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On Some Real Valued $I-$Convergent $\Lambda-$Summable Difference Sequence Spaces Defined by Sequences of Orlicz Functions

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Abstract: In this article we introduce some new difference sequence spaces using $I-$convergent and Orlicz function, and study some basic topological and algebraic properties of these spaces. Also we investigate the relations between these spaces.

Keywords: Ideal, I-convergent, Orlicz function, difference sequence

1 Introduction

The notion of $I-$convergence initially introduced by Kostyrko, Salat and Wilczynski [1]. Later on, it was futher investigated from the sequence space point of view and linked with the summability theory by Salat, Tripathy and Ziman [2,3], Tripathy and Hazarika [4,5,6] and Kumar and Kumar [7], Aiyub [26], Khan et.al [27] and many others authors.

Let X be a non-empty set, then a family of sets $I \subset 2^X$ (the class of all subsets of X) is called an *ideal* if and only if for each $A,B \in I$, we have $A \cup B \in I$ and for each $A \in I$ and each $B \subset A$, we have $B \in I$. A non-empty family of sets $F \subset 2^X$ is a *filter* on X if and only if $\phi \notin F$, for each $A,B \in F$, we have $A \cap B \in F$ and each $A \in F$ and each $A \subset B$, we have $B \in F$. An ideal I is called *non-trivial ideal* if $I \neq \phi$ and $X \notin I$. Clearly $I \subset 2^X$ is a non-trivial ideal if and only if $F = F(I) = \{X - A : A \in I\}$ is a filter on X. A non-trivial ideal $I \subset 2^X$ is called *admissible* if and only if $\{\{x\} : x \in X\} \subset I$. A non-trivial ideal I is *maximal* if there cannot exists any non-trivial ideal $J \neq I$ containing I as a subset. Further details on ideals of 2^X can be found in Kostyrko, et.al [1].

Lemma 1.1. ([1, Lemma 5.1]) If $I \subset 2^N$ is a maximal ideal, then for each $A \subset N$ we have either $A \in I$ or $N - A \in I$.

Example 1.2. If we take $I = I_f = \{A \subseteq N : A$ is a finite subset$\}$. Then I_f is a non-trivial admissible ideal of N and the corresponding convergence coincide with the usual convergence.

Example 1.3. If we take $I = I_\delta = \{A \subseteq N : \delta(A) = 0\}$ where $\delta(A)$ denote the asymptotic density of the set A. Then I_δ is a non-trivial admissible ideal of N and the corresponding convergence coincide with the statistical convergence.

Kızmaz [8] defined the difference sequence spaces $\ell_\infty(\Delta), c(\Delta)$ and $c_0(\Delta)$ as follows: For $Z = \ell_\infty, c$ and c_0

$$Z(\Delta) = \{x = (x_k) : (\Delta x_k) \in \Delta Z\},$$

where $\Delta x = (\Delta x_k) = (x_k - x_{k+1})$, for all $k \in N$. The above spaces are Banach spaces, normed by

$$\|x\| = |x_1| + \sup_k \|\Delta x_k\|.$$

The idea of Kizmaz [8] was applied to introduce different type of difference sequence spaces and study their different properties by Tripathy ([9,10]), Et and Esi [11] and many others.

Recall in [12] that an *Orlicz function* M is continuous, convex, nondecreasing function is defined for $x > 0$ such that $M(0) = 0$ and $M(x) > 0$. If convexity of Orlicz function is replaced by $M(x+y) \leq M(x) + M(y)$ then this function is called the *modulus function* and characterized by Ruckle [13]. An Orlicz function M is said to satisfy

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Δ_2-*condition* for all values of u, if there exists $K>0$ such that $M(2u)\leq KM(u)$, $u\geq 0$.

Lemma 1.4. Let M be an Orlicz function which satisfies Δ_2-condition and let $0<\delta<1$. Then for each $t\geq\delta$, we have $M(t)<K\delta^{-1}tM(2)$ for some constant $K>0$.

