Information Sciences Letters

Volume 5
Issue 3 *Sep. 2016*

Article 2

2016

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Information Sciences Letters
An International Journal

http://dx.doi.org/10.18576/isl/050302

N-Group SU-Action and its Applications to N-Group Theory

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Received: 22 Jul. 2016, Revised: 10 Aug.2016, Accepted: 22 Aug. 2016
Published online: 1 Sep. 2016

Abstract: In this paper, we define a new type of N-group action, called N-group soft union (SU) action on a soft set. This new concept illustrates how a soft set effects on an N-group structure in the mean of union and inclusion of sets and it functions as a bridge among soft set theory, set theory and N-group theory. Furthermore, we derive its basic properties with illustrative examples, investigate the relationship between N-group SI-action defined in [32] and N-group SU-action and obtain some analog of classical N-group theoretic concepts for N-group SU-action. Finally, we give the applications of N-group SU-actions to N-group theory.

Keywords: Soft sets, N-group SI-action, N-group SU-action, N-ideal SU-action, soft pre-image, soft anti image, α-inclusion.

1 Introduction

Molodtsov [23] introduced soft set theory in 1999 for dealing with uncertainties and it continues to experience tremendous growth and diversification in the mean of algebraic structures as in [1,2,10,14,15,16,18,19,26,28,29,30,31,34].

Operations of soft sets have been studied by some authors. Maji et al. [20] presented some definitions on soft sets and based on the analysis of several operations on soft sets Ali et al. [3] introduced several operations of soft sets and Sezgin and Atagün [27] studied on soft set operations as well. Moreover, soft set relations and functions [4] and soft mappings [22] with many related concepts were discussed. The theory of soft set also has a wide range of applications especially in soft decision making as in the following studies: [5,6,13,21,24].

Sezgin et al. [32] introduced a new concept to the literature of N-group, called *N-group soft intersection action* and abbreviated as "*N-group SI-action*". In this paper, we define a new type of N-group action on a soft set, which we call *N-group soft union action* and abbreviate as "*N-group SU-action*". While N-group SI-action is based on the inclusion relation and intersection of sets, N-group SU-action is based on the inclusion relation and union of sets. Since N-group SU-action gathers soft set theory, set theory and N-group theory, it is useful in improving the soft set theory with respect to N-group structures. Based on this new concept, we then introduce the concepts of N-ideal SU-action and we show that if N is a zero-symmetric near-ring, then every N-ideal SU-action over U is an N-group SU-action over U. Moreover, we investigate these notions with respect to soft pre-image, soft anti image and α-inclusion of soft sets and obtain a significant relationship between N-group SI-action and N-group SU-action. Finally, we give some applications of N-group SU-action to N-group theory.

2 Preliminaries

In this section, we recall some basic notions relevant to N-groups and soft sets. By a *near-ring*, we shall mean an algebraic system $(N,+,.)$, where

N1)$(N,+)$ forms a group (not necessarily abelian)
N2)$(N,.)$ forms a semigroup and
N3)$(a+b)c = ac+bc$ for all $a,b,c \in N$ (i.e. we study on right near-rings.)

Throughout this paper, N will always denote a right near-ring. A normal subgroup I of N is called a left ideal of N

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if $n(s+i)-ns \in I$ for all $n,s \in N$ and $i \in I$ and denoted by $I \lhd_\ell N$.

Let $(\Gamma,+)$ be a group and

$$\mu : N \times \Gamma \to \Gamma$$
$$(n,\gamma) \to n\gamma$$

(Γ,μ) is called a *near-ring module* or *N-group* if $\forall x,y \in N$, $\forall \gamma \in \Gamma$,

i)$x(y\gamma) = (xy)\gamma$ and
ii)$(x+y)\gamma = x\gamma + y\gamma$.

It is denoted by Γ. Clearly N itself is an N-group by natural operation. Let G be a group, written additively but not necessarily abelian, and let $M(G)$ be the set $\{f | f : G \to G\}$ of all functions from G to G. An addition operation can be defined on $M(G)$: given f, g in $M(G)$, then the mapping $f+g$ from G to G is given by $(f+g)(x) = f(x)+g(x)$ for all x in G. Then $(M(G),+)$ is also a group, which is abelian if and only if G is abelian. Taking the composition of mappings as the product , $M(G)$ becomes a near-ring. Let G be a group. Then, under the operation below:

$$\mu : M(G) \times G \to G$$
$$(f,a) \quad \to f(a)$$

G is an $M(G)$-group. For a near-ring N, the zero-symmetric part of N denoted by N_0 is defined by $N_0 = \{n \in N \mid n0 = 0\}$. A subgroup Δ of Γ with $N\Delta \subseteq \Delta$ is said to be an *N-subgroup* of Γ and denoted by $\Delta \leq_N \Gamma$. A normal subgroup Δ of Γ is called an *N-ideal* of Γ and denoted by $\Delta \trianglelefteq_N \Gamma$, if $\forall \gamma \in \Gamma$, $\forall \delta \in \Delta$, $\forall n \in N$, $n(\gamma+\delta) - n\gamma \in \Delta$. Let N be a near-ring, Γ and Ψ two N-groups. Then, $h : \Gamma \to \Psi$ is called an *N-homomorphism* if $\forall \gamma,\delta \in \Gamma$, $\forall n \in N$,

i)$h(\gamma+\delta) = h(\gamma)+h(\delta)$ and
ii)$h(n\gamma) = nh(\gamma)$.

