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Receding Horizon Chaos Synchronization Method

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Abstract: This article proposes a new synchronization method, called a receding horizon synchronization (RHS) method, for a general class of chaotic systems. A new linear matrix inequality (LMI) condition on the finite terminal weighting matrix is proposed for chaotic systems under which non-increasing monotonicity of the optimal cost is guaranteed. It is shown that the proposed terminal inequality condition guarantees the closed-loop stability of the RHS method for chaotic systems. As an application of the proposed method, the RHS problem for Chua's chaotic system is investigated.

Keywords: receding horizon control (RHC); chaos synchronization; cost monotonicity; linear matrix inequality (LMI)

1. Introduction

Synchronization for chaotic dynamic systems has received much interest among scientists since a scheme to synchronize two identical nonlinear chaotic systems was introduced in [1]. It has been widely investigated in several fields including chemical, physical, and ecological systems [2]. In the literature, several synchronization schemes, such as OGY method [3], variable structure control [4], parameters adaptive control [5,6], observer-based control [7], active control [8,9], time-delay feedback approach [10], backstepping design technique [11,12], complete synchronization [13], have been applied to the chaos synchronization successfully.

Receding horizon control (RHC) scheme has been widely studied as an excellent feedback strategy [14–19]. RHC has made an important impact on industrial controls and is being increasingly applied in process controls. Various advantages are known for RHC, including the ability to handle time-varying and nonlinear systems, input/output constraint, uncertainty, and so on. The first method to guarantee the stability of the RHC is to impose an infinite terminal weighting. It is known that this method is equivalent to setting a zero terminal weighting matrix for the inverse Riccati equation [14,15]. We call this method the terminal equality condition. Since the requirement for infinite terminal weighting is too demanding, studies of finite terminal weighting matrices have been made [16–19]. Although there are many advantages of RHC, to the best of our knowledge, the RHC based synchronization method for chaotic systems has not been established in the literature so far. This situation motivates our investigation.

In this paper, a new synchronization method based on the receding horizon control is proposed for chaotic systems. This method is called a receding horizon synchronization (RHS) method. First, we propose a new linear matrix inequality (LMI) condition on the finite terminal weighting matrix of the receding horizon cost function. Under this condition, non-increasing monotonicity of the optimal cost is shown to be guaranteed. Based on this LMI condition, we propose the RHS method for chaotic systems which guarantees the closed-loop asymptotic stability of the synchronization error system. We present a numerical example to illustrate the effectiveness of the proposed synchronization method.

This paper is organized as follows. In Section 2, we formulate the problem. In Section 3, an LMI condition for non-increasing monotonicity of the optimal cost is pro-

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posed. In Section 4, a new RHS method for chaotic systems is proposed. In Section 5, a numerical example is given, and finally, conclusions are presented in Section 6.

2. Problem Formulation

Consider a class of chaotic systems described by the following nonlinear differential equation:

$$\dot{x}(t) = Ax(t) + Bf(x(t)) \tag{1}$$

where $x(t) \in R^n$ is the state vector, $f(x(t)) \in R^n$ is a nonlinear function vector satisfying the global Lipschitz condition with Lipschitz constant $L_f > 0$, $A \in R^{n\times n}$ and $B \in R^{n\times n}$ are known constant matrices. The system (1) is considered as a drive system. The synchronization problem of system (1) is considered using the drive-response configuration. According to the drive-response concept, the controlled response system is given by

$$\dot{z}(t) = Az(t) + Bf(z(t)) + Cu(t) \tag{2}$$

where $z(t) \in R^n$ and $u(t) \in R^m$ are the state vector and the control input of the controlled response system, respectively, and $C \in R^{n\times m}$ is a known constant matrix. In fact, the matrix C is chosen arbitrarily. In this paper, we design a feedback control input $u(t)$ via the RHC scheme. In order to design the feedback control input $u(t)$, we need information on states of drive and response systems. Thus, the control input $u(t)$ in (2) depends on states of drive and response systems. Define the synchronization error $e(t) = z(t) - x(t)$. Then we obtain the synchronization error system