Two Orlicz functions M_1 and M_2 are said to be *equivalent* if there exist positive constants α,β and x_0 such that

$$M_1(\alpha)\leq M_2(x)\leq M_1(\beta)$$

for all x with $0\leq x<x_0$.

Lindenstrauss and Tzafriri [14] studied some Orlicz type sequence spaces defined as follows:

$$\ell_M=\left\{(x_k)\in w:\sum_{k=1}^{\infty}M\left(\frac{|x_k|}{\rho}\right)<\infty,\text{ for some }\rho>0\right\}.$$

The space ℓ_M with the norm

$$||x||=\inf\left\{\rho>0:\sum_{k=1}^{\infty}M\left(\frac{|x_k|}{\rho}\right)\leq 1\right\}$$

becomes a Banach space which is called an *Orlicz sequence space*. The space ℓ_M is closely related to the space ℓ_p which is an Orlicz sequence space with $M(t)=|t|^p$, for $1\leq p<\infty$.

In the later stage, different classes of Orlicz sequence spaces were introduced and studied by Parashar and Choudhary [15], Esi and Et [16], Et, Altin, Choudhary and Tripathy [17], Altinok, Altin and Isik [18], Gungor, Et and Altin [19], Hazarika et.al [24], Hazarika and Esi [25], Esi and Ozdemir [28] and many others.

Throughout the article N and R denote the set of positive integers and set of real numbers, respectively. The zero sequence is denoted by θ.

A sequence space E is said to be *solid (or normal)* if $(y_k)\in E$ whenever $(x_k)\in E$ and $|y_k|\leq|x_k|$ for all $k\in N$.

Lemma 1.5. ([20, page 53]) A sequence space E is normal implies E is monotone.

Let $n\in N$ and X^n be a real vector space of dimension n. A real-valued function $\|.,\ldots,.\|$ on X^n satisfying the following four conditions:

(i) $\|x_1,x_2,\ldots,x_n\|=0$ if and only if $x_1,x_2,\ldots,x_n$ are linearly dependent,

(ii) $\|x_1,x_2,\ldots,x_n\|$ is invariant under permutation,

(iii) $\|\alpha x_1,x_2,\ldots,x_n\|=|\alpha|\,\|x_1,x_2,\ldots,x_n\|$, $\alpha\in R$,

(iv) $\|x_1+x_1^{\prime},x_2,\ldots,x_n\|\leq\|x_1,x_2,\ldots,x_n\|+\|x_1^{\prime},x_2,\ldots,x_n\|$

is called an $n-$*norm* on X^n, and the pair $(X^n,\|.,\ldots,.\|)$ is called an $n-$*normed space* [21].

A trivial example of $n-$normed space is $X^n=R^n$ equipped with the following Euclidean $n-$norm:

$$\|x_1,x_2,\ldots,x_n\|_E=abs\left(\begin{vmatrix}x_{11}\ldots x_{1n}\\ \ldots\\ x_{n1}\ldots x_{nn}\end{vmatrix}\right)$$

where $x_i=(x_{i1},\ldots,x_{in})\in R^n$ for each $i=1,2,\ldots,n$.

Lemma 1.6. Every n-normed space is an $(n-r)-$normed space for all $r=1,2,\ldots,n-1$. In particular every n-normed space is a normed space.

Lemma 1.7. A standard $n-$normed space is complete if and only if it is complete with respect to usual norm $\|.\|=\sqrt{<,>}$.

Lemma 1.8. On a standard n-normed space X, the derived from $(n-1)-$norm $\|..,.,\ldots\|_\infty$ defined with respect to the orthogonal set $\{e_1,e_2,\ldots,e_n\}$ is equivalent to the standard $(n-1)-$norm $\|..,.,\ldots\|_S$. To be precise, for all $z_1,z_2,\ldots,z_{n-1}\in X$, we have

$$\begin{aligned}\|z_1,z_2,\ldots,z_{n-1}\|_\infty&\leq\|z_1,z_2,\ldots,z_{n-1}\|_S\\&\leq\sqrt{n}\|z_1,z_2,\ldots,z_{n-1}\|_\infty,\end{aligned}$$

where $\|z_1,z_2,\ldots,z_{n-1}\|_\infty=\max_{1\leq i\leq n}\{\|z_1,z_2,\ldots,z_{n-1},e_i\|_S\}$.