For all undefined concepts and notions we refer to [25]. From now on, U refers to an initial universe, E is a set of parameters, $P(U)$ is the power set of U and $A,B,C \subseteq E$.

Definition 1.[6,23] *A soft set f_A over U is a set defined by*

$$f_A : E \to P(U) \text{ such that } f_A(x) = \emptyset \text{ if } x \notin A.$$

Here, f_A is also called approximate function. A soft set over U can be represented by the set of ordered pairs

$$f_A = \{(x, f_A(x)) : x \in E, f_A(x) \in P(U)\}.$$

It is clear to see that a soft set is a parametrized family of subsets of the set U. It is worth noting that the sets $f_A(x)$ may be arbitrary. Some of them may be empty, some may have nonempty intersection. If we define more than one soft set in a subset A of the set of parameters E, then the soft sets will be denoted by f_A, g_A, h_A etc. If we define more than one soft set in some subsets A, B, C etc. of parameters E, then the soft sets will be denoted by f_A, f_B, f_C etc., respectively. We refer to [6,11,12,20,23] for further details.

Definition 2.[6] *Let f_A and f_B be soft sets over U. Then, f_A is a soft subset of f_B, denoted by $f_A \tilde{\subseteq} f_B$, if $f_A(x) \subseteq f_B(x)$ for all $x \in E$.*

Complement of the soft set f_A over U, denoted by f_A^c, is defined as $f_A^c(\alpha) = U \setminus f_A(\alpha)$ for all $\alpha \in E$.

Definition 3.[6] *Let f_A and f_B be soft sets over U. Then, union of f_A and f_B, denoted by $f_A \tilde{\cup} f_B$, is defined as $f_A \tilde{\cup} f_B = f_{A\tilde{\cup}B}$, where $f_{A\tilde{\cup}B}(x) = f_A(x) \cup f_B(x)$ for all $x \in E$.*

Intersection of f_A and f_B, denoted by $f_A \tilde{\cap} f_B$, is defined as $f_A \tilde{\cap} f_B = f_{A\tilde{\cap}B}$, where $f_{A\tilde{\cap}B}(x) = f_A(x) \cap f_B(x)$ for all $x \in E$.

Definition 4.[6] *Let f_A and f_B be soft sets over U. Then, $\vee$-product of f_A and f_B, denoted by $f_A \vee f_B$, is defined as $f_A \vee f_B = f_{A\vee B}$, where $f_{A\vee B}(x,y) = f_A(x) \cup f_B(y)$ for all $(x,y) \in E \times E$.*

$\wedge$-product of f_A and f_B, denoted by $f_A \wedge f_B$, is defined as $f_A \wedge f_B = f_{A\wedge B}$, where $f_{A\wedge B}(x,y) = f_A(x) \cap f_B(y)$ for all $(x,y) \in E \times E$.

Definition 5.[7] *Let f_A and f_B be soft sets over the common universe U and Ψ be a function from A to B. Then, soft image of f_A under Ψ, denoted by $\Psi(f_A)$, is a soft set over U by*

$$(\Psi(f_A))(b) = \begin{cases} \bigcup\{f_A(a) \mid a \in A \text{ and } \Psi(a) = b\}, & \text{if } \Psi^{-1}(b) \neq \emptyset, \\ \emptyset, & \text{otherwise} \end{cases}$$

for all $b \in B$. And soft pre-image (or soft inverse image) of f_B under Ψ, denoted by $\Psi^{-1}(f_B)$, is a soft set over U by $(\Psi^{-1}(f_B))(a) = f_B(\Psi(a))$ for all $a \in A$.

Definition 6.[8] *Let f_A and f_B be soft sets over the common universe U and Ψ be a function from A to B. Then, soft anti image of f_A under Ψ, denoted by $\Psi^\star(f_A)$, is a soft set over U by $(\Psi^\star(f_A))(b) = \begin{cases} \bigcap\{f_A(a) \mid a \in A \text{ and } \Psi(a) = b\}, & \text{if } \Psi^{-1}(b) \neq \emptyset, \\ \emptyset, & \text{otherwise} \end{cases}$ for all $b \in B$.*

Theorem 1.[8] *Let f_A and f_B be soft sets over U, f_A^c, f_B^c be their complements, respectively and Ψ be a function from A to B. Then,*

i)$\Psi^{-1}(f_B^c) = (\Psi^{-1}(f_B))^c$.
ii)$\Psi(f_A^c) = (\Psi^\star(f_A))^c$ and $\Psi^\star(f_A^c) = (\Psi(f_A))^c$.

