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On Fuzzy Three Level Large Scale Linear Programming Problem

O. E. Emam[1,*], *A. M. Abdo*[1] *and A. A. Abohany*[2]

[1] Department of Information Systems, Faculty of Computer Science and Information, Helwan University , P.O. Box 11795, Egypt
[2] Director of Electronic Document Management Systems, Human Resources Behalf, Telecom Egypt, Smart Village, B7, Egypt

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Abstract: This paper suggests an algorithm to solve a three level large scale linear programming problem with fuzzy numbers, where all coefficients of the objective functions are symmetric trapezoidal fuzzy numbers. A three-level programming problem can be thought as a static version of the Stackelberg strategy. The suggested algorithm uses a linear ranking function at each level to define a crisp model which is equivalent to the fuzzy number, then all decision makers attempts to optimize its problem separately as a large scale programming problem using Dantzig and Wolfe decomposition method. Therefore, we handle the optimization process through a series of sub problems that can be solved independently. Finally, a numerical example is given to clarify the main results developed in this paper.

Keywords: Large scale problems; linear programming; ranking; three-level programming; decomposition algorithm.
MSC 2000: 90C06; 90C05; 90C99.

1 Introduction

The basic concept of the three level programming (TLP) technique is that a first level decision maker (FLDM) sets his goals and/or decisions and then asks each subordinate level of the organization for their optima which are calculated in isolation; the second-level decision maker (SLDM) decision is then submitted and modified by the FLDM; Finally, the third-level decision maker (TLDM) decision is submitted and modified by the FLDM and SLDM with consideration of the overall benefit for the organization; and the process continued until an optimal solution is reached [8].

Most studies in multilevel field are focused on bi-level optimization problem [9,13,14,15,16]. Saraj and Safaei [16] introduced an approach to solve a fuzzy linear fractional bilevel multi-objective programming problem (FLFBLMOP) by using of Taylor series and Kuhn-Tucker conditions approach. The Taylor series is an expansion of a series that represents a function. By using the Kuhn-Tucker conditions, the problem is reduced to a single objective.

Commonly, when formulating a large scale programming model which closely describes and represents the real world decision situations, various factors of the real system should be reflected in the description of the objective functions and constraints. Naturally, these objective functions and constraints involve many parameters and the experts may assign them different values, which can be represented by fuzzy numbers .

In the traditional approaches of large scale systems, parameters are fixed at some values in an experimental and/or subjective manner through the experts' understanding of the nature of the parameters. In practice, however, it is natural to consider that the possible values of these parameters are often only ambiguously known to experts' understanding of the parameters as fuzzy numerical data, which can be represented by means of fuzzy subsets of the real line known as fuzzy numbers [2,3,4].

Fuzzy linear programming problems have an essential in fuzzy modeling which can formulate uncertainty in actual environment. Afterwards many authors [1,6,10,11,17] considered various types of the fuzzy linear programming and propose several approaches for solving these problems.

Recently, notable studies have been done in the area of large scale linear and nonlinear programming problems with block angular structure based on Dantzig and Wolfe decomposition method [3,12].

* Corresponding author e-mail: emam_o_e@yahoo.com

This paper is organized as follows: we start in Section 2 by formulation the model of the three level large scale linear programming problem with fuzzy numbers. In Section 3, presents a decomposition algorithm for a three level large scale linear programming problems with fuzzy numbers based on linear ranking function. An algorithm followed by a flowchart for solving a three level large scale linear programming problem with fuzzy numbers is suggested in Section 4 and Section 5. In addition a numerical example is provided in Section 6 to clarify the results. Finally, conclusion and future works are reported in Section 7.