Some counter examples for n-normed spaces can be found in Dutta et.al [29].

2 Some New Sequence Spaces

The following well-known inequality will be used throughout the article. Let $p=(p_k)$ be any sequence of positive real numbers with $0\leq p_k\leq\sup_k p_k=G$, $D=\max\{1,2^{G-1}\}$ then

$$|a_k+b_k|^{p_k}\leq D\left(|a_k|^{p_k}+|b_k|^{p_k}\right)$$

for all $k\in N$ and $a_k,b_k\in C$. Also $|a_k|^{p_k}\leq\max\left\{1,|a_k|^G\right\}$ for all $a_k\in C$.

The main aim of this article is to introduce the following sequence spaces and examine topological and algebraic properties of the resulting sequence spaces.

In paper [22], Mursaleen and Noman introduced the notion of λ-convergent and λ-bounded sequences as follows: Let $\lambda=(\lambda_k)_{k=0}^{\infty}$ be a strictly increasing sequence of positive real numbers tending to infinity, that is

$$0<\lambda_o<\lambda_1<\ldots\text{ and }\lambda_k\to\infty\text{ as }k\to\infty$$

and let

$$\Lambda_k(x)=\frac{1}{\lambda_k}\sum_{m=1}^{k}(\lambda_m-\lambda_{m-1})x_m.$$

Let I be an admissible ideal of N and let $p=(p_k)$ be a bounded sequence of positive real numbers for all $k\in N$. Let $\mathbf{M}=(M_k)$ be a sequence of Orlicz functions and $(X,\|.,\ldots,.\|)$ be an $n-$normed space. Further $w(n-X)$ denotes $X-$valued sequence space, we define the following sequence spaces as follows:

$$c^I(\mathbf{M},\Lambda,p,\|...,...\|)_\Delta = \Big\{ (x_k) \in w(n-X): \ \Big\{ k \in N: \left[M_k \left(\left\| \frac{\Lambda_k(\Delta x) - L}{\rho}, z_1, z_2, ..., z_{n-1} \right\| \right) \right]^{p_k} \geq \varepsilon \Big\} \in I, \text{ for some } \rho > 0 \text{ and } L \text{ and for every } z_1, z_2, ..., z_{n-1} \in X \Big\},$$

$$c_o^I(\mathbf{M},\Lambda,p,\|...,...\|)_\Delta = \Big\{ (x_k) \in w(n-X): \ \Big\{ k \in N: \left[M_k \left(\left\| \frac{\Lambda_k(\Delta x)}{\rho}, z_1, z_2, ..., z_{n-1} \right\| \right) \right]^{p_k} \geq \varepsilon \Big\} \in I, \text{ for some } \rho > 0 \text{ and for every } z_1, z_2, ..., z_{n-1} \in X \Big\},$$

$$\ell_\infty^I(\mathbf{M},\Lambda,p,\|...,...\|)_\Delta = \Big\{ (x_k) \in w(n-X): \exists K > 0 \text{ such that } \Big\{ k \in N: \left[M_k \left(\left\| \frac{\Lambda_k(\Delta x)}{\rho}, z_1, z_2, ..., z_{n-1} \right\| \right) \right]^{p_k} \geq K \Big\} \in I, \text{ for some } \rho > 0 \text{ and for every } z_1, z_2, ..., z_{n-1} \in X \Big\}$$

and

$$\ell_\infty(\mathbf{M},\Lambda,p,\|...,...\|)_\Delta = \Big\{ (x_k) \in w(n-X): \exists K > 0 \text{ such that } \sup_k \left[M_k \left(\left\| \frac{\Lambda_k(\Delta x)}{\rho}, z_1, z_2, ..., z_{n-1} \right\| \right) \right]^{p_k} \leq K, \text{ for some } \rho > 0 \text{ and for every } z_1, z_2, ..., z_{n-1} \in X \Big\}.$$

3 Main Results

In this section we examine the basic topological and algebraic properties of these spaces and obtain the inclusion relation between these spaces.