Definition 7.[9] *Let f_A be a soft set over U and α be a subset of U. Then, upper α-inclusion of f_A, denoted by $f_A^{\supseteq\alpha}$, and lower α-inclusion of f_A, denoted by $f_A^{\subseteq\alpha}$, are defined as*

$$f_A^{\supseteq\alpha} = \{x \in A \mid f_A(x) \supseteq \alpha\} \text{ and } f_A^{\subseteq\alpha} = \{x \in A \mid f_A(x) \subseteq \alpha\},$$

respectively.

3 N-group SU-actions and N-ideal SU-actions

In this section, we first define *N-group soft union actions*, abbreviated as *N-group SU-actions* and N-ideal SU-actions with illustrative examples. We then study their basic properties with respect to soft set operations.

Definition 8. *Let Γ be an N-group and f_Γ be a soft set over U. Then, f_Γ is called a N-group SU-action over U if it satisfies the following properties:*

i) $f_\Gamma(x+y) \subseteq f_\Gamma(x) \cup f_\Gamma(y)$,
ii) $f_\Gamma(-x) = f_\Gamma(x)$,
iii) $f_\Gamma(nx) \subseteq f_\Gamma(x)$

for all $x,y \in \Gamma$ and $n \in N$.

Example 1. Let $N = \{0,1,2,3\}$ be the (right) near-ring due to [25] (Near-rings of low order (D-5)) with the following tables:

+	0	1	2	3
0	0	1	2	3
1	1	2	3	0
2	2	3	0	1
3	3	0	1	2

.	0	1	2	3
0	0	0	0	0
1	0	1	1	0
2	0	2	2	0
3	0	3	3	0

Let $\Gamma = N$ be the sets of parameters and $U = \left\{ \begin{bmatrix} x & 0 \\ x & 0 \end{bmatrix} \mid x,y \in \mathbb{Z}_4 \right\}$, 2×2 matrices with $\mathbb{Z}_4$ terms, is the universal set. We construct a soft set f_Γ over U by

$$f_\Gamma(0) = \left\{ \begin{bmatrix} 2 & 0 \\ 2 & 0 \end{bmatrix} \right\}$$

$$f_\Gamma(1) = f_\Gamma(2) = f_\Gamma(3) = \left\{ \begin{bmatrix} 1 & 0 \\ 1 & 0 \end{bmatrix}, \begin{bmatrix} 2 & 0 \\ 2 & 0 \end{bmatrix}, \begin{bmatrix} 3 & 0 \\ 3 & 0 \end{bmatrix} \right\}.$$

Then, one can easily show that the soft set f_Γ is an N-group SU-action over U.

Example 2. In Example 1, assume that Γ is again the set of parameters and $U = S_4$ is the universal set. We define a soft set f_Γ by

$$f_\Gamma(0) = \{e\}, \quad f_\Gamma(1) = \{e,(13)(24)\},$$
$$f_\Gamma(2) = \{e,(12)(34),(1234),(2134)\} \text{ and}$$
$$f_\Gamma(3) = \{e,(13)(24),(134)\}.$$

Since $f_\Gamma(2\cdot 1) = f_\Gamma(2) \nsubseteq f_\Gamma(1)$, f_Γ is not an N-group SU-action over U.

It is known that if $N = N_0$, then $n0_\Gamma = 0_\Gamma$ for all $n \in N$. Therefore, if N is a zero-symmetric near-ring and if we take $\Gamma = \{0_\Gamma\}$, then f_Γ is an N-group SU-action over U no matter how f_Γ is defined and no matter what U is.

Proposition 1. *Let f_Γ be an N-group SU-action over U. Then, $f_\Gamma(0_\Gamma) \subseteq f_\Gamma(x)$ for all $x \in \Gamma$.*

Proof. Assume that f_Γ is an N-group SU-action over U. Then, for all $x \in \Gamma$, $f_\Gamma(0_\Gamma) = f_\Gamma(x-x) \subseteq f_\Gamma(x) \cup f_\Gamma(-x) = f_\Gamma(x) \cup f_\Gamma(x) = f_\Gamma(x)$.

Theorem 2. *Let Γ be an N-group and f_Γ be a soft set over U. Then, f_Γ is an N-group SU-action over U if and only if*

i) $f_\Gamma(x-y) \subseteq f_\Gamma(x) \cup f_\Gamma(y)$
ii) $f_\Gamma(nx) \subseteq f_\Gamma(x)$

for all $x,y \in \Gamma$ and $n \in N$.

Proof. Suppose that f_Γ is an N-group SU-action over. Then, by Definition 8, $f_\Gamma(xy) \subseteq f_\Gamma(y)$ and $f_\Gamma(x-y) \subseteq f_\Gamma(x) \cup f_\Gamma(-y) = f_\Gamma(x) \cup f_\Gamma(y)$ for all $x,y \in \Gamma$.
Conversely, assume that $f_\Gamma(xy) \subseteq f_\Gamma(y)$ and $f_\Gamma(x-y) \subseteq f_\Gamma(x) \cup f_\Gamma(y)$ for all $x,y \in \Gamma$. If we choose $x = 0_\Gamma$, then

$$f_\Gamma(0_\Gamma - y) = f_\Gamma(-y) \subseteq f_\Gamma(0_\Gamma) \cup f_\Gamma(y) = f_\Gamma(y)$$

by Proposition 1. Similarly, $f_\Gamma(y) = f_\Gamma(-(-y)) \subseteq f_\Gamma(-y)$, thus $f_\Gamma(-y) = f_\Gamma(y)$ for all $y \in \Gamma$. Also, by assumption $f_\Gamma(x+y) \subseteq f_\Gamma(x) \cup f_\Gamma(-y) = f_\Gamma(x) \cup f_\Gamma(y)$. Thus, the proof is completed.

