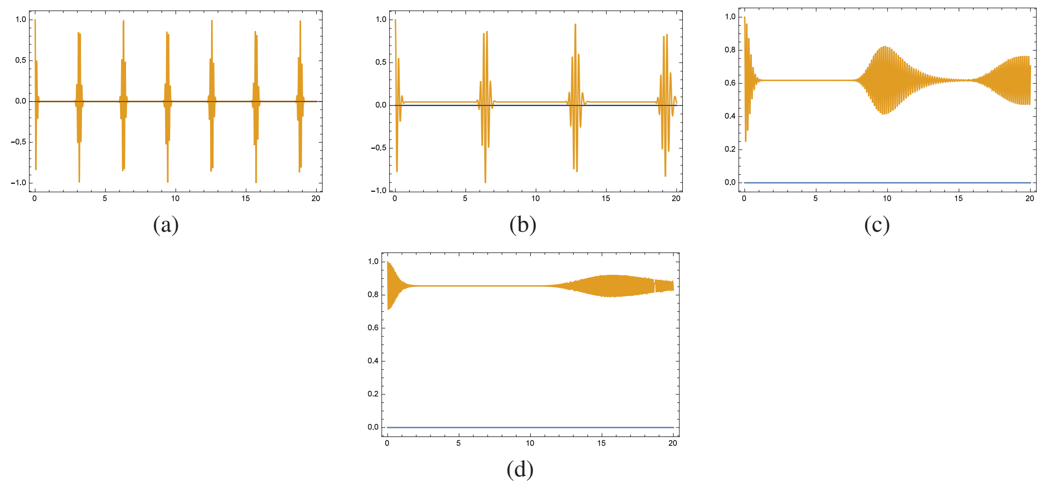


Fig. 1: The atomic population inversion evolution vs the scaled time, $\bar{m} = 25$ and $r_j = \sqrt{\frac{\gamma_j}{\beta_j}}$. (a)$\chi = 0$, $\varepsilon = 0$, $r_1 = r_2 = 0$ and $\Delta = 0$, (b) $\chi = 0$, $\varepsilon = \pi\bar{\lambda}$, $r_1 = r_2 = 1\bar{\lambda}$ and $\Delta = 0$, (c) $\chi = 0$, $\varepsilon = 10\pi\bar{\lambda}$, $r_1 = r_2 = 1\bar{\lambda}$ and $\Delta = 0$, (d) $\chi = 0$, $\varepsilon = 20\pi\bar{\lambda}$, $r_1 = r_2 = 1\bar{\lambda}$ and $\Delta = 0$.