2 Problem Formulation and Solution Concept

The three level large scale linear programming problem with fuzzy numbers in objective functions (TLLSLPPFN) may be formulated as follows:

[First Level]

$$\underset{x_1,x_2}{Max}\tilde{F}_1(x) = \underset{x_1,x_2}{Max}\sum_{j=1}^{m}\tilde{c}_{1j}x_j, \tag{1}$$

where $x_3,\ldots,x_m$ solves

[Second Level]

$$\underset{x_3,x_4}{Max}\tilde{F}_2(x) = \underset{x_3,x_4}{Max}\sum_{j=1}^{m}\tilde{c}_{2j}x_j, \tag{2}$$

where $x_5,\ldots,x_m$ solves

[Third Level]

$$\underset{x_5,x_6}{Max}\tilde{F}_3(x) = \underset{x_5,x_6}{Max}\sum_{j=1}^{m}\tilde{c}_{3j}x_j, \tag{3}$$

where $x_7,\ldots,x_m$ solves

Subject to

$$x \in G. \tag{4}$$

where

$$\begin{aligned} G = \{a_{01}x_1 + a_{02}x_2 \quad & + a_{0m}x_m \le b_0, \\ d_1x_1 \quad & \le b_1, \\ d_2x_2 \quad & \le b_2, \\ & d_mx_m \le b_m, \\ & x_1,\ldots,x_m \ge 0\}. \end{aligned}$$

In the above problem (1)-(4), $x_j \in R, (j = 1,2,\ldots,m)$ be a real vector variables, $\tilde{c}_{ij}$ an n-dimensional row vector of fuzzy parameters in the objective functions, G is the large scale linear constraint set where, $b = (b_0,\ldots,b_m)^T$ is $(m+1)$ vector, and $a_{01},\ldots,a_{0m},d_1,\ldots,d_m$ are constants.

Therefore $F_i : R^m \to R, (i = 1,2,3)$ be the first level objective function, the second level objective function, and the third level objective function, respectively. Moreover, FLDM has x_1,x_2 indicating the first decision level choice, SLDM and TLDM have x_3,x_4 and x_5,x_6 indicating the second decision level choice and the third decision level choice, respectively.

Definition 1. For any $(x_1,x_2 \in G_1 = \{x_1,x_2|\ (x_1,\ldots,x_m) \in G\})$ given by FLDM and $(x_3,x_4 \in G_2 = \{x_3,x_4|(x_1,\ldots,x_m) \in G\})$ given by SLDM, if the decision-making variable $(x_5,x_6 \in G_3 = \{x_5,x_6|(x_1,\ldots,x_m) \in G\})$ is the Pareto optimal solution of the TLDM, then $(x_1,\ldots,x_m)$ is a feasible solution of TLLSLPPFN.

Definition 2. If $x^* \in R^m$ is a feasible solution of the TLLSLPPFN; no other feasible solution $x \in G$ exists, such that $\tilde{F}_1(x^*) \le \tilde{F}_1(x)$; so x^* is the Pareto optimal solution of the TLLSLPPFN.

3 A Decomposition Technique for the TLLSLPPFN Based on Linear Ranking

To solve the TLLSLPPFN using a decomposition technique based on a linear ranking technique, the FLDM convert fuzzy number form into equivalent crisp form by using linear ranking function [17], then he/she gets the optimal solution using the decomposition method [7] to break the large scale problem into n-sub problems that can be solved directly. Then by inserting the FLDM decision variable to the SLDM for him/her to seek the optimal solution using linear ranking function [17] combine with the decomposition method [7]. Finally the TLDM do the same action till he obtains the optimal solution of his problem which is the optimal solution to TLLSLPPFN.

Definition 3. [17] $\tilde{A} = (a,b,c,d) \in F(R)$, then a linear ranking function is defined as $\Re(\tilde{A}) = a+b+\frac{1}{2}(d-c)$.

Definition 4. [17] Let $\tilde{A}(a_1,b_1,c_1,d_1)$ and $\tilde{B}(a_2,b_2,c_2,d_2)$ be two trapezoidal fuzzy numbers and $x \in G$. A convenient method for comparing of the fuzzy numbers is by using of ranking functions. A ranking function is a map from $F(R)$ into the real line. So , the orders on $F(R)$ as follows:

1. $\tilde{A} \geq \tilde{B}$ if and only if $\Re(\tilde{A}) \geq \Re(\tilde{B})$.
2. $\tilde{A} > \tilde{B}$ if and only if $\Re(\tilde{A}) > \Re(\tilde{B})$.
3. $\tilde{A} = \tilde{B}$ if and only if $\Re(\tilde{A}) = \Re(\tilde{B})$

where $\tilde{A}$ and $\tilde{B}$ are in $F(R)$.