Theorem 3.1. If $\{\Lambda_k(\Delta x), z_1, z_2, ..., z_{n-1}\}$ is a linearly dependent set in $(X, \|...,...\|)$ for all but finite k, where $x = (x_k) \in w(n-X)$ and $\inf_k p_k > 0$, then

(a) $I-\lim_{k\to\infty} \left[M_k \left(\left\| \frac{\Lambda_k(\Delta x)}{\rho}, z_1, z_2, ..., z_{n-1} \right\| \right) \right]^{p_k} = 0$, for some $\rho > 0$,

(b) $\sup_k \left[M_k \left(\left\| \frac{\Lambda_k(\Delta x)}{\rho}, z_1, z_2, ..., z_{n-1} \right\| \right) \right]^{p_k} < \infty$, for some $\rho > 0$.

Proof. (a) Suppose that $\{\Lambda_k(\Delta x), z_1, z_2, ..., z_{n-1}\}$ *is a linearly dependent set in* $(X, \|...,...\|)$ for all but finite k. Then we have

$$\|\Lambda_k(\Delta x), z_1, z_2, ..., z_{n-1}\| \to 0 \text{ as } k \to \infty.$$

Since M_k is continuous for all k and $0 \leq p_k \leq \sup_k p_k = G < \infty$ for each k, then we have

$$I - \lim_{k\to\infty} \left[M_k \left(\left\| \frac{\Lambda_k(\Delta x)}{\rho}, z_1, z_2, ..., z_{n-1} \right\| \right) \right]^{p_k} = 0,$$

for some $\rho > 0$.

(b) The proof of this part is similar to part (a).

Theorem 3.2. $c_o^I(\mathbf{M},\Lambda,p,\|...,...\|)_\Delta$, $c^I(\mathbf{M},\Lambda,p,\|...,...\|)_\Delta$ *and* $\ell_\infty^I(\mathbf{M},\Lambda,p,\|...,...\|)_\Delta$ are linear spaces.

Proof. We will proved the result for the space $c_o^I(\mathbf{M},\Lambda,p,\|...,...\|)_\Delta$ only, and the others can be proved in similar way. Let $x = (x_k)$ and $y = (y_k)$ be two elements in $c_o^I(\mathbf{M},\Lambda,p,\|...,...\|)_\Delta$. Then for every $z_1, z_2, ..., z_{n-1} \in X$ there exist $\rho_1 > 0$ and $\rho_2 > 0$ such that

$$A_{\frac{\varepsilon}{2}} = \Big\{ k \in N: \left[M_k \left(\left\| \frac{\Lambda_k(\Delta x)}{\rho}, z_1, z_2, ..., z_{n-1} \right\| \right) \right]^{p_k} \geq \frac{\varepsilon}{2} \Big\} \in I$$

and

$$B_{\frac{\varepsilon}{2}} = \Big\{ k \in N: \left[M_k \left(\left\| \frac{\Lambda_k(\Delta y)}{\rho}, z_1, z_2, ..., z_{n-1} \right\| \right) \right]^{p_k} \geq \frac{\varepsilon}{2} \Big\} \in I.$$