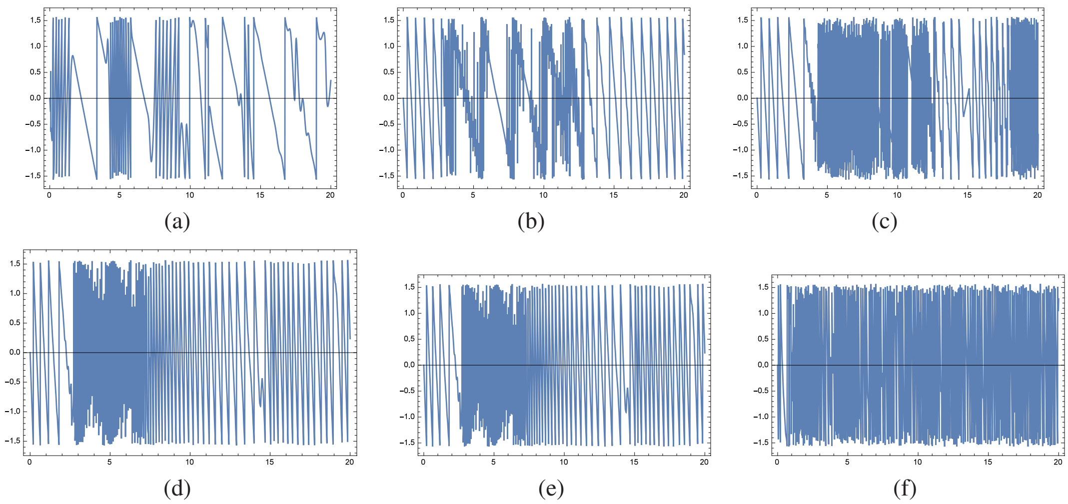


Fig. 2: The Pancharatnam phase evolution vs the scaled time, $\bar{m} = 10$, and $r_j = \sqrt{\frac{\gamma_j}{\beta_j}}$. (a) $\chi = 0$, $\varepsilon = \pi\bar{\lambda}$, $r_1 = r_2 = 0.5\bar{\lambda}$, and $\Delta = 0$, (b) $\chi = 0$, $\varepsilon = 12\pi\bar{\lambda}$, $r_1 = r_2 = 0.5\bar{\lambda}$, and $\Delta = 0$, (c) $\chi = 0$, $\varepsilon = 20\pi\bar{\lambda}$, $r_1 = r_2 = 0.5\bar{\lambda}$, and $\Delta = 0$, (d) $\chi = 0.01\bar{\lambda}$, $\varepsilon = \pi\bar{\lambda}$, $r_1 = r_2 = 0.5\bar{\lambda}$, and $\Delta = 0$, (e) $\chi = 0.01\bar{\lambda}$, $\varepsilon = 12\pi\bar{\lambda}$, $r_1 = r_2 = 0.5\bar{\lambda}$, and $\Delta = 0$.

the latter phase and the initial prepared coherence phase[24].

In fig (2), We display the evolution of Pancharatnam phase Φ (t) vs the scaled time $\bar{\lambda}t$ in different cases for the system parameters with constant values of $r_1 = r_2 = 0.5\bar{\lambda}$ and $\Delta = 0$. Generally, we take notice that the phase Φ(t) is bounded between $\frac{-1}{2}\pi$ and $\frac{1}{2}\pi$, shaped like a saw-toothed variation, and the periodicity is slightly shown. In fig (2(a)), we put $\chi = 0$, $\varepsilon = \pi\bar{\lambda}$, and we note's that many artificial phase jumps are observed. In fig (2(b),2(c)), we put $\chi = 0$, $\varepsilon = 12\pi$. By comparing the result with fig(2(a)), we note that the phase jumps became sharper and the chaotic behavior increases and longer period of time is wasted to be achieved. In fig(2(d),2(e),2(f)), we set ($\chi = 0.01\bar{\lambda}$, $\varepsilon = 12\pi\bar{\lambda}$), ($\chi = 0.05\bar{\lambda}$, $\varepsilon = 12\pi\bar{\lambda}$), ($\chi = 0.1\bar{\lambda}$, $\varepsilon = 12\pi\bar{\lambda}$), respectively. We note that as χ increases, the chaotic behavior becomes more observable.