3.1 The First-Level Decision-Maker (FLDM) Problem

The first-level decision-maker problem of the (TLLSLPPFN) is as follows:

[First Level]

$$\underset{x_1,x_2}{Max} \tilde{F}_1(x) = \underset{x_1,x_2}{Max} \sum_{j=1}^{m} \tilde{c}_{1j} x_j, \tag{5}$$

Subject to

$x \in G$.

The FLDM transforms fuzzy numbers form into equivalent crisp form by using of linear ranking function equation.

The FLDM problem can be reduced to the following form [17]:

[First Level]

$$Max F_1(x) = Max \sum_{j=1}^{m} c_{1j} x_j, \tag{6}$$

Subject to

$x \in G$.

To obtain the optimal solution of the FLDM problem, the FLDM solves his problem by the decomposition technique [7].

The decomposition principle is based on representing the TLLSLPP in terms of the extreme points of the sets $d_j x_j \leq b_j, x_j \geq 0, j = 1,2,\ldots,m$. To do so, the solution space described by each $d_j x_j \leq b_j, x_j \geq 0, j = 1,2,\ldots,m$ must be bounded and closed, for more details see [7].

To obtain the optimal solution of the FLDM problem of the TLLSLPPFN is as follows: Suppose that the extreme points of $d_j x_j \leq b_j, x_j \geq 0$ are defined as $\hat{x}_{jk}, k = 1,2,3$, where x_j defined by:

$$x_j = \sum_{k=1}^{k_j} \beta_{jk} \hat{x}_{jk}, \;\; j = 1,\ldots,m. \tag{7}$$

and $\beta_{jk} \geq 0$, for all k and $\sum_{k=1}^{k_j} \beta_{jk} = 1$.

Now, the FLDM problem in terms of the extreme points to obtain the following master problem of the FLDM are formulated as stated in [7]:

$$Max \sum_{k=1}^{k_1} c_{11} \hat{x}_{1k} \beta_{1k} + \sum_{k=1}^{k_2} c_{12} \hat{x}_{2k} \beta_{2k} + \cdots + \sum_{k=1}^{k_n} c_{1n} \hat{x}_{nk} \beta_{nk}, \tag{8}$$

Subject to

$$\sum_{k=1}^{k_1} a_{01}\hat{x}_{1k}\beta_{1k}+\sum_{k=1}^{k_2} a_{02}\hat{x}_{2k}\beta_{2k}+\cdots+\sum_{k=1}^{k_n} a_{0n}\hat{x}_{nk}\beta_{nk}\le b_0,$$
$$\sum_{k=1}^{k_1}\beta_{1k}=1,$$
$$\sum_{k=1}^{k_2}\beta_{2k}=1,$$
$$\sum_{k=1}^{k_n}\beta_{nk}=1,$$
$$\beta_{jk}\ge 0, \text{for all } j \text{ and } k.$$

The new variables in the FLDM problem are β_{jk} which determined using Balinski's algorithm [5]. Once their optimal values β^*_{jk} are obtained, then the optimal solution to the original problem can be found by back substitution as follow:

$$x_j=\sum_{k=1}^{k_1}\beta^*_{jk}\hat{x}_{jk}, j=1,2,3. \tag{9}$$

It may appear that the solution of the FLDM problem requires prior determination of all extreme points $\hat{x}_{jk}$.

To solve the FLDM problem by the revised simplex method, it must determine the entering and leaving variables at each iteration. Let us start first with the entering variables.

Given C_B and B^{-1} of the current basis of the FLDM problem, then for non-basic β_{jk}:

$$z_{jk}-c_{jk}=C_B B^{-1}P_{jk}-c_{jk}, \tag{10}$$

Where

$$C_{jk}=C_j\hat{x}_{jk} \quad\text{and}\quad P_{jk}=\begin{bmatrix} a_j\hat{x}_{jk}\\ 0\\ 1\\ 0\end{bmatrix}. \tag{11}$$

Now, to decide which of the variables β_{jk} should enter the solution it must determine:

$$z^*_{jk}-c^*_{jk}=\min\{z_{jk}-c_{jk}\}. \tag{12}$$

Consequently, if $z^*_{jk}-c^*_{jk}\le 0$, then according to the maximization optimality condition, β^*_{jk} must enter the solution; otherwise, the optimal has been reached.

