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Dynamics of Classical and Quantum Information on Spin-chains with Multiple Interactions

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Abstract: The classical and quantum information is very senstive to any internal and external noise. In this paper we invistigated the effcet of the Calogero-Moser (CM) type interaction, Dzyaloshinskii-Moriya (DM) interaction, thermalization and uniform and nonuniform magnetic fields on the dynamics of both classical and quantum information. The spin-chain model is used to describe the system in the thermal state. The egenerated correlation is quantified by the Shannon entropy for the classical information and the Von Neuman entropy for the quantum one. The obtained results showed that the quantum information is more influenced by the distance between the spins.

Keywords: Shannon entropy, Von Neuman entropy, Spin-chain systems, thermal state, uniform and nonuniform magnetic field.

1 Introduction

The recent years witnessed a big revolution in the area of quantum information. The quantum computer may will be available commercially shortly [1,2], also the quantum communication and quantum networks have very speed progress and also will be turned on very soon [3,4]. Classical and quantum information are very sensitive to the internal and external noises [5,6,7,8,9,10].

The spin-chain model is used to simulate the quantum system perfectly from the quantum information, computation, communication [11], meteorology [12] and quantum thermodynamics [13]. Classical and quantum information in XX spins [14], the thermal state in XY spin-chain model is studied in [15] and in th time domain is studied by Burrell & Osborne [16]. Sarandy studied the dynamics of the classical information and the quantum discord in XXZ spin-chains [17]. Also, the classical and quantum correlations dynmaics over the XXX [18] and XYZ [19] spin-chain models are investigated. The effect of the internal (spin-orbit coupling and Dzyaloshinskii-Moriya (DM) interaction) and external (uniform and non-uniform magnetic fields) interactions on the classical and quantum information of spin-chains is studied by several groups [20,21,22].

The previous studies showed that the long-range interactions plays an important roles in the area of quantum information [23,24,25]. The quantum information is quantified using quantum discord between to spin with the effect of the Calogero-Moser type interaction [25]. Moreover, the possibility of using the long range interactions between the spins as quantum communication channels is investigated [26,27,28,29]. The entanglement under the effect of long-range interactions is investigated in several papers. Sultani et. al., studied the quantum entanglement between two spins in the Ising model with an added Dzyaloshinsky-Moriya (DM) interaction and in the presence of the transverse magnetic field [30,31]. The quantum state transfer and entanglement renormalization using Long-Range interactions is investigated by Eldredge et. al. [32]. Our motivation in this paper is to extend the previous work by studying the effect of long range interaction with different types of noise in the dynamics of the classical and quantum information of the spin chain system.

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This paper is organized as follow: the mathematical model and it is solution is addressed in section (2). The results and discussion is presented in section (2). We conclude our results in section (4).

2 The Model and it's solution for a two-qubit spin-chain

The Hamiltonian H for a two-qubit anisotropic Heisenberg model with z-component interaction parameter D_z is

$$H = J_z\sigma_1^z\sigma_2^z + D_z(\sigma_1^x\sigma_2^y - \sigma_1^y\sigma_2^x) + (B+b)\sigma_1^z + (B-b)\sigma_2^z + J(R)(\sigma_1^x\sigma_2^x + \sigma_1^y\sigma_2^y) \qquad (1)$$

where J_z is the real coupling constant, D_z is the z-component DM interaction parameter, and $\sigma^i (i = x,y,z)$ are Pauli matrices. B is the homogeneous part of the magnetic field and b show the inhomogeneous. The DM-interaction and external magnetic fields are thought to be along the z-direction. $J(R)$ is the spin interaction coupling which will be characterized as far as CM interactions. All the parameters are dimensionless.
In matrix form we can write the system Hamiltonian Eq.1 as follows:

$$H = \begin{pmatrix} 2B+\mathrm{Jz} & 0 & 0 & 0 \\ 0 & 2b-\mathrm{Jz} & 2(-i\mathrm{Dz}+J(R)) & 0 \\ 0 & 2(i\mathrm{Dz}+J(R)) & -2b-\mathrm{Jz} & 0 \\ 0 & 0 & 0 & \mathrm{Jz}-2B \end{pmatrix} \qquad (2)$$