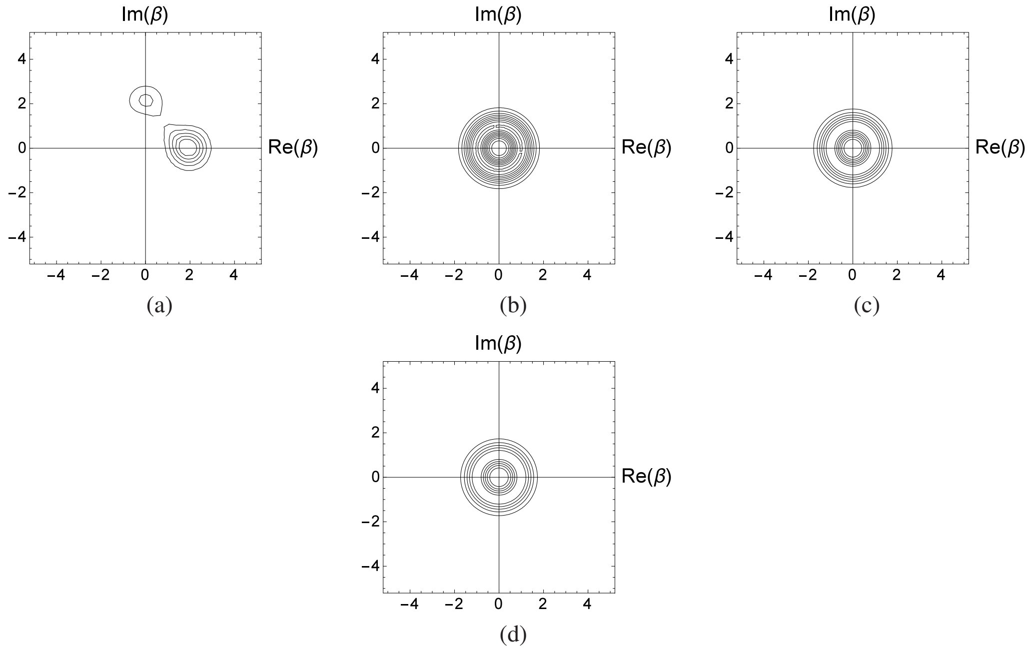


Fig. 3: The contour figures of Husimi Q function in the subspace $\beta_1 = \beta_2 = \beta$ in the complex β-plane where $x = Re(\beta)$ and $y = Im(\beta)$ $\bar{m} = 10$, $\chi = 0$, $r_1 = r_2 = 1$ and $\Delta = 0$. (a) $\varepsilon = 0$, (b) $\varepsilon = 5\pi\bar{\lambda}$, (c) $\varepsilon = 10\pi\bar{\lambda}$, (d) $\varepsilon = 15\pi\bar{\lambda}$.

5 Q-function Husimi

The expectation value of the reduced field density operator in the coherent phase space is easily presented by the Q-function . So, it is broadly used to study the field classical properties. It has no singularities.There exists for all density operator is limited and even larger than or equal to zero. Furthermore, the the Q-function width presents a measure for the EMF squeezing. Consequently, to study the Q-function behavior [43], that is presented as

$$Q(\beta_1, \beta_2, t) = \frac{1}{\pi^2} \langle \beta_1, \beta_2 | \hat{\rho}_F | \beta_1, \beta_2 \rangle \tag{28}$$

where $\hat{\rho}_F = tr_A\big(|\Psi(t)\rangle\langle\Psi(t)|\big)$ refers to the reduced density operator for field and $|\beta_\ell\rangle$ is the field coherent state with the definition in eq (12) but when β_j is spanned in the complex β_j-plane where $\beta_j = x_j + iy_j$.

In figures (3 and 4), we display the contour of Husimi Q-function in the subspace $\beta_1 = \beta_2 = \beta$ in the complex (phase space) β-plane where $x = Re(\beta)$ and $y = Im(\beta)$. In figure (3), we set $\chi = 1\lambda$ for various values of the time-modulation parameter ε. In figure (3(a)), when $\varepsilon = 0$, the Husimi Q-function is split into two fully separated peaks which reflects the strong effect of the Kerr medium that will be explained later. Also, we note that as ε value increases, the density of lines in the contour plot changes dramatically, as we may explain this result as to occur due to the dependence of atom when the field interact with it in a coupling torque in the case of time-dependent . In fig (4(a)), we observe that when $\chi = 0.01$ the contour plot consists of a single peak, while we grow the value of χ as we note in fig (4(b),4(c)), the contour is the shape of a banana because of the widening in the distribution. Generally, the distribution is moving in a counter clock-wise until it reached a semi ring-like in fig(4(d)). In fig (4(f)) the contour splits into four connected peaks.