Theorem 3. *Let f_Γ be an N-group SU-action over U. If $f_\Gamma(x-y) = f_\Gamma(0_\Gamma)$ for any $x,y \in \Gamma$, then $f_\Gamma(x) = f_\Gamma(y)$.*

Proof. Assume that $f_\Gamma(x-y) = f_\Gamma(0_\Gamma)$ for any $x,y \in \Gamma$. Then,

$$\begin{aligned} f_\Gamma(x) &= f_\Gamma(x-y+y) \\ &\subseteq f_\Gamma(x-y) \cup f_\Gamma(y) \\ &= f_\Gamma(0_\Gamma) \cup f_\Gamma(y) \\ &= f_\Gamma(y) \end{aligned}$$

and accordingly

$$\begin{aligned} f_\Gamma(y) &= f_\Gamma((y-x)+x) \\ &\subseteq f_\Gamma(y-x) \cup f_\Gamma(x) \\ &= f_\Gamma(-(y-x)) \cup f_\Gamma(x) \\ &= f_\Gamma(0_\Gamma) \cup f_\Gamma(x) \\ &= f_\Gamma(x). \end{aligned}$$

Thus, $f_\Gamma(x) = f_\Gamma(y)$, completing the proof.

It is known that if Γ is an N-group, then $(\Gamma,+)$ is a group but not necessarily abelian. That is, for any $x,y \in \Gamma$, $x+y$ needs not be equal to $y+x$. However, we have the following:

Theorem 4. *Let f_Γ be an N-group SU-action over U and $x \in \Gamma$. Then, for all $y \in \Gamma$*

$$f_\Gamma(x) = f_\Gamma(0_\Gamma) \Leftrightarrow f_\Gamma(x+y) = f_\Gamma(y+x) = f_\Gamma(y)$$

Proof. Suppose that $f_\Gamma(x+y) = f_\Gamma(y+x) = f_\Gamma(y)$ for all $y \in \Gamma$. Then by choosing $y = 0_\Gamma$, we obtain that $f_\Gamma(x) = f_\Gamma(0_\Gamma)$. Conversely, assume that $f_\Gamma(x) = f_\Gamma(0_\Gamma)$. Then, by Proposition 1, we have

$$f_\Gamma(0_\Gamma) = f_\Gamma(x) \subseteq f_\Gamma(y), \quad \forall y \in \Gamma. \tag{1}$$

Since f_Γ is an N-group SU-action over U, then

$$f_\Gamma(x+y) \subseteq f_\Gamma(x) \cup f_\Gamma(y) = f_\Gamma(y), \quad \forall y \in \Gamma.$$

Furthermore, for all $y \in \Gamma$

$$\begin{aligned} f_\Gamma(y) &= f_\Gamma((-x)+x)+y) \\ &= f_\Gamma(-x+(x+y)) \\ &\subseteq f_\Gamma(-x) \cup f_\Gamma(x+y) \\ &= f_\Gamma(x) \cup f_\Gamma(x+y) \\ &= f_\Gamma(x+y) \end{aligned}$$

Because, by (1), $f_\Gamma(x) \subseteq f_\Gamma(y)$ for all $y \in \Gamma$ and $x, y \in \Gamma$ implies that $x + y \in \Gamma$. Thus, $f_\Gamma(x) \subseteq f_\Gamma(x+y)$ and sit follows that $f_\Gamma(x+y) = f_\Gamma(y)$ for all $y \in \Gamma$. Now, let $x \in \Gamma$. Then, for all $y \in \Gamma$

$$\begin{aligned} f_\Gamma(y+x) &= f_\Gamma(y+x+(y-y)) \\ &= f_\Gamma(y+(x+y)-y) \\ &\subseteq f_\Gamma(y) \cup f_\Gamma(x+y) \cup f_\Gamma(y) \\ &= f_\Gamma(y) \cup f_\Gamma(x+y) \\ &= f_\Gamma(y), \end{aligned}$$

since $f_\Gamma(x+y) = f_\Gamma(y)$. Moreover, for all $y \in \Gamma$,

$$\begin{aligned} f_\Gamma(y) &= f_\Gamma(y+(x-x)) \\ &= f_\Gamma((y+x)-x) \\ &\subseteq f_\Gamma(y+x) \cup f_\Gamma(x) \\ &= f_\Gamma(y+x) \end{aligned}$$

by (1). It follows that $f_\Gamma(y+x) = f_\Gamma(y)$, so $f_\Gamma(x+y) = f_\Gamma(y+x) = f_\Gamma(y)$ for all $y \in \Gamma$.