We consider the first type of the CM model, which is a version of the Haldane-Shastry model with exchange interaction $J = 1/R^2$.
The eigenvalues for this type of CM interaction are given by:

$$E_1 = 2B + \mathrm{Jz}$$
$$E_2 = \mathrm{Jz} - 2B$$
$$E_3 = 2\sqrt{b^2 + \mathrm{Dz}^2 + G^2} - \mathrm{Jz}$$
$$E_4 - 2\sqrt{b^2 + \mathrm{Dz}^2 + G^2} - \mathrm{Jz}$$
$$|\psi_1\rangle = |00\rangle$$
$$|\psi_2\rangle = |11\rangle$$
$$|\psi_3\rangle = \frac{1}{\sqrt{1+MM^*}}(M|01\rangle + |10\rangle)$$
$$|\psi_4\rangle = \frac{1}{\sqrt{1+NN^*}}(M|01\rangle + |10\rangle)$$

where

$$M = \frac{\sqrt{b^2 + \mathrm{Dz}^2 + G^2} + b}{G + i\mathrm{Dz}}$$
$$N = \frac{b - \sqrt{b^2 + \mathrm{Dz}^2 + G^2}}{G + i\mathrm{Dz}} \qquad (3)$$
$$G = \frac{1}{R^2}$$

Once we get the eigenvalues and eigenvectors of the system Hamiltonian, we can obtain the dynamics of the density operator in the thermal state as follows:

$$\rho(T) = \frac{1}{z}\sum_i (\exp(\frac{-E_i}{T}))|\psi_i\rangle\langle\psi_i| \qquad (4)$$

where $z = Tr(\rho(T))^{\cdot}$ In the basis $\{|00\rangle\ ,|01\rangle,\ |10\rangle,\ |11\rangle\}$we can obtain the density operator of the system as follows:

$$\rho(T) = \frac{1}{z}\begin{pmatrix} \rho_{11} & 0 & 0 & 0 \\ 0 & \rho_{22} & \rho_{23} & 0 \\ 0 & \rho_{32} & \rho_{33} & 0 \\ 0 & 0 & 0 & \rho_{44} \end{pmatrix} \qquad (5)$$

has matrix elements given in Eq. 5 as follows:

$$\rho_{11} = e^{2B+\mathrm{Jz}}$$
$$\rho_{22} = e^{-\mathrm{Jz}}\left(\frac{b\sinh\left(2\sqrt{b^2+\mathrm{Dz}^2+G^2}\right)}{\sqrt{b^2+\mathrm{Dz}^2+G^2}} + \cosh\left(2\sqrt{b^2+\mathrm{Dz}^2+G^2}\right)\right)$$
$$\rho_{23} = \frac{e^{-\mathrm{Jz}}(G - i\mathrm{Dz})\sinh\left(2\sqrt{b^2+\mathrm{Dz}^2+G^2}\right)}{\sqrt{b^2+\mathrm{Dz}^2+G^2}}$$
$$\rho_{32}\frac{e^{-\mathrm{Jz}}(G + i\mathrm{Dz})\sinh\left(2\sqrt{b^2+\mathrm{Dz}^2+G^2}\right)}{\sqrt{b^2+\mathrm{Dz}^2+G^2}} \qquad (6)$$
$$\rho_{33} = e^{-\mathrm{Jz}}\left(\cosh\left(2\sqrt{b^2+\mathrm{Dz}^2+G^2}\right) - \frac{b\sinh\left(2\sqrt{b^2+\mathrm{Dz}^2+G^2}\right)}{\sqrt{b^2+\mathrm{Dz}^2+G^2}}\right)$$
$$\rho_{44} = e^{\mathrm{Jz}-2B}$$
$$z = 2e^{-\mathrm{Jz}}\cosh\left(2\sqrt{b^2+\mathrm{Dz}^2+G^2}\right) + 2e^{\mathrm{Jz}}\cosh(2B)$$

By using the $\rho(T)$ as in eq.5 it is easy to study the dynamics of the classical and quantum information using the Shannon and Von Neumann entropies as detailed in the following section.