6 Second-order correlation

The study statistics of photon for our quantum system state to be sub-Poissonian, Poissonian, or super-Poissonian is very noteworthy of the effect of non-classical . To discuss the effect for the normalized correlation function $g_j^{(2)}(t)$ of second-order, the subscript j refers to the j's field-mode defined as [44]

$$g_j^{(2)}(t) = \frac{\langle \hat{a}_j^{\dagger 2} \hat{a}_j^2 \rangle}{\langle \hat{a}_j^{\dagger} \hat{a}_j \rangle^2}, \qquad j = 1, 2. \tag{29}$$

The distribution of sub-Poissonian is to be if $g_j^{(2)}(t) < 1$ as the EMF, the detecting probability of an incident pair of two photons is less than it could be for a coherent field represented by the Poissonian distribution for a non-classical effect . Otherwise, the super-Poissonian distribution is to be if $g_j^{(2)}(t) > 1$ as the light field and the standard for the coherent state (Poissonian distribution of photon) in a classical effect if

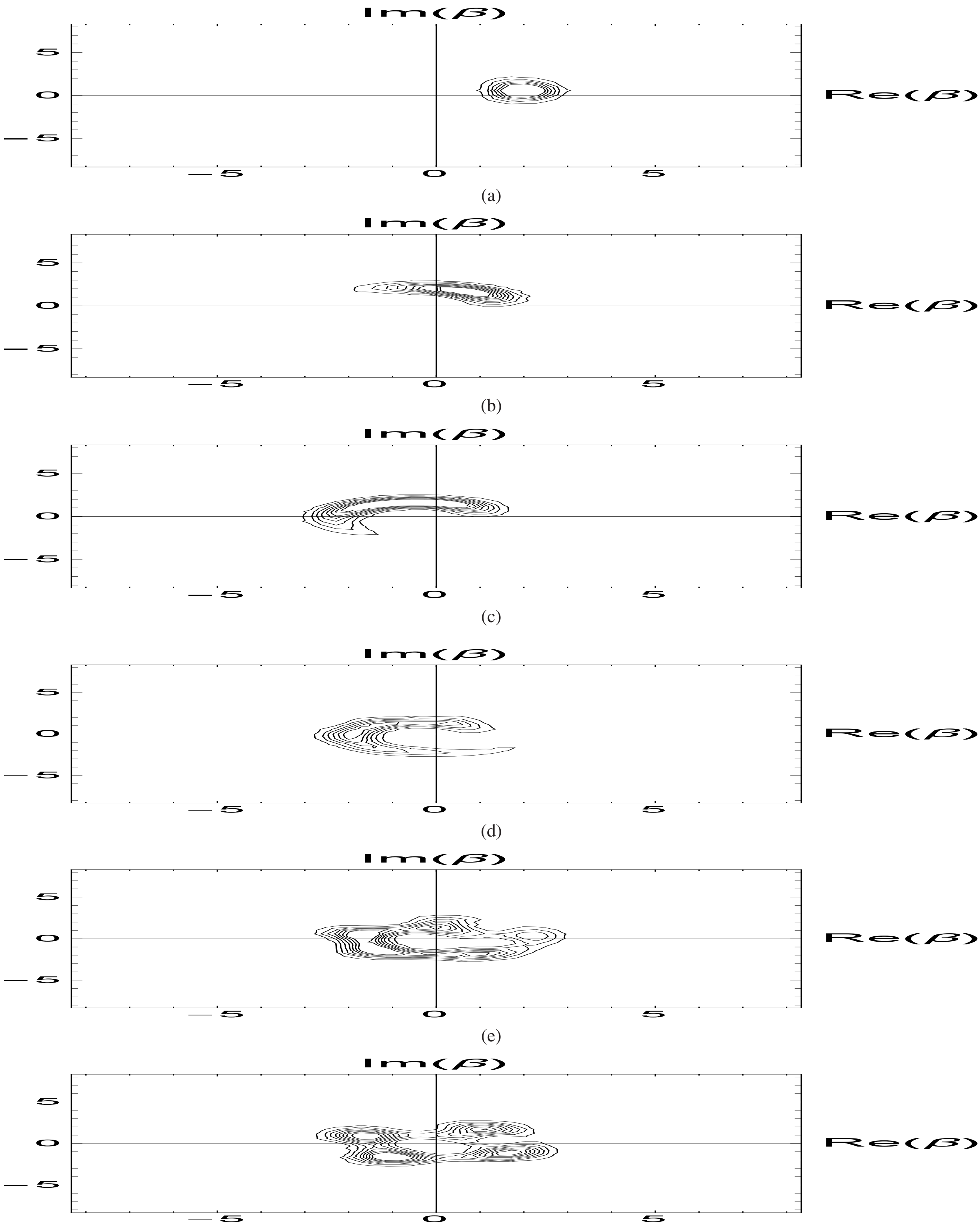
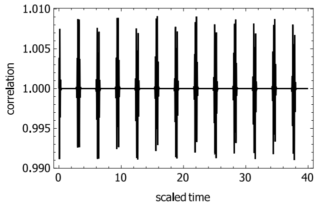
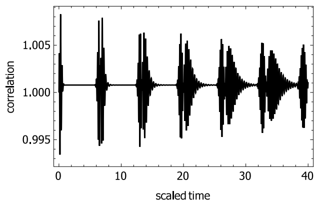