$$\dot{e}(t) = Ae(t) + B(f(z(t)) - f(x(t))) + Cu(t). \tag{3}$$

For the design of the RHS controller, the following finite horizon cost is associated with the synchronization error system (3):

$$J(e(t_0), t_0, t_1) = \int_{t_0}^{t_1} [e^T(t)Qe(t) + u^T(t)Ru(t)]dt + e^T(t_1)Q_f e(t_1), \tag{4}$$

where $t_0 > 0$ is the initial time, t_1 is the final time, $Q > 0$, $R > 0$, and $Q_f = Q_f^T > 0$. The optimal control minimizing the cost function (4) and the corresponding optimal cost will be denoted by $u^*(t)$, $(t_0 \le t \le t_1)$, and $J^*(e(t_0), t_0, t_1)$, respectively. The RHS controller is then obtained by minimizing the cost function (4) with the initial time t_0 and the terminal time t_1 replaced by the current time t and the future time $t+T$, respectively, where $T > 0$ is a constant. The stability of the proposed RHS controller depends on the choice of the terminal weighting matrix Q_f. In this paper, we show that the RHS controller with the cost function (4) guarantees the asymptotic stability under an LMI condition on the finite terminal weighting matrix Q_f.

3. Monotonicity of the Optimal Cost

In this section, we obtain a new LMI condition for the finite terminal weighting matrix Q_f under which the non-increasing cost monotonicity is guaranteed.

Theorem 1. *Assume that there exist $X = X^T > 0$ and $Y \in R^{m\times n}$ such that*

$$\begin{bmatrix} [1,1] & X & Y^T & X & B \\ X & -Q^{-1} & 0 & 0 & 0 \\ Y & 0 & -R^{-1} & 0 & 0 \\ X & 0 & 0 & -\frac{1}{L_f^2}I & 0 \\ B^T & 0 & 0 & 0 & -I \end{bmatrix} \le 0, \tag{5}$$

where $[1,1] = (AX + CY) + (AX + CY)^T$. Then, the optimal cost $J^*(e(\tau), \tau, \sigma)$ satisfies the following relation:

$$\frac{\partial J^*(e(\tau), \tau, \sigma)}{\partial \sigma} \le 0, \qquad \tau \le \sigma. \tag{6}$$

Furthermore, Q_f is given by $Q_f = X^{-1}$.

Proof: $\frac{\partial J^*(e(\tau),\tau,\sigma)}{\partial \sigma}$ satisfies the following relation:

$$\begin{aligned} &\frac{\partial J^*(e(\tau), \tau, \sigma)}{\partial \sigma} \\ &= \lim_{\Delta\to 0} \frac{1}{\Delta}\{J^*(e(\tau), \tau, \sigma + \Delta) - J^*(e(\tau), \tau, \sigma)\} \\ &= \lim_{\Delta\to 0} \frac{1}{\Delta}\Big\{ \int_\tau^\sigma [e_1^T(t)Qe_1(t) + u_1^T(t)Ru_1(t)]dt \\ &+ J^*(e_1(\sigma), \sigma, \sigma + \Delta) - \int_\tau^\sigma [e_2^T(t)Qe_2(t) \\ &+ u_2^T(t)Ru_2(t)]dt - e_2^T(\sigma)Q_f e_2(\sigma)\Big\}, \end{aligned} \tag{7}$$

where $u_1(t)$ and $u_2(t)$ are the optimal controls to minimize $J(e(\tau), \tau, \sigma + \Delta)$ and $J(e(\tau), \tau, \sigma)$, respectively. If $u_1(\cdot)$ is replaced by $u_2(\cdot)$ up to σ and $u_1(t) = Ke_2(t)$ for $t \ge \sigma$, then