The SLDM and the TLDM solve their master problem by the decomposition method [7] as the FLDM.

3.2 The Second-Level Decision-Maker (SLDM) Problem

According to the mechanism of the TLLSLPPFN, the FLDM variables x_1^F, x_2^F must be given to the SLDM; hence, the SLDM problem of the (TLLSLPPFN) can be written as follows :

[Second Level]

$$\underset{x_3,x_4}{Max}\tilde{F}_2(x)=\underset{x_3,x_4}{Max}\sum_{j=1}^{m}\tilde{c}_{2j}x_j, \tag{13}$$

Subject to

$x\in G,$

where

$G_2=(x_1^F,x_2^F,\ldots,c_m).$

The SLDM problem can be reduced to the following form [17] :

[Second Level]

$$Max F_2(x)=Max\sum_{j=1}^{m}c_{2j}x_j, \tag{14}$$

Subject to
$x \in G_2$.

To obtain the optimal solution of the SLDM problem; the SLDM solves his problem by the decomposition technique [7] as the FLDM.

3.3 The Third-Level Decision-Maker (TLDM) Problem

According to the mechanism of the TLLSLPPFN, the SLDM variables $x_1^F, x_2^F, x_3^S, x_4^S$ must be given to the TLDM; hence, the TLDM problem can be written as follows:
[Third Level]

$$\underset{x_5,x_6}{Max} \tilde{F}_3(x) = \underset{x_5,x_6}{Max} \sum_{j=1}^{m} \tilde{c}_{3j} x_j, \tag{15}$$

Subject to
$x \in G_3$
where
$G_3 = (x_1^F, x_2^F, x_3^S, x_4^S, \ldots, x_m)$.
The TLDM problem can be reduced to the following form [17] :
[Third Level]

$$Max\, F_3(x) = Max \sum_{j=1}^{m} c_{3j} x_j, \tag{16}$$

Subject to
$x \in G_3$.
To obtain the optimal solution of the TLDM problem; the TLDM solves his problem by the decomposition technique [7] as the FLDM.
Now the optimal solution $(x_1^F, x_2^F, x_3^S, x_4^S, x_5^T, x_6^T, \ldots, x_m^T)$ of the TLDM is the optimal solution of the TLLSLPPFN.

4 An Algorithm for Solving a TLLSLPPFN

A solution algorithm to solve (TLLSLPPFN) is described in a series of steps. This algorithm uses linear ranking function to convert the fuzzy number into real line to overcome the complexity nature of the three level large scale linear programming problem with fuzzy numbers, and uses the constraint method of the three level optimization to facility the large scale nature. Inserting the variables value of every higher level decision maker to his lower level decision maker break the difficulty faces the problem.

The suggested algorithm can be summarized in the following manner.

Step 1. Formulate FLDM problem in form (8) and solve it.

Step 2. If FLDM obtain his optimal solution set $(x_1, x_2) = (x_1^F, x_2^F)$ go to Step 9 , otherwise go to Step 3.

Step 3. Compute $\Re(\tilde{A})$ for all the cofficients of the problem (1)-(4) where $\tilde{A}$ is trpezoidal Fuzzy number, go to Step 4.

Step 4. Set $k = 1$.

Step 5. Compute $z_{jk} - c_{jk} = C_B B^{-1} P_{jk} - C_{jk}$, go to Step 6.

Step 6. If $z_{jk}^* - c_{jk}^* \leq 0$, then go to Step 7; otherwise, the optimal solution has been reached, go to Step 8.

Step 7. Set $k = k + 1$, go to Step 5.

Step 8. The decision maker use arithmetic on fuzzy numbers $x\tilde{A} = (xa_1, xb_1, xc_1, xd_1)$, $\tilde{A} + \tilde{B} = (a_1 - a_2, b_1 - b_2, c_1 + c_2, d_1 + d_2)$ to transform crisp form into equivalent fuzzy numbers form, go to Step 2.

Step 9. If SLDM obtain his optimal solution set $(x_1, x_2, x_3, x_4) = (x_1^F, x_2^F, x_3^S, x_4^S)$ then go to Step 11 , otherwise go to Step 10.