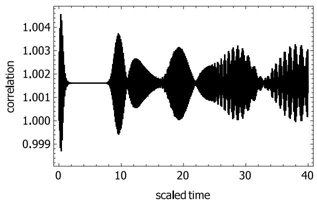
Fig. 4: The contour figures of the Q function in the subspace $\beta_1 = \beta_2 = \beta$ in the complex β-plane where $x = Re(\beta)$ and $y = Im(\beta)$, $\bar{m} = 10$, $\varepsilon = \pi\bar{\lambda}$, $\beta_1 = 1\bar{\lambda}$, $\beta_2 = 5\bar{\lambda}$, $\gamma_1 = 1\bar{\lambda}$, $\gamma_2 = 0.2\bar{\lambda}$, and $\Delta = 25\bar{\lambda}$. (a) $\chi = 0.01\bar{\lambda}$, (b) $\chi = 0.2\bar{\lambda}$, (c) $\chi = 0.4\bar{\lambda}$, (d) $\chi = 0.6\bar{\lambda}$, (e) $\chi = 0.8\bar{\lambda}$, (e) $\chi = 1.0\bar{\lambda}$.



(a)



(b)



(c)

Fig. 5: The correlation function of the second-order $g_1^{(2)}(t)$ verse the scaled time $\bar{\lambda}t$ when $\bar{m} = 25$. (a) $\varepsilon = 0.0$, (b) $\varepsilon = 3\pi\bar{\lambda}$, (c) $\varepsilon = 12\pi\bar{\lambda}$.

$g_j^{(2)}(t) = 1$. At $g_j^{(2)}(t) = 2$, the system exhibits in the meantime a thermal statistics , and at $g_j^{(2)}(t) > 2$ in super-thermal.

The expectation value of $\hat{a}_1^{\dagger}\hat{a}_1$ in single-photon process and the first field-mode operator $\hat{a}_1^{\dagger 2}\hat{a}_1^2$ are respectively, given as

$$\langle \hat{a}_1^{\dagger}(t)\hat{a}_1(t)\rangle = \sum_{m_1=0}^{+\infty}\sum_{m_2=0}^{+\infty} |q_{m_1}|^2 |q_{m_2}|^2 \Big[m_1 \left(|A(t)|^2 + |B(t)|^2\right) + (m_1+2)\left(|C(t)|^2 + |D(t)|^2\right)\Big], \tag{30}$$

Now, the calculations of the second-order correlations function $g_1^{(2)}(t)$ is presented by figures (5). In fig.(5(a)), we note that the oscillation baseline becomes around 1.0 which refers to the investigated quantum system is displaying Poissonian distribution for $\varepsilon = 0$. In fig.(5(b) and 5(c)), we note that the oscillation is super-Poissonian distribution means that the baseline is being shifted upwards as the value of ε increases.

7 Perspective

In this paper, we investigated the time-dependent interaction coupling torque between a two two-level atoms with a two-mode EMF in an ideal cavity. We investigated the atomic population inversion and we note's that the oscillation baseline shifts above, the number of revivals decreases and revivals be squeezed due to the increase in the value of the time-modulation coupling parameter ε. We studied the Pancharatnam phase, we observe that by increasing the value of χ and ε enhances the number of artificial phase jumps and the chaotic behavior period. The squeeze of the distribution in counter clock-wise direction and the change of the distribution from single peak to semi-separated multiple peaks consequently the effect of the Kerr-like medium on Husimi Q-function . Also, we note that the system changes from displaying a Poissonian distribution to super-Poissonian distribution as ε increases when we investigated the correlation function of the second-order.