$$\begin{aligned} &\frac{\partial J^*(e(\tau), \tau, \sigma)}{\partial \sigma} \\ &\le \lim_{\Delta\to 0} \frac{1}{\Delta}\Big\{ \int_\tau^\sigma [e_2^T(t)Qe_2(t) + u_2^T(t)Ru_2(t)]dt \\ &+ J(e_2(\sigma), \sigma, \sigma + \Delta) \\ &- \int_\tau^\sigma [e_2^T(t)Qe_2(t) + u_2^T(t)Ru_2(t)]dt - e_2^T(\sigma)Q_f e_2(\sigma)\Big\} \\ &= \lim_{\Delta\to 0} \frac{1}{\Delta}\Big\{ \int_\sigma^{\sigma+\Delta} [e_2^T(t)Qe_2(t) + e_2^T(t)K^T RKe_2(t)]dt \\ &+ e_2^T(\sigma + \Delta)Q_f e_2(\sigma + \Delta) - e_2^T(\sigma)Q_f e_2(\sigma)\Big\} \\ &= e_2^T(\sigma)Qx_2(\sigma) + e_2^T(\sigma)K^T RKe_2(\sigma) \\ &+ \frac{d}{d\sigma}\{e_2^T(\sigma)Q_f e_2(\sigma)\} \\ &= e_2^T(\sigma)Qx_2(\sigma) + e_2^T(\sigma)K^T RKe_2(\sigma) \\ &+ \dot{e}_2^T(\sigma)Q_f e_2(\sigma) + e_2^T(\sigma)Q_f \dot{e}_2(\sigma). \end{aligned} \tag{8}$$

By using (3), it can be shown that

$$\begin{aligned}&\frac{\partial J^*(e(\tau),\tau,\sigma)}{\partial\sigma}\\ &\le e_2^T(\sigma)Qe_2(\sigma)+e_2^T(\sigma)K^TRKe_2(\sigma)\\ &+e_2^T(\sigma)Q_f[Ae_2(\sigma)+B(f(z_2(\sigma))-f(x_2(\sigma)))\\ &+CKe_2(\sigma)]+[Ae_2(\sigma)+B(f(z_2(\sigma))-f(x_2(\sigma)))\\ &+CKe_2(\sigma)]^TQ_fe_2(\sigma)\\ &=e_2^T(\sigma)Qe_2(\sigma)+e_2^T(\sigma)K^TRKe_2(\sigma)+e_2^T(\sigma)[Q_fA\\ &+Q_fCK+K^TC^TQ_f+A^TQ_f]e_2(\sigma)\\ &+e_2^T(\sigma)Q_fB(f(z_2(\sigma))-f(x_2(\sigma)))\\ &+(f(z_2(\sigma))-f(x_2(\sigma)))^TB^TQ_fe_2(\sigma). \qquad (9)\end{aligned}$$

If we use the inequality $X^TY+Y^TX\le X^T\Lambda X+Y^T\Lambda^{-1}Y$, which is valid for any matrices $X\in R^{n\times m}$, $Y\in R^{n\times m}$, $\Lambda=\Lambda^T>0$, $\Lambda\in R^{n\times n}$, we have

$$\begin{aligned}&e_2^T(\sigma)Q_fB(f(z_2(\sigma))-f(x_2(\sigma)))\\ &+(f(z_2(\sigma))-f(x_2(\sigma)))^TB^TQ_fe_2(\sigma)\\ &\le e_2^T(\sigma)Q_fBB^TQ_fe_2(\sigma)\\ &+(f(z_2(\sigma))-f(x_2(\sigma)))^T(f(z_2(\sigma))-f(x_2(\sigma)))\\ &\le e_2^T(\sigma)[Q_fBB^TQ_f+L_f^2I]e_2(\sigma). \qquad (10)\end{aligned}$$

Using (10), we have

$$\begin{aligned}&\frac{\partial J^*(e(\tau),\tau,\sigma)}{\partial\sigma}\\ &\le e_2^T(\sigma)\big[Q+K^TRK+Q_fA+Q_fCK+K^TC^TQ_f\\ &+A^TQ_f+Q_fBB^TQ_f+L_f^2I\big]e_2(\sigma). \qquad (11)\end{aligned}$$