In [32], Sezgin et al. showed that $\wedge$-product of two N-group SI-actions over U is an N-group SI-action. However, we have the following for N-group SU-actions:

Theorem 5. *If f_Γ and f_Δ are N-group SU-actions over U, then so is $f_\Gamma \vee f_\Delta$ over U.*

Proof. By Definition 4, let $f_\Gamma \vee f_\Delta = f_{\Gamma\vee\Delta}$, where $f_{\Gamma\vee\Delta}(x,y) = f_\Gamma(x) \cup f_\Delta(y)$ for all $(x,y) \in E \times E$. Since Γ and Δ are N-groups, then $\Gamma \times \Delta$ is an $N \times N$-group. So, let $(x_1,y_1),(x_2,y_2) \in \Gamma \times \Delta$ and $(n_1,n_2) \in N \times N$. Then,

$$\begin{aligned} f_{\Gamma\vee\Delta}((x_1,y_1)-(x_2,y_2)) &= f_{\Gamma\vee\Delta}(x_1-x_2, y_1-y_2) \\ &= f_\Gamma(x_1-x_2) \cup f_\Delta(y_1-y_2) \\ &\subseteq (f_\Gamma(x_1) \cup f_\Gamma(x_2)) \cup (f_\Delta(y_1) \cup f_\Delta(y_2)) \\ &= (f_\Gamma(x_1) \cup f_\Delta(y_1)) \cup (f_\Gamma(x_2) \cup f_\Delta(y_2)) \\ &= f_{\Gamma\vee\Delta}(x_1,y_1) \cup f_{\Gamma\vee\Delta}(x_2,y_2) \end{aligned}$$

$$\begin{aligned} f_{\Gamma\vee\Delta}((n_1,n_2)(x_1,y_1)) &= f_{\Gamma\vee\Delta}(n_1x_1, n_2y_1) \\ &= f_\Gamma(n_1x_1) \cup f_\Delta(n_2y_1) \\ &\subseteq f_\Gamma(x_1) \cup f_\Delta(y_1) \\ &= f_{\Gamma\vee\Delta}(x_1,y_1) \end{aligned}$$

Thus, $f_\Gamma \vee f_\Delta$ is an N-group SU-action over U.

In [32], Sezgin et al. showed that if f_Γ and h_Γ are two N-group SI-actions over U, then so is $f_\Gamma \tilde{\cap} h_\Gamma$ over U. However, we have the following for N-group SU-actions.

Theorem 6. *If f_Γ and h_Γ are two N-group SU-actions over U, then so is $f_\Gamma \tilde{\cup} h_\Gamma$ over U.*

Proof. Let $x, y \in \Gamma$ and $n \in N$, then

$$\begin{aligned} (f_\Gamma \tilde{\cup} h_\Gamma)(x-y) &= f_\Gamma(x-y) \cup h_\Gamma(x-y) \\ &\subseteq (f_\Gamma(x) \cup f_\Gamma(y)) \cup (h_\Gamma(x) \cup h_\Gamma(y)) \\ &= (f_\Gamma(x) \cup h_\Gamma(x)) \cup (f_\Gamma(y) \cup h_\Gamma(y)) \\ &= (f_\Gamma \tilde{\cup} h_\Gamma)(x) \cup (f_\Gamma \tilde{\cup} h_\Gamma)(y), \end{aligned}$$

$$\begin{aligned} (f_\Gamma \tilde{\cup} h_\Gamma)(nx) &= f_\Gamma(nx) \cup h_\Gamma(nx) \\ &\subseteq f_\Gamma(x) \cup h_\Gamma(x) \\ &= (f_\Gamma \tilde{\cup} h_\Gamma)(x) \end{aligned}$$

Therefore, $f_\Gamma \tilde{\cup} h_\Gamma$ is an N-group SU-action over U.

Definition 9. *Let Γ be an N-group and f_Γ be an N-group SU-action over U. Then, f_Γ is called an N-ideal SU-action of Γ over U if it satisfies the following properties:*

i) $f_\Gamma(x+y) \subseteq f_\Gamma(x) \cup f_\Gamma(y)$,
ii) $f_\Gamma(-x) = f_\Gamma(x)$,
iii) $f_\Gamma(x+y-x) \subseteq f_\Gamma(y)$,
iv) $f_\Gamma(n(x+y)-nx) \subseteq f_\Gamma(y)$,

for all $x, y \in \Gamma$ and $n \in N$. Here, note that $f_\Gamma(x+y) \subseteq f_\Gamma(x) \cup f_\Gamma(y)$ and $f_\Gamma(-x) = f_\Gamma(x)$ imply $f_\Gamma(x-y) \subseteq f_\Gamma(x) \cup f_\Gamma(y)$.

Example 3. Let $N = \{0,1,2,3\}$ be the (right) near-ring due to [25] (Near-rings of low order (D-10)) with the following tables:

+	0	1	2	3
0	0	1	2	3
1	1	2	3	0
2	2	3	0	1
3	3	0	1	2

.	0	1	2	3
0	0	0	0	0
1	0	1	2	1
2	0	2	0	2
3	0	3	2	3

Let $\Gamma = N$ be the sets of parameters and $U = D_3$, dihedral group, be the universal set. We define a soft set f_Γ over U by

$$f_\Gamma(0) = \{e,x\},\ f_\Gamma(1) = f_\Gamma(3) = \{e,x,yx,yx^2\},\ f_\Gamma(2) = \{e,x,yx^2\}.$$

Then, one can show that f_Γ is an N-ideal SU-action of Γ over U.