Step 10. Formulate SLDM problem in form (8), go to Step 3.

Step 11. If the TLDM obtain the optimal solution go to Step 13 , otherwise go to Step 12.

Step 12. Formulate TLDM in form (8), go to Step 3.

Step 13. Put $(x_1^F, x_2^F, x_3^S, x_4^S, x_5^T, x_6^T, \ldots, x_m^T$ is an optimal solution for three-level large scale linear programming problem with fuzzy numbers, then stop.

5 A Flowchart for Solving a TLLSLPPFN

A flowchart to explain the suggested algorithm for solving a three-level large scale linear programming problem with fuzzy numbers is described as follows.

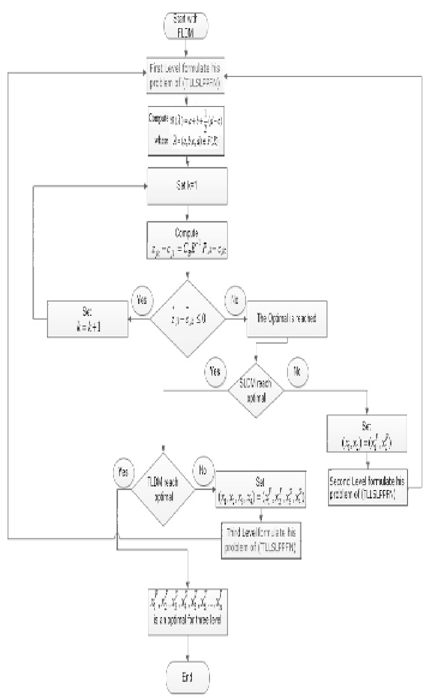

Fig. 1: A three level large scale linear programming problem with fuzzy numbers in objective functions flowchart.

Remark 1. For TLLSLPPFN, the winqsb package is suggested as a basic solution tool.

6 Numerical Example

[First Level]
$\underset{x_1,x_2}{Max}\tilde{F}_1(x_1,x_2) = \underset{x_1,x_2}{Max}(2,3,1,1)x_1 + (1,3,1,1)x_2 + (1,2,1,1)x_5,$
where x_3,x_4,x_5,x_6 solves

[Second Level]
$\underset{x_3,x_4}{Max}\tilde{F}_2(x) = \underset{x_3,x_4}{Max}(3,5,1,1)x_3 + (2,3,1,1)x_4 + (1,2,1,1)x_5 + (1,2,1,1)x_6,$
where x_5,x_6 solves

[Third Level]
$\underset{x_5,x_6}{Max}\tilde{F}_3(x_5,x_6) = \underset{x_5,x_6}{Max}(1,2,1,1)x_3 + (3,5,1,1)x_5 + (2,4,1,1)x_6.$
Subject to

$x_1 + x_2 + x_3 + x_4 + x_5 + x_6 \leq 50,$
$2x_1 + x_2 \leq 40,$
$5x_3 + x_4 \leq 12,$
$x_5 + x_6 \leq 20,$
$x_5 + 5x_6 \leq 80,$
$x_1,x_2,x_3,x_4,x_5,x_6 \geq 10.$

Firstly, the FLDM solves his problem as follow:
1- Applying ranking function $\Re(\tilde{A}) = a + b + \frac{1}{2}(d - c)$ where $\tilde{A} = (a,b,c,d) \in F(R)$ to transform the fuzzy number form into equivalent crisp form. So, the problem reduces to

$MaxF_1(x) = Max5x_2 + 3x_5$
subject to
$x \in G.$

2- Set $k = 1$, so the slack variable x_7 convert common constraint into equation x_8,x_9,x_{10} are artificial variables as :

$$x_1 + x_2 + x_3 + X_4 + x_5 + x_6 + x_7 = 50.$$

Let's identify iteration 0 as:

$$X_B = (X_7,x_8,x_9,x_{10})^T, X_B = (50,1,1,1)^T, C_B = (0,-M,-M,-M), B = 1, B^{-1} = 1.$$

Now iteration 1 for sub problem 1 where j=1 is :
$Min\, w_1 = -5x_1 - 4x_1 - M,$
Subject to
$2x_1 + x_2 \leq 40,$
$x_1,x_2 \geq 0,$
$\hat{x}_{11} = (0,40),\ \ w_1^* = -160 - M.$
For sub problem 2 where $j = 2$
$Min\, w_2 = -M,$
Subject to
$5x_3 + x_4 \leq 12,$
$x_3,x_4 \geq 0,$
$\hat{x}_{21} = (0,0),\ \ w_2^* = -M.$
For sub problem 3 where $j = 3$
$Min\, w_3 = -x_5 - x_6 - M,$
Subject to
$x_5 + x_6 \leq 20,$
$x_5 + 5x_6 \leq 80,$
$x_5,x_6 \geq 0.$
$\hat{x}_{31} = (20,0),\ \ w_3^* = -60 - M.$
After 4 iterations the FLDM obtain his optimal solution
$(x_1^F,x_2^F,x_3^F,x_4^F,x_5^F,x_6^F) = (10,20,0,0,20,0).$

Now set $(x_1^F, x_2^F) = (10, 20)$ to the SLDM constraints.
Secondly, the SLDM solves his/her problem as follows:
Applying ranking function $\Re(\hat{A}) = a + b + \frac{1}{2}(d - c)$ where $\tilde{A} = (a, b, c, d) \in F(R)$ to transform the fuzzy number form into equivalent crisp form. So, the problem reduces to
$Max\, F_2(X) = Max\, 8x_3 + 5x_4 + 3x_5 + 3x_6$
Subject to

$x \in G_2$.

The SLDM do the same action like FLDM till he obtains the optimal solution $(x_3^S, x_4^S, x_5^S, x_6^S) = (0, 12, 8, 0)$, now set $(x_3^S, x_4^S) = (0, 12)$ to the TLDM constraints.
Finally, the TLDM solves his/her problem as follows:
Applying ranking function $\Re(\hat{A}) = a + b + \frac{1}{2}(d - c)$ where $\tilde{A} = (a, b, c, d) \in F(R)$ to transform the fuzzy number form into equivalent crisp form. So, the problem reduces to
$Max\, F_3(x_5, x_6) = Max\, 8x_5 + 6x_6$,
Subject to

$x \in G_3$.

The TLDM do the same action like FLDM and SLDM till he obtain the optimal solution $(x_5^T, x_6^T) = (8, 0)$.
SO $(x_1^F, x_2^F, x_3^S, x_4^S, x_5^T, x_6^T) = (10, 20, 0, 12, 8, 0)$ is the optimal solution for three level large scale linear programming problem with fuzzy numbers.
Where $\tilde{F}_1 = (48, 106, 38, 38), \tilde{F}_2 = (32, 52, 20, 20)$ and $\tilde{F}_3 = (24, 40, 8, 8)$.

7 Conclusion

This paper suggested an algorithm to solve a three level large scale linear programming problem with fuzzy numbers, where all coefficients of the objective functions are trapezoidal fuzzy numbers. A three-level programming problem can be thought as a static version of the Stackelberg strategy. The suggested algorithm used a linear ranking function at each level to define a crisp model which is equivalent to the fuzzy number, then all decision makers' attempts to optimize its problem separately as a large scale programming problem using Dantzig and Wolfe decomposition method. Therefore, we handled the optimization process through a series of sub problems that can be solved independently. The solution algorithm has a few features:

1. It combines both a decomposition algorithm and linear ranking function to obtain an optimal solution solution for the TLLSLPPFN.
2. The objective functions and constraints involve many parameters and the descision makers may assign them different values.
3. It can be efficiently coded.
4. It is found that the decomposition based method generally leads better results than the traditional simplex-based methods. Especially, the efficiency of the decomposition-based method increased sharply with the scale of the problem.

Finally, a numerical example was given to clarify the main results developed in this paper. However, there are many other aspects, which should by explored and studied in the area of a large scale multi-level optimization such as:
1- Large scale multi-level fractional programming problem with fuzzy parameters in the objective functions and in the constraints and with integrality conditions.
2- Large scale multi-level fractional programming problem with stochastic parameters in the objective functions and in the constraints and with integrality conditions.
3- Large scale multi-level fractional programming problem with rough parameters in the objective functions and in the constraints and with integraity conditions.

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