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Appendix

we can solve the system

$$i\begin{bmatrix}\dot{\bar{A}}(t)\\ \dot{B}(t)\\ \dot{C}(t)\\ \dot{\bar{D}}(t)\end{bmatrix}=\begin{bmatrix}k_1 & \nu_2 & \nu_1 & 0\\ \nu_2 & \Gamma_2 & 0 & \nu_1\\ \nu_1 & 0 & \Gamma_3 & \nu_2\\ 0 & \nu_1 & \nu_2 & k_2\end{bmatrix}\begin{bmatrix}\bar{A}(t)\\ B(t)\\ C(t)\\ \bar{D}(t)\end{bmatrix} \tag{31}$$

where

$$\bar{A}(t)=A(t)e^{i(\Delta-\varepsilon)t},$$
$$\bar{D}(t)=D(t)e^{-i(\Delta-\varepsilon)t}.$$

This coupled system of differential equations can be solved analytically. The energy eigenvalues $\Xi_\mu(t)$ of the system in equation (1), can be formulated as follows;

$$\Xi_\mu(t)=-\frac{b}{4}\pm h\pm\sqrt{\frac{q}{h}-2p-4h^2}, \tag{32}$$

with

$$h=\frac{1}{2}\sqrt{\frac{1}{3}\left(\mathscr{Q}+\frac{\Delta_1}{\mathscr{Q}}\right)},\quad \mathscr{Q}=\left(\frac{\Delta_2+\sqrt{\Delta_2^2-4\Delta_1^3}}{2}\right)^{1/3},$$
$$p=\frac{8c-3b^2}{8},\qquad q=\frac{b^3+8d-4bc}{8},$$
$$\Delta_1=c^2-3bd+12e,$$
$$\Delta_2=2c^3-9bcd+27(b^2e+d^2)-72ce, \tag{33}$$

$$b=-i(\kappa_1+\kappa_2+\kappa_3+\kappa_4),$$
$$c=\Delta^2+2\nu_1^2+2\nu_2^2+\Delta\kappa_1-\kappa_1\kappa_2-\kappa_1\kappa_3-\kappa_2\kappa_3-\Delta\kappa_4$$
$$-\kappa_1\kappa_4-\kappa_2\kappa_4-\kappa_3\kappa_4-2\Delta\varpi-\varpi\kappa_1+\varpi\kappa_4+\varpi^2,$$
$$d=-i\Big(\Delta\kappa_2+\Delta\kappa_1\kappa_2+\Delta^2\kappa_3+\Delta\kappa_1\kappa_3-\kappa_1\kappa_2\kappa_3$$
$$-\Delta\kappa_2\kappa_4-\kappa_1\kappa_2\kappa_4-\Delta\kappa_3\kappa_4-\kappa_1\kappa_3\kappa_4-\kappa_2\kappa_3\kappa_4$$
$$+\nu_1^2(\kappa_1+\kappa_2+\kappa_3+\kappa_4)+\nu_2^2(\kappa_1+\kappa_2+\kappa_3+\kappa_4)$$
$$-(\kappa_2+\kappa_3)(2\Delta+\kappa_1-\kappa_4)\varpi+\varpi^2(\kappa_2+\kappa_3)\Big),$$
$$e=\nu_1^4+\Big(\nu_2^2-\kappa_2(\Delta+\kappa_1-\varpi)\Big)\Big(\nu_2^2+\kappa_3(\Delta-\kappa_4-\varpi)\Big)$$
$$-\nu_1^2\Big(2\nu_2^2+\Delta(\kappa_3-\kappa_2)+\kappa_3(\kappa_1-\varpi)+\kappa_2(\kappa_4+\varpi)\Big). \tag{34}$$