Example 4. Let $N = \{0,a,b,c\}$ be the (right) near-ring per scheme 2 ([25], p. 408) under the operations defined by the following tables:

+	0	a	b	c
0	0	a	b	c
a	a	0	c	b
b	b	c	0	a
c	c	b	a	0

.	0	a	b	c
0	0	0	0	0
a	0	0	a	a
b	0	a	b	b
c	0	a	c	c

Let $\Gamma = N$ be the sets of parameters and $U = \mathbb{Z}^-$ be the universal set. We define a soft set f_Γ over U by $f_\Gamma(0) = \{-3\}$, $f_\Gamma(a) = \{-3,-5,-9\}$, $f_\Gamma(b) = \{-3,-5,-9,-11,-15\}$,

$$f_\Gamma(c) = \{-3,-11,-15\}.$$

Since $f_\Gamma(a(c+c)-ac) = f_\Gamma(a0-ac) = f_\Gamma(0-a) = f_\Gamma(0+a) = f_\Gamma(a) \nsubseteq f_\Gamma(c)$, f_Γ is not an N-ideal SU-action of Γ over U.

It is known that if N is a zero-symmetric near-ring, then every N-ideal of Γ is also an N-subgroup of Γ [25]. Here, we have an analog for this case:

Theorem 7. *Let N be a zero-symmetric near-ring. Then, every N-ideal SU-action over U is an N-group SU-action over U.*

Proof. Let f_Γ be an N-ideal SU-action of Γ over U. Since $f_\Gamma(n(x+y)-nx) \subseteq f_\Gamma(y)$, for all $x,y \in \Gamma$ and $n \in N$, in particular for $x = 0_\Gamma$, it follows that $f_\Gamma(n(0_\Gamma+y)-n0_\Gamma) = f_\Gamma(ny-0_\Gamma) = f_\Gamma(ny) \subseteq f_\Gamma(y)$. Since the other conditions is satisfied by Definition 9, f_Γ is an N-group SU-action over U.

In [32], Sezgin et al. showed that $\wedge$-product of two N-ideal SI-actions over U is an N-ideal SI-action over U. However, we have the following for N-ideal SU-action:

Theorem 8. *If f_Γ is an N-ideal SU-action of Γ and f_Δ is an N-ideal SU-action of Δ over U, then $f_\Gamma \vee f_\Delta$ is an N-ideal SU-action of $\Gamma \times \Delta$ over U.*

Proof. Let $(x_1,y_1),(x_2,y_2)$ and $(n_1,n_2) \in N \times N$. Then $f_{\Gamma\vee\Delta}((x_1,y_1)-(x_2,y_2)) \subseteq f_{\Gamma\vee\Delta}(x_1,y_1) \cup f_{\Gamma\vee\Delta}(x_2,y_2)$ can be shown similar to Theorem 5. Now,

$$\begin{aligned} f_{\Gamma\vee\Delta}((x_1,y_1)+(x_2,y_2)-(x_1,y_1)) &= f_{\Gamma\vee\Delta}(x_1+x_2-x_1,y_1+y_2-y_1) \\ &= f_\Gamma(x_1+x_2-x_1) \cup f_\Delta(y_1+y_2-y_1) \\ &\subseteq f_\Gamma(x_2) \cup f_\Delta(y_2) \\ &= f_{\Gamma\vee\Delta}(x_2,y_2), \end{aligned}$$

and

$$\begin{aligned} & f_{\Gamma\vee\Delta}((n_1,n_2)((x_1,y_1)+(x_2,y_2))-(n_1,n_2)(x_1,y_1)) \\ &= f_{\Gamma\vee\Delta}(n_1(x_1+x_2)-n_1x_1,n_2(y_1+y_2)-n_2y_1) \\ &= f_\Gamma(n_1(x_1+x_2)-n_1x_1) \cup f_\Delta(n_2(y_1+y_2)-n_2y_1) \\ &\subseteq f_\Gamma(x_2) \cup f_\Delta(y_2) \\ &= f_{\Gamma\vee\Delta}(x_2,y_2). \end{aligned}$$

Therefore, $f_\Gamma \vee f_\Delta$ is an N-ideal SU-action of $\Gamma \times \Delta$ over U.