If the following matrix inequality is satisfied:

$$\begin{aligned}&Q+K^TRK+Q_fA+Q_fCK+K^TC^TQ_f\\ &\quad+A^TQ_f+Q_fBB^TQ_f+L_f^2I\le 0, \qquad (12)\end{aligned}$$

it is clear that $\frac{\partial J^*(e(\tau),\tau,\sigma)}{\partial\sigma}\le 0$. From Schur complement, the negative semi-definite of (12) is equivalent to

$$\begin{bmatrix}(1,1) & I & K^T & I & Q_fB\\ I & -Q^{-1} & 0 & 0 & 0\\ K & 0 & -R^{-1} & 0 & 0\\ I & 0 & 0 & -\frac{1}{L_f^2}I & 0\\ B^TQ_f & 0 & 0 & 0 & -I\end{bmatrix}\le 0, \qquad (13)$$

where $(1,1)=Q_f(A+CK)+(A+CK)^TQ_f$. Pre- and post-multiplying (13) by $diag(Q_f^{-1},I,I,I,I)$ and introducing change of variables such as $X=Q_f^{-1}$ and $Y=KQ_f^{-1}$, (13) is equivalently changed into the LMI (5). This completes the proof. ∎

In the following theorem, it will be shown that the monotonicity of the optimal cost holds for all subsequent times if it holds once.

Theorem 2. *If $\frac{\partial J^*(e(\tau'),\tau',\sigma)}{\partial\sigma}\le 0$ for some τ', then $\frac{\partial J^*(e(\tau''),\tau'',\sigma)}{\partial\sigma}\le 0$ where $\tau'\le\tau''\le\sigma$.*

Proof: $\frac{\partial J^*(e(\tau'),\tau',\sigma)}{\partial\sigma}$ satisfies the following relation:

$$\begin{aligned}&\frac{\partial J^*(e(\tau'),\tau',\sigma)}{\partial\sigma}\\ &=\lim_{\Delta\to 0}\frac{1}{\Delta}\{J^*(e(\tau'),\tau',\sigma+\Delta)-J^*(e(\tau'),\tau',\sigma)\}\\ &=\lim_{\Delta\to 0}\frac{1}{\Delta}\bigg\{\int_{\tau'}^{\tau''}[e_1^T(t)Qe_1(t)+u_1^T(t)Ru_1(t)]dt\\ &+J^*(e_1(\tau''),\tau'',\sigma+\Delta)\\ &-\int_{\tau'}^{\tau''}[e_2^T(t)Qe_2(t)+u_2^T(t)Ru_2(t)]dt\\ &-J^*(e_2(\tau''),\tau'',\sigma)\bigg\}, \qquad (14)\end{aligned}$$

where $u_1(t)$ and $u_2(t)$ are the optimal controls to minimize $J(e(\tau'),\tau',\sigma+\Delta)$ and $J(e(\tau'),\tau',\sigma)$, respectively. If $u_2(\cdot)$ is replaced by $u_1(\cdot)$ up to τ'', then we have

$$\begin{aligned}&\frac{\partial J^*(e(\tau'),\tau',\sigma)}{\partial\sigma}\\ &\ge\lim_{\Delta\to 0}\frac{1}{\Delta}\Big\{J^*(e_1(\tau''),\tau'',\sigma+\Delta)-J^*(e_1(\tau''),\tau'',\sigma)\Big\}\\ &=\frac{\partial J^*(e_1(\tau''),\tau'',\sigma)}{\partial\sigma}. \qquad (15)\end{aligned}$$

$\frac{\partial J^*(e(\tau'),\tau',\sigma)}{\partial\sigma}\le 0$ implies $\frac{\partial J^*(e(\tau''),\tau'',\sigma)}{\partial\sigma}\le 0$. This completes the proof. ∎

4. Closed-loop Stability of Receding Horizon Synchronization Controller

The RHS controller is obtained by replacing t_0 and t_1 by t and $t+T$, respectively, where T denotes the horizon length satisfying $0<T<\infty$. The stability of the RHS controller is given in the following theorem:

Theorem 3. *If $\frac{\partial J^*(e(t),t,\sigma)}{\partial\sigma}\Big|_{\sigma=t+T}\le 0$, the synchronization error system (3) with the RHS controller is asymptotically stable.*

Proof: $J^*(e(t),t,t+T)$ is given by

$$\begin{aligned}&J^*(e(t),t,t+T)\\ &=\int_t^{t+\mu}[e^{*T}(t)Qe^*(t)+u^{*T}(t)Ru^*(t)]dt\\ &+J^*(e(t+\mu),t+\mu,t+T). \qquad (16)\end{aligned}$$