Let α, β be two scalars. Since $\mathbf{M} = (M_k)$ is a sequence of continuous functions, the following inequality holds:

$$\begin{aligned} &\left[M_k \left(\left\| \frac{\Lambda_k(\Delta(\alpha x + \beta y))}{|\alpha|\rho_1 + |\beta|\rho_2}, z_1, z_2, ..., z_{n-1} \right\| \right) \right]^{p_k} \\ &\leq D \frac{|\alpha|\rho_1}{|\alpha|\rho_1 + |\beta|\rho_2} \left[M_k \left(\left\| \frac{\Lambda_k(\Delta(x))}{\rho_1}, z_1, z_2, ..., z_{n-1} \right\| \right) \right]^{p_k} \\ &\quad + D \frac{|\beta|\rho_2}{|\alpha|\rho_1 + |\beta|\rho_2} \left[M_k \left(\left\| \frac{\Lambda_k(\Delta(y))}{\rho_2}, z_1, z_2, ..., z_{n-1} \right\| \right) \right]^{p_k} \\ &\leq DK \left[M_k \left(\left\| \frac{\Lambda_k(\Delta(x))}{\rho_1}, z_1, z_2, ..., z_{n-1} \right\| \right) \right]^{p_k} \\ &\quad + DK \left[M_k \left(\left\| \frac{\Lambda_k(\Delta(y))}{\rho_2}, z_1, z_2, ..., z_{n-1} \right\| \right) \right]^{p_k}, \end{aligned}$$

where $K = \max \Big\{ 1, \left(\frac{|\alpha|\rho_1}{|\alpha|\rho_1 + |\beta|\rho_2} \right)^G, \left(\frac{|\beta|\rho_2}{|\alpha|\rho_1 + |\beta|\rho_2} \right)^G \Big\}$.

From the above relation we obtain the following:

$$\begin{aligned} &\Big\{ k \in N: \left[M_k \left(\left\| \frac{\Lambda_k(\Delta(\alpha x + \beta y))}{|\alpha|\rho_1 + |\beta|\rho_2}, z_1, z_2, ..., z_{n-1} \right\| \right) \right]^{p_k} \geq \varepsilon \Big\} \subseteq \\ &\Big\{ k \in N: DK \left[M_k \left(\left\| \frac{\Lambda_k(\Delta(x))}{\rho_1}, z_1, z_2, ..., z_{n-1} \right\| \right) \right]^{p_k} \geq \frac{\varepsilon}{2} \Big\} \\ &\cup \Big\{ k \in N: DK \left[M_k \left(\left\| \frac{\Lambda_k(\Delta(y))}{\rho_2}, z_1, z_2, ..., z_{n-1} \right\| \right) \right]^{p_k} \geq \frac{\varepsilon}{2} \Big\} \in I. \end{aligned}$$

This completes the proof.

Remark 3.3. It is easy to verify that the space $\ell_\infty(\mathbf{M},\Lambda,p,\|...,...\|)_\Delta$ is a linear space.

The proof of the following theorem is similar to [23, Theorem 2.2], therefore we omit it.

Theorem 3.4. The space $\ell_\infty(\mathbf{M},\Lambda,p,\|...,...\|)_\Delta$ is a paranormed space (not totally paranormed) with the paranorm g defined by

$$g(x)=\inf\left\{\rho^{\frac{p_k}{G}}:\sup_k\left[M_k\left(\left\|\frac{\Lambda_k(\Delta x)}{\rho},z_1,z_2,...,z_{n-1}\right\|\right)\right]^{p_k}\leq 1,\right.$$
$$\left.\text{for some } \rho>0 \text{ and for every } z_1,z_2,...,z_{n-1}\in X\right\},$$

where $G=\max\{1,\sup p_k\}$.

Theorem 3.5. Let $\mathbf{M}=(M_k)$ and $\mathbf{S}=(S_k)$ be sequences of Orlicz functions. Then the following hold:

(i) $c_o^I(\mathbf{S},\Lambda,p,\|...,...\|)_\Delta\subseteq c_o^I(\mathbf{M}\circ\mathbf{S},\Lambda,p,\|...,...\|)_\Delta$, provided $p=(p_k)$ be such that $G_0=\inf p_k>0$.