3 Results and Discussion

In this section we studied the dynamics of the classical (Shannon entropy) and quantum information (Von Neumann entropy) over the system and it is dependence on the isotropic and anisotropic interactions.

In fig. (1) we investigated the dynamics of the Shannon and Von Neumann entropies as a function of the distance between the qubits “R” and the temperature ”T” in the presence of several interactions (Spin-orbit interactions, uniform and non-uniform magnetic fields, Calogero-Moser (CM) type interaction, Dzyaloshinskii-Moriya (DM) interaction) where $J_z = D_z = B = b = 1$ where (a) for the Shannon entropy and (b) for the Von Neumann entropy. We can see from fig. (1)-a that the classical information is started from $S_H = 0.5$ at R = 0 and continued until R = 0.2, the Shannon entropy increased suddenly and reached its maximum value $S_H = 1$ at R = 0.4. Also, we can see that by increasing the temperature the disturbance of the system increased and consequently the classical information increased. In fig. (1)-b the quantum

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information has the same behavior with the classical information, except the initial period of the figure, where the quantum information is initially started equal to zero and with distance increased the Von Neumann entropy increased and reached the maximum value as in figure (1)-*a*. Also, the temperature have the same effect as with classical information.

4 Conclusion

In this paper a model of spin-chain model with several types of interactions is designed and analytically solved. The dynamics of the classical and quantum correlation are investigated under the effect of the system parameters. It is found that the long range interactions have bad effect on the both quantum and classical information but by increasing the temperature the classical and quantum information recovered again.

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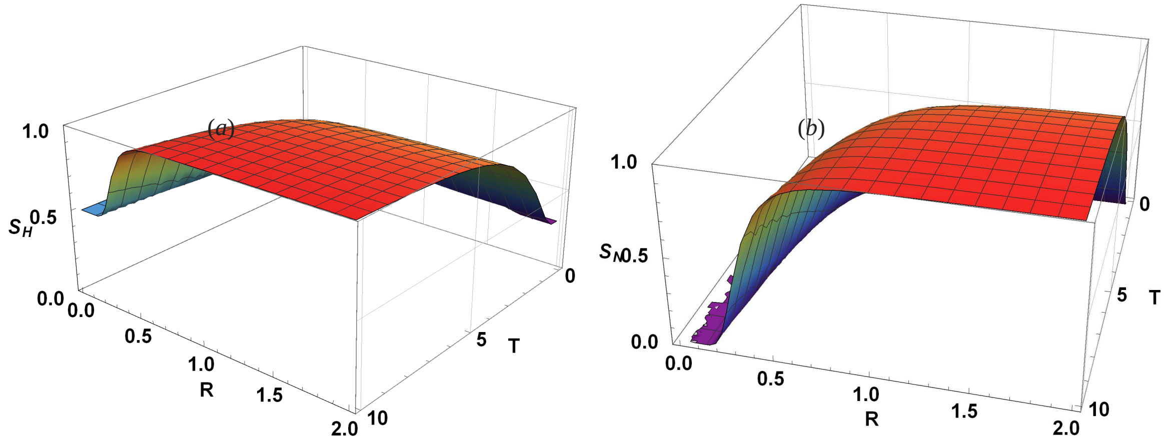

Fig. 1: The dynamics of the information (*a*) Shannon entropy (*b*) Von Neumann entropy as a function of distance between the Qubits R and the thermalization T.

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