By applying the method of *Newton interpolation* [?] for getting the matrix exponential, which states that the eigenvalues η_j $(j=1,2,..,m)$ of the matrix B, where m is the matrix dimension,is given by:

$$e^{tB}=e^{\eta_1 t}\mathscr{I}+\sum_{j=2}^{m}\left[\eta_1,...,\eta_j\right]\Pi_{\kappa=1}^{j-1}(B-\eta_\kappa\mathscr{I}), \tag{35}$$

wherever the unitary matrix is to be $\mathscr{I}$ and the divided differences $\left[\eta_1,...,\eta_j\right]$ is function of t and defined recursively by:

$$[\eta_1,\eta_2]=\frac{e^{\eta_1 t}-e^{\eta_2 t}}{\eta_1-\eta_2} \tag{36}$$

$$[\eta_1,...,\eta_{\kappa+1}]=\frac{[\eta_1,...,\eta_\kappa]-[\eta_2,...,\eta_\kappa]}{\eta_1-\eta_{\kappa+1}},\qquad \kappa\geq 2. \tag{37}$$

We obtain $e^{-i\mathscr{M}t}$ where $B=-i\mathscr{M}$ and the eigenvalues of B are defined in eq.(31) by using the previous method. Then

$$e^{-it\mathscr{M}}=e^{\Xi_1 t}\mathscr{I}+[\Xi_1,\Xi_2](-i\mathscr{M}-\Xi_1\mathscr{I})$$
$$+[\Xi_1,\mathscr{E}_3](-i\mathscr{M}-\Xi_1\mathscr{I})(-i\mathscr{M}-\Xi_2\mathscr{I})$$
$$+[\Xi_1,\Xi_4](-i\mathscr{M}-\Xi_1\mathscr{I})(-i\mathscr{M}-\Xi_2\mathscr{I})(-i\mathscr{M}-\Xi_3\mathscr{I}), \tag{38}$$

where the divided differences are formulated as:

$$[\Xi_1,\mathscr{E}_2]=\frac{e^{\Xi_1 t}-e^{\Xi_2 t}}{\Xi_1-\Xi_2}, \tag{39}$$

$$[\Xi_1,\Xi_3]=\frac{[\Xi_1,\Xi_2]-[\Xi_2,\Xi_3]}{\Xi_1-\Xi_3}$$
$$=\frac{e^{\Xi_1 t}-e^{\Xi_2 t}}{(\Xi_1-\Xi_2)(\Xi_1-\Xi_3)}-\frac{e^{\Xi_2 t}-e^{\Xi_3 t}}{(\Xi_1-\Xi_2)(\Xi_2-\Xi_3)}, \tag{40}$$

$$[\Xi_1,\Xi_4]=\frac{[\Xi_1,\mathscr{E}_3]-[\Xi_3,\Xi_4]}{\Xi_1-\Xi_4}$$
$$=\frac{e^{\Xi_1 t}-e^{\Xi_2 t}}{(\Xi_1-\Xi_2)(\Xi_1-\Xi_3)(\Xi_1-\Xi_4)}$$
$$-\frac{e^{\Xi_2 t}-e^{\Xi_3 t}}{(\Xi_1-\Xi_2)(\Xi_2-\Xi_3)(\Xi_1-\Xi_4)}$$
$$-\frac{e^{\Xi_3 t}-e^{\Xi_4 t}}{(\Xi_1-\Xi_4)(\Xi_3-\Xi_4)}. \tag{41}$$

the probability amplitudes are formulated as:

$$\begin{bmatrix}A^{(m_1,m_2)}(t)e^{i(\Delta-\varepsilon)t}\\ B^{(m_1,m_2)}(t)\\ C^{(m_1,m_2)}(t)\\ D^{(m_1,m_2)}(t)e^{-i(\Delta-\varepsilon)t}\end{bmatrix}=e^{-it\mathscr{M}}\begin{bmatrix}A^{(m_1,m_2)}(0)\\ B^{(m_1,m_2)}(0)\\ C^{(m_1,m_2)}(0)\\ D^{(m_1,m_2)}(0)\end{bmatrix} \tag{42}$$

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