(ii) $c_o^I(\mathbf{M},\Lambda,p,\|...,...\|)_\Delta\cap c_o^I(\mathbf{S},\Lambda,p,\|...,...\|)_\Delta\subseteq c_o^I(\mathbf{M}+\mathbf{S},\Lambda,p,\|...,...\|)_\Delta$.

Proof. (i) Let $\varepsilon>0$ be given. Choose $\varepsilon_1>0$ such that $\max\left\{\varepsilon_1^G,\varepsilon_1^{G_0}\right\}<\varepsilon$. Choose $0<\delta<1$ such that $0<t<\delta$ implies that $M_k(t)<\varepsilon_1$ for each $k\in N$. Let $x=(x_k)$ be any element in $c_o^I(\mathbf{S},\Lambda,p,\|...,...\|)_\Delta$. For every $z_1,z_2,...,z_{n-1}\in X$, put

$$A_\delta=\left\{k\in N:\left[S_k\left(\left\|\frac{\Lambda_k(\Delta x)}{\rho},z_1,z_2,...,z_{n-1}\right\|\right)\right]^{p_k}\geq\delta^G\right\}.$$

Then by the definition of ideal we have $A_\delta\in I$. If $k\notin A_\delta$ we have

$$\left[S_k\left(\left\|\frac{\Lambda_k(\Delta x)}{\rho},z_1,z_2,...,z_{n-1}\right\|\right)\right]^{p_k}<\delta^G,\text{ for } k=1,2,...,n$$
$$\Rightarrow S_k\left(\left\|\frac{\Lambda_k(\Delta x)}{\rho},z_1,z_2,...,z_{n-1}\right\|\right)<\delta, \quad (3.1)$$

Since $\mathbf{M}=(M_k)$ is a sequence of continuous functions, from the relation (3.1) we have

$$M_k\left(S_k\left(\left\|\frac{\Lambda_k(\Delta x)}{\rho},z_1,z_2,...,z_{n-1}\right\|\right)\right)<\varepsilon_1,$$

for $k=1,2,3,...,n$. Consequently we get

$$\left[M_k\left(S_k\left(\left\|\frac{\Lambda_k(\Delta x)}{\rho},z_1,z_2,...,z_{n-1}\right\|\right)\right)\right]^{p_k}<\max\left\{\varepsilon_1^G,\varepsilon_1^{G_0}\right\}<\varepsilon$$
$$\Rightarrow\left[M_k\left(S_k\left(\left\|\frac{\Lambda_k(\Delta x)}{\rho},z_1,z_2,...,z_{n-1}\right\|\right)\right)\right]^{p_k}<\varepsilon.$$

This implies that

$$\left\{k\in N:\left[M_k\left(S_k\left(\left\|\frac{\Lambda_k(\Delta x)}{\rho},z_1,z_2,...,z_{n-1}\right\|\right)\right)\right]^{p_k}\geq\varepsilon\right\}$$
$$\subseteq A_\delta\in I.$$

This completes the proof.

(ii) Let $x=(x_k)\in c_o^I(\mathbf{M},\Lambda,p,\|...,...\|)_\Delta\cap c_o^I(\mathbf{S},\Lambda,p,\|...,...\|)_\Delta$. Then by the following inequality the result follows:

$$\left[(M_k+S_k)\left(\left\|\frac{\Lambda_k(\Delta x)}{\rho},z_1,z_2,...,z_{n-1}\right\|\right)\right]^{p_k}$$
$$\leq D\left[M_k\left(\left\|\frac{\Lambda_k(\Delta x)}{\rho},z_1,z_2,...,z_{n-1}\right\|\right)\right]^{p_k}$$
$$+D\left[S_k\left(\left\|\frac{\Lambda_k(\Delta x)}{\rho},z_1,z_2,...,z_{n-1}\right\|\right)\right]^{p_k}.$$

The proof of the following theorems are easy and so omitted.