According to Theorem 2, $\frac{\partial J^*(e(t),t,\sigma)}{\partial\sigma}\Big|_{\sigma=t+T}\le 0$ implies $\frac{\partial J^*(e(t+\mu),t+\mu,\sigma)}{\partial\sigma}\Big|_{\sigma=t+T}\le 0$ for any $0<\mu<T$. Hence,

we have

$$\begin{aligned} & J^*(e(t), t, t+T) \\ & \geq \int_t^{t+\mu} [e^{*T}(t)Qe^*(t) + u^{*T}(t)Ru^*(t)]dt \\ & + J^*(e(t+\mu), t+\mu, t+T+\mu), \end{aligned} \quad (17)$$

which means that $J^*(e(t), t, t+T)$ is strictly decreasing. Therefore, $J^*(e(t), t, t+T) \to c > 0$ as $t \to \infty$. Furthermore, from (17), it is clear that $\int_t^{t+\mu}[e^{*T}(t)Qe^*(t) + u^{*T}(t)Ru^*(t)]dt \to 0$ as $t \to \infty$. Finally, $e(t) \to 0$ and $u(t) \to 0$ as $t \to \infty$. This completes the proof. ∎

This result states that the non-increasing monotonicity of the optimal cost is a sufficient condition for the stability of the RHS controller. Based on Theorem 1, we obtain the following result on the stability of the RHS controller with the finite terminal weighting matrix Q_f.

Corollary 1. *Assume that the finite terminal weighting matrix Q_f in (4) satisfies the LMI condition (5). Then, the synchronization error system (3) with the RHS controller is asymptotically stable.*

Proof: The existence of Q_f satisfying the LMI condition (5) guarantees $\frac{\partial J^*(e(t),t,\sigma)}{\partial \sigma}\Big|_{\sigma=t+T} \leq 0$. Thus, the closed-loop stability follows from Theorem 3. This completes the proof. ∎

5. Numerical Example

In this section, to verify and demonstrate the effectiveness of the proposed method, we discuss the simulation result for synchronizing Chua's chaotic system. Consider the following Chua's chaotic system:

$$\begin{aligned} \dot{x}_1(t) &= -10x_1(t) + 10x_2(t) + \Big[-0.69x_1(t) \\ &\quad - \frac{0.59}{2}(|x_1(t)+1| - |x_1(t)-1|) \Big], \\ \dot{x}_2(t) &= x_1(t) - x_2(t) + x_3(t), \\ \dot{x}_3(t) &= -15x_2(t) - 0.0385x_3(t). \end{aligned} \quad (18)$$

The Chua's chaotic system (18) is rewritten as

$$\dot{x}(t) = Ax(t) + Bf(x(t)), \quad (19)$$

where

$$A = \begin{bmatrix} -10.69 & 10 & 0 \\ 1 & -1 & 1 \\ 0 & -15 & -0.0385 \end{bmatrix}, \quad B = \begin{bmatrix} -0.59 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix}, \quad f(x(t)) = \begin{bmatrix} \frac{1}{2}(|x_1(t)+1| - |x_1(t)-1|) \\ 0 \\ 0 \end{bmatrix}.$$

For the numerical simulation, we use the following parameters:

$$L_f = 1, \quad C = \begin{bmatrix} 1 \\ 1 \\ 0 \end{bmatrix}, \quad Q = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}, \quad R = 1, \quad T = 0.5,$$

where $L_f = 1$ is obtained from the relation $\|f(x(t))\| \leq \|x(t)\|$. In this simulation, we use $T = 0.5$. If T is a big positive constant, the computational burden to obtain the RHS controller may increase very much. In this case, we need to use the high performance hardware for the implementation of the proposed RHS controller. Applying Theorem 1 to the Chua's chaotic system (19) yields

$$X = \begin{bmatrix} 0.0576 & 0.0021 & 0.0323 \\ 0.0021 & 0.0235 & 0.0332 \\ 0.0323 & 0.0332 & 0.2377 \end{bmatrix}, \quad Y = \begin{bmatrix} 0.0470 & -0.3578 & 0.0543 \end{bmatrix}.$$