In [32], Sezgin et al. showed that if f_Γ and h_Γ are two N-ideal SI-actions of Γ over U, then so is $f_\Gamma \widetilde{\cap} h_\Gamma$ over U. However, we have the following for N-ideal SU-actions:

Theorem 9. *If f_Γ and h_Γ are two N-ideal SU-actions of Γ over U, then $f_\Gamma \widetilde{\cup} h_\Gamma$ is an N-ideal SU-action of Γ over U.*

Proof. Let $x,y \in \Gamma$ and $n \in N$. Then,

$$(f_\Gamma \widetilde{\cup} h_\Gamma)(x-y) \subseteq (f_\Gamma \widetilde{\cup} h_\Gamma)(x) \cup (f_\Gamma \widetilde{\cup} h_\Gamma)(y)$$

can be shown similar to Theorem 6. Now,

$$\begin{aligned} (f_\Gamma \widetilde{\cup} h_\Gamma)(x+y-x) &= f_\Gamma(x+y-x) \cup h_\Gamma(x+y-x) \\ &\subseteq f_\Gamma(y) \cup h_\Gamma(y) \\ &= (f_\Gamma \widetilde{\cup} h_\Gamma)(y) \end{aligned}$$

$$\begin{aligned} (f_N \widetilde{\cup} h_N)(n(x+y)-nx) &= f_N(n(x+y)-nx) \cup h_N(n(x+y)-nx) \\ &\subseteq f_N(y) \cup h_N(y) \\ &= (f_N \widetilde{\cup} h_N)(y) \end{aligned}$$

Therefore, $f_\Gamma \widetilde{\cup} h_\Gamma$ is an N-ideal SU-action of Γ over U.

4 Applications of N-group SU-actions and N-ideal SU-actions

In this section, first we obtain the relation between N-ideal SI-action and N-ideal SU-action of an N-group over U and then give the applications of soft pre-image, soft anti image, lower α-inclusion of soft sets and N-homomorphism to N-group theory with respect to N-group SU-actions and N-ideal SU-actions.

Theorem 10. *Let f_Γ be a soft set over U. Then, f_Γ is an N-ideal SU-action of Γ over U if and only if f_Γ^c is an N-ideal SI-action of Γ over U.*

Proof. Let f_Γ be an N-ideal SU-action of Γ over U. Then, for all $x,y \in \Gamma$ and $n \in N$,

$$\begin{aligned} f_\Gamma^c(x-y) &= U \setminus f_\Gamma(x-y) \\ &\supseteq U \setminus ((f_\Gamma(x) \cup f_\Gamma(y)) \\ &= (U \setminus f_\Gamma(x)) \cap (U \setminus f_\Gamma(y)) \\ &= f_\Gamma^c(x) \cap f_\Gamma^c(y), \end{aligned}$$

Also,

$$\begin{aligned} f_\Gamma^c(x+y-x) &= U \setminus f_\Gamma(x+y-x) \\ &\supseteq U \setminus (f_\Gamma(y)) \\ &= f_\Gamma^c(y) \end{aligned}$$

Furthermore,

$$\begin{aligned} f_\Gamma^c(n(x+y)-nx) &= U \setminus f_\Gamma(n(x+y)-nx) \\ &\supseteq U \setminus (f_\Gamma(y)) \\ &= f_\Gamma^c(y) \end{aligned}$$

which shows that f_Γ^c is an N-ideal SI-action of Γ over U. The converse can be shown similarly.

Theorem 11. *If f_Γ is an N-ideal SU-action of Γ over U, then $\Gamma_f = \{x \in \Gamma : f_\Gamma(x) = f_\Gamma(0_\Gamma)\}$ is an N-ideal of Γ.*

Proof. It is obvious that $0_\Gamma \in \Gamma_f \subseteq \Gamma$. We need to show that (i) $x-y \in \Gamma_f$, (ii) $\gamma+x-\gamma \in \Gamma_f$ and (iii) $n(\gamma+x)-n\gamma \in \Gamma_f$ for all $x,y \in \Gamma_f$ and $n \in N$ and $\gamma \in \Gamma$. If $x,y \in \Gamma_f$, then $f_\Gamma(x) = f_\Gamma(y) = f_\Gamma(0_\Gamma)$. By Proposition 1,

$$f_\Gamma(0_\Gamma) \subseteq f_\Gamma(x-y),\ f_\Gamma(0_\Gamma) \subseteq f_\Gamma(\gamma+x-\gamma) \text{ and } f_\Gamma(0_\Gamma) \subseteq f_\Gamma(n(\gamma+x)-n\gamma)$$

for all $n \in N$, $x,y \in \Gamma_f$ and $\gamma \in \Gamma$. Since f_Γ is an N-ideal SU-action of Γ over U, then for all $n \in N$, $x,y \in \Gamma_f$ and $\gamma \in \Gamma$

$$\begin{aligned} (i)\ & f_\Gamma(x-y) \subseteq f_\Gamma(x) \cup f_\Gamma(y) = f_\Gamma(0_\Gamma), \\ (ii)\ & f_\Gamma(\gamma+x-\gamma) \subseteq f_\Gamma(x) = f_\Gamma(0_\Gamma) \text{ and} \\ (iii)\ & f_\Gamma(n(\gamma+x)-n\gamma) \subseteq f_\Gamma(x) = f_\Gamma(0_\Gamma). \end{aligned}$$

Hence,

$$f_\Gamma(x-y) = f_\Gamma(0_\Gamma),\ f_\Gamma(\gamma+x-\gamma) = f_\Gamma(0_\Gamma) \text{ and } f_\Gamma(n(\gamma+x)-n\gamma) = f_\Gamma(0_\Gamma)$$

for all $n \in N$, $x,y \in \Gamma_f$ and $\gamma \in \Gamma$. Therefore, Γ_f is an N-ideal of Γ.