Theorem 3.6. Let $0<p_k\leq q_k$ *and* $\left(\frac{q_k}{p_k}\right)$ is bounded, then

$$c_o^I(\mathbf{M},\Lambda,q,\|...,...\|)_\Delta\subseteq c_o^I(\mathbf{M},\Lambda,p,\|...,...\|)_\Delta.$$

Theorem 3.7. For any two sequences $p=(p_k)$ and $q=(q_k)$ of positive real numbers, then the following holds:

$$Z(\mathbf{M},\Lambda,p,\|...,...\|)_\Delta\cap Z(\mathbf{M},\Lambda,q,\|...,...\|)_\Delta\neq\phi,$$

for $Z=c^I,c_o^I,\ell_\infty^I$ and ℓ_∞.

Proposition 3.8. The sequence spaces $Z(\mathbf{M},\Lambda,p,\|...,...\|)_\Delta$ are normal as well as monotone for $Z=c_0^I$ and ℓ_∞^I.

Proof. We shall give the prove of the proposition for $c_o^I(\mathbf{M},\Lambda,p,\|...,...\|)_\Delta$ only. Let $x=(x_k)\in c_o^I(\mathbf{M},\Lambda,p,\|...,...\|)_\Delta$ and $y=(y_k)$ be such that $|y_k|\leq|x_k|$ for all $k\in N$. Then for given $\varepsilon>0$ we have

$$B=\left\{k\in N:\left[M_k\left(\left\|\frac{\Lambda_k(\Delta x)}{\rho},z_1,z_2,...,z_{n-1}\right\|\right)\right]^{p_k}\geq\varepsilon\right\}\in I.$$

Again the set

$$E=\left\{k\in N:\left[M_k\left(\left\|\frac{\Lambda_k(\Delta y)}{\rho},z_1,z_2,...,z_{n-1}\right\|\right)\right]^{p_k}\geq\varepsilon\right\}\subseteq B.$$

Hence $E\in I$ and so $y=(y_k)\in c_o^I(\mathbf{M},\Lambda,p,\|...,...\|)_\Delta$. Thus the space $c_o^I(\mathbf{M},\Lambda,p,\|...,...\|)_\Delta$ is normal. Also from the Lemma 1.5., it follows that $c_o^I(\mathbf{M},\Lambda,p,\|...,...\|)_\Delta$ is monotone.

In view of Lemma 1.8., we state the following theorem.

Theorem 3.9. Let X be a standard $n-$normed space and $\{e_1,e_2,...,e_n\}$ be an orthonormal set in X. Then the following holds:

(a) $c^I(M,\Lambda,p,\|...,...\|_\infty)_\Delta=c^I(M,\Lambda,p,\|...,...\|_{n-1})_\Delta$,
(b) $c_o^I(M,\Lambda,p,\|...,...\|_\infty)_\Delta=c_o^I(M,\Lambda,p,\|...,...\|_{n-1})_\Delta$,
(c) $\ell_\infty^I(M,\Lambda,p,\|...,...\|_\infty)_\Delta=\ell_\infty^I(M,\Lambda,p,\|...,...\|_{n-1})_\Delta$,
(d) $\ell_\infty(M,\Lambda,p,\|...,...\|_\infty)_\Delta=\ell_\infty(M,\Lambda,p,\|...,...\|_{n-1})_\Delta$,

where $\|...,...\|_\infty$ is derived $(n-1)-$norm defined with respect to the set $\{e_1,e_2,...,e_n\}$ and $\|...,...\|_{n-1}$ is the standard $(n-1)-$norm on X.

Theorem 3.10. The spaces $c^I(\mathbf{M},\Lambda,p,\|...,...\|)_\Delta$ and $c^I(\mathbf{M},p,\|...,...\|_\infty)_\Delta$ are equivalent as topological spaces.