In this section, in order to solve the LMI feasibility problem in Theorem 1, we utilized MATLAB LMI Control Toolbox [20], which implements state-of-the-art interior-point algorithms. Figure 1 shows state trajectories when the initial states are given by

$$\begin{bmatrix} x_1(0) \\ x_2(0) \\ x_3(0) \end{bmatrix} = \begin{bmatrix} 3.8 \\ 3.1 \\ 2.5 \end{bmatrix}, \quad \begin{bmatrix} z_1(0) \\ z_2(0) \\ z_3(0) \end{bmatrix} = \begin{bmatrix} 1.9 \\ 1.2 \\ -1.1 \end{bmatrix}.$$

From this figure, it can be seen that drive and response systems are indeed achieving chaos synchronization. Figure 2 shows that the proposed RHS method guarantees the asymptotic stability of the synchronization error system.

6. Conclusion

In this paper, we have proposed the RHS controller, which is a new synchronization controller, for chaotic systems. A new LMI condition on the finite terminal weighting matrix was proposed, which guaranteed the monotonicity of the optimal cost. Under this condition, it was shown that the asymptotic stability of the RHS method is guaranteed. Furthermore, the synchronization for the Chua's chaotic system was given to demonstrate the effectiveness of the proposed synchronization method.

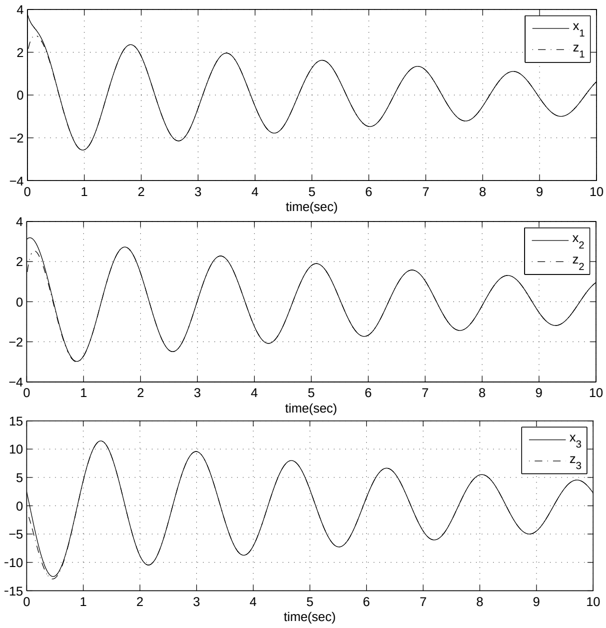

Figure 1 State trajectories

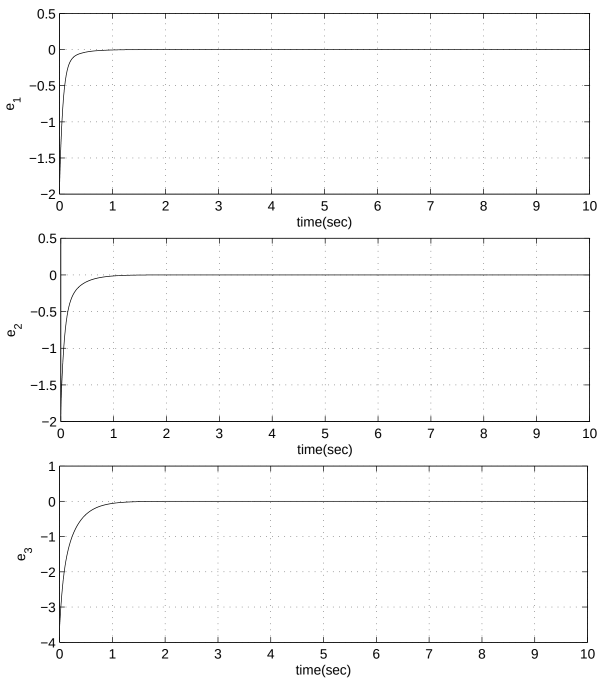

Figure 2 Synchronization errors

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