Theorem 12. *[32] Let f_Γ be a soft set over U and α be a subset of U such that $\emptyset \subseteq \alpha \subseteq f_\Gamma(0_\Gamma)$. If f_Γ is an N-ideal SI-action over U, then $f_\Gamma^{\supseteq\alpha}$ is an N-ideal of Γ.*

Theorem 13. *Let f_Γ be a soft set over U and α be a subset of U such that $\emptyset \subseteq f_\Gamma(0_\Gamma) \subseteq \alpha$. If f_Γ is an N-ideal SU-action of Γ over U, then $f_\Gamma^{\subseteq\alpha}$ is an ideal of Γ.*

Proof. Since $f_\Gamma(0_\Gamma) \subseteq \alpha$, then $0_\Gamma \in f_\Gamma^{\subseteq\alpha}$ and $\emptyset \neq f_\Gamma^{\subseteq\alpha} \subseteq \Gamma$. Let $x,y \in f_\Gamma^{\subseteq\alpha}$, then

$$f_\Gamma(x) \subseteq \alpha \text{ and } f_\Gamma(y) \subseteq \alpha.$$

We need to show that $(i)\ x-y \in f_{\Gamma}^{\subseteq\alpha}$, $(ii)\ \gamma+x-\gamma \in f_{\Gamma}^{\subseteq\alpha}$ and $(iii)\ n(\gamma+x)-n\gamma \in f_{\Gamma}^{\subseteq\alpha}$ for all $x,y \in f_{\Gamma}^{\subseteq\alpha}$, $n \in N$ and $\gamma \in \Gamma$. Since f_Γ is an N-ideal SI-action of Γ over U, it follows that

$$f_\Gamma(x-y) \subseteq f_\Gamma(x) \cup f_\Gamma(y) \subseteq \alpha \cup \alpha = \alpha,$$
$$f_\Gamma(\gamma+x-\gamma) \subseteq f_\Gamma(x) \subseteq \alpha \text{ and}$$
$$f_\Gamma(n(\gamma+x)-n) \subseteq f_\Gamma(x) \subseteq \alpha.$$

Thus, the proof is completed.

Theorem 14.[32] *Let f_Γ and f_Δ be soft sets over U and Ψ be an N-isomorphism from Γ to Δ. If f_Γ is an N-ideal SI-action of Γ over U, then $\Psi(f_\Gamma)$ is an N-ideal SI-action of Δ over U.*

Theorem 15. *Let f_Γ and f_Δ be soft sets over U and Ψ be an N-isomorphism from Γ to Δ. If f_Γ is an N-ideal SU-action of Γ over U, then $\Psi^\star(f_\Gamma)$ is an N-ideal SU-action of Δ over U.*

Proof. Let f_Γ be an N-ideal SU-action of Γ over U. Then, f_Γ^c is an N-ideal SI-action of Γ over U by Theorem 10 and $\Psi(f_\Gamma^c)$ is an N-ideal SI-action of Δ over U by Theorem 14. Thus, $\Psi(f_\Gamma^c) = (\Psi^\star(f_\Gamma))^c$ is an N-ideal SI-action of Δ over U by Theorem 1 (ii). Therefore, $\Psi^\star(f_\Gamma)$ is an N-ideal SU-action of Δ over U by Theorem 10.

Theorem 16.[32] *Let f_Γ and f_Δ be soft sets over U and Ψ be an N-homomorphism from N to Δ. If f_Δ is an N-ideal SI-action of Δ over U, then $\Psi^{-1}(f_\Delta)$ is an N-ideal SI-action of Γ over U.*

Theorem 17. *Let f_Γ and f_Δ be soft sets over U and Ψ be an N-homomorphism from Γ to Δ. If f_Δ is an N-ideal SU-action of Δ over U, then $\Psi^{-1}(f_\Delta)$ is an N-ideal SU-action of Γ over U.*

Proof. Let f_Δ be an N-ideal SU-action of Δ over U. Then, f_Δ^c is an N-ideal SI-action of Δ over U by Theorem 10 and $\Psi^{-1}(f_\Delta^c)$ is an N-ideal SI-action of Γ over U by Theorem 16. Thus, $\Psi^{-1}(f_\Delta^c) = (\Psi^{-1}(f_\Delta))^c$ is an N-ideal SI-action of Γ over U by Theorem 1 (i). Therefore, $\Psi^{-1}(f_\Delta)$ is an N-ideal SU-action of Γ over U by Theorem 10.

5 Conclusion

In this paper, we have defined a new kind of N-group action on a soft set, called N-group SU-action. This new concept is very functional for obtaining results in the mean of N-group structure, since it brings the soft sets, sets and N-groups together. Based on the definition, we have introduced the concept of N-ideal SU-action of an N-group. We have then investigated this notion with respect to soft pre-image, soft anti image and lower α-inclusion of soft sets. Finally, we obtain the relationship between N-group SI-action and N-group SU-action and give some applications of these new concepts to N-group theory. To extend this study, one can further study the other algebraic structures such as algebras in view of their SU-actions.

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