Proof. Consider the mapping $T:c^I(\mathbf{M},\Lambda,p,\|...,...\|)_\Delta\to c^I(\mathbf{M},p,\|...,...\|_\infty)_\Delta$ defined by $T(x)=(\Lambda_k(\Delta x))$ for each $x=(x_k)\in c^I(\mathbf{M},\Lambda,p,\|...,...\|)_\Delta$. Then clearly T is a linear homeomorphism and the proof follows.

4 Conclusion:

In this paper defined some new difference sequence spaces combining the concepts of Orlicz function ad I-convergence. Further, we proved some topological and algebraic properties of resulting spaces. This notion can be used for further generalization of such spaces.

References

[1] P. Kostyrko, T. Šalàt, W. Wilczyński, Real Analysis Exchange **26(2)**, 669-686 (2000-2001).

[2] T. Šalàt, B. C. Tripathy, M. Ziman, Tatra Mt. Math. Publ **28**, 279-286 (2004).

[3] T. Šalàt, B. C. Tripathy, M. Ziman, Indian Jour. Pure Applied Math **17**, 45-54 (2005).

[4] B. C. Tripathy and B. Hazarika, Math. Ineq. Appl. **11(3)**, 543-548 (2008).

[5] B. C. Tripathy and B. Hazarika, Math. Slovaca **59(4)**, 485-494 (2009).

[6] B. C. Tripathy and B. Hazarika, Acta Math. Appl. Sinica **27(1)**, 149-154 (2011).

[7] V. Kumar and K. Kumar, Inform. Sci. **178**, 4670-4678 (2008).

[8] H. Kizmaz, Canad. Math. Bull. **24**, 169-176 (1981).

[9] B. C. Tripathy, Demonstratio Math. **36(4)**, 867-872 (2003).

[10] B. C. Tripathy, Internat. J. Math. Sci. **2(1)**, 159-166 (2003).

[11] M. Et and A. Esi, Bull. Malaysian Math. Sci. Soc. (Second Series) **23**, 1-8 (2000).

[12] M. A. Krasnoselski and Y. B. Rutitskii, Groningen, Netherland, (1961.

[13] W. H. Ruckle, Canad. J. Math. **25**, 973-978 (1973).

[14] J. Lindenstrauss and L. Tzafriri, Israel J. Math. **101**, 379-390 (1971).

[15] S. D. Parashar and B. Choudhury, Indian J. Pure Appl. Math. **25(14)**, 419-428 (1994).

[16] A. Esi and M. Et, Indian J. Pure Appl. Math. **31(8)**, 967-972 (2000).

[17] M. Et, Y. Altin, B. Choudhary, B. C. Tripathy, Math. Inequal. Appl. **9(2)**, 335-342 (2006).

[18] H. Altinok, Y. Altin, M. Isik, Indian J. Pure Appl. Math. **39(1)**, 49-58 (2008).

[19] M. Gungor, M. Et, Y. Altin, Applied Mathematics and Computation **157(2)**, 561-571 (2004).

[20] P. K. Kamthan and M. Gupta, Marcel Dekker, 1980.

[21] H. Gunawan and M. Mashadi, Internat. J. Math. Math. Sci. **27(10)**, 631-639 (2001).

[22] M. Mursaleen and A. K. Noman, Thai J. Math. **8**, 311-329 (2010).

[23] A. Esi, Kyunpook Math. J. **52**, 137-147 (2012).

[24] B. Hazarika, K. Tamang, B. K. Singh, Journal of Egyptian Mathematical Society **22**, 413-419 (2014).

[25] B. Hazarika and A. Esi, Acta Scientarium.Technology **35(3)**, 527-537 (2013).

[26] M. Aiyub, Inf.Sci.Lett. **4(3)**, 135-140 (2015).

[27] A. V. Khan, N. Khan, Yasmeen and L. Liu, Cogent Mathematics **3(1)**, 1-9 (2016).

[28] A. Esi and M. K. Ozdemir, Math. Slovaca **63(4)**, 829-838 (2013).

[29] H. Dutta, B. S. Reddy, S. S. Cheng, Applied Mathematics E-Notes **10**, 199-209 (2010).

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