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On Some Generalized Structures in Fuzzy Soft Topological Spaces

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Abstract: Molodstov introduced soft sets to deal with uncertainties and not clear objects. Researchers developed the relationship between soft sets and information systems. Fuzzification of the notions of soft sets and soft topology have been discussed in the literature. In this paper, we define and establish the topological structures of fuzzy soft semi-open sets and fuzzy soft semi-closed sets. We introduce and explore fuzzy soft semi-interior, fuzzy soft semi-closure and fuzzy soft semi-exterior. The relationship between fuzzy soft semi-open(closed), fuzzy soft semi-interior(closure) and fuzzy soft open(closed)sets, fuzzy soft interior(closure) introduced by Chang [C. L. Chang, Journal of Mathematical Analysis and Applications, **24(1)**, 182- 190(1968)], Tanay and Kandemir [B. Tanay and M. B. Kandemir, Computers and Mathematics with Applications, **61(10)**, 2952- 2957(2011)] and further studied by Varol and Aygun [B. P. Varol and H. Aygun, Hacettepe Journal of Mathematics and Statistics, **41(3)**, 407-419 (2012)] have been discussed. We also initiate and discuss the characterizations and properties of fuzzy soft semi-boundary and establish the behavior of fuzzy soft semi-boundary with fuzzy soft semi-interior, fuzzy soft semi-exterior, and fuzzy soft semi-closure. Moreover, we define and explore the characterizations and properties of fuzzy soft semi-continuous and fuzzy soft semi-open mappings in fuzzy soft topological spaces. Theses findings are not only important but also will provide the base for further study on fuzzy soft topology towards practical life applications.

Keywords: Fuzzy soft sets, Fuzzy soft topology, Fuzzy soft semi-open(closed), Fuzzy soft semi-interior(exterior), Fuzzy soft semi-boundary, Fuzzy soft semi-continuous(open) function.

1 Introduction

The notion of fuzzy set was introduced by Zadeh in his classical paper [1]. By presenting this concept, he provided a natural foundation for treating mathematically the fuzzy phenomena, which exists previously in our real world. Chang [2] generalized the basic concepts of general topology in fuzzy setting and established the new and modern theory of fuzzy topology. Solving problems in practical life, fuzzy topology found to be very useful.
Molodtsov [3] introduced soft sets theory as a new mathematical tool to deal with uncertainties and unclear defined objects. In [4], he mentioned several directions for the applications of soft sets such as smoothness of functions, game theory, operations research, Riemann-integration, Perron integration, probability and theory of measurement for modelling the problems in engineering physics, computer science, economics, social sciences and medical sciences. In soft systems a very general frame work has been provided with the involvement of parameters. Recently, researches on soft set theory and its applications in various disciplines and real life problem are now catching momentum and progressing rapidly in different areas. Maji et. al [5,6] defined and studied several basic notions of soft set theory and discussed in detail the application of soft set theory in decision making problems. Xiao et. al [7] and Pei et. al [8] studied the relationship between soft sets and information systems. Kostek [9] presented the criteria to measure sound quality using approach of soft sets. Kong et. al [10] presented the normal parameter reduction of soft sets. Mushrif et. al [11] used the notions of soft set theory to develop the remarkable method for the classification of natural textures. Many researchers contributed towards the algebraic structures of soft set theory.
Shabir and Naz [12] initiated the study of soft topological spaces and defined basic notions of soft topological spaces. After that, Hussain et. al [13] [14], Aygunoglu

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et.al [15], Zoultrana et. al [16] added many notions and concepts towards the properties of soft topological spaces. Recently, Bin Chen [17,18] introduced and explored the properties of soft semi-open sets and soft-semi-closed sets in soft topological spaces. S. Hussain [19] continued to explore the properties of soft semi-open , soft semi-closed sets. He defined and established the properties and characterizations of soft semi-exterior, soft semi-boundary, soft semi-open neighborhood and soft semi-open neighborhood systems in soft topological spaces. Moreover he discussed the characterizations and properties of soft semi-interior, soft semi-exterior, soft semi-closure and soft semi-boundary in soft topological spaces.
Maji et. al [20] introduced the notion of fuzzy soft sets by combining the notion of fuzzy sets and soft sets. Dua et. al [21] fuzzified the very useful 9-intersection Egenhofer model [22,23] for depicting topological relations in geographic information systems. Ahmad and Kharral [24] explored the properties of fuzzy soft sets and defined the concept of a mapping on fuzzy soft sets. Also Feng et.al [25], Aktas and Cagman [26] and Ali et. al [27] improved the work of Maji et. al [20].
Tanay and Kendemir [28] initiated the concept of fuzzy soft topology using the fuzzy soft sets introduced by Chang [2] and explored its basic properties. Varol and Aygun [29] studied the topological structures of fuzzy soft set theory. Gundaz and Bayramov [30] explored the structural properties of fuzzy soft topological spaces.
In this paper, We define and establish the topological structures of fuzzy soft semi-open sets and fuzzy soft semi-closed sets. We introduce and explore the fuzzy soft semi-interior, fuzzy soft semi-closure and fuzzy soft semi-exterior. We discuss the relationship between fuzzy soft semi-open(closed), fuzzy soft semi-interior(closure) and fuzzy soft open(closed)sets, fuzzy soft interior(closure) introduced by Chang [2], Tanay et. al [28] and further studied by B. P. Varol and H. Aygun [29]. We also define and explore the characterizations and properties of fuzzy soft semi-boundary in fuzzy soft topological spaces. Moreover, we establish the behavior of fuzzy soft semi-boundary with fuzzy soft semi-interior, fuzzy soft semi-exterior, and fuzzy soft semi-closure. In last section, we define and explore the characterizations and properties of fuzzy soft semi-continuous and fuzzy soft semi-open mappings in fuzzy soft topological spaces.

2 Preliminaries

First we recall some definitions and results which will use in the sequel.

Definition 2.1 [1]. A fuzzy set f on X is a mapping $f : X \to I = [0,1]$. The value $f(x)$ represents the degree of membership of $x \in X$ in the fuzzy set f, for $x \in X$.

Definition 2.2 [3]. Let X be an initial universe and E be a set of parameters. Let $P(X)$ denotes the power set of X and A be a non-empty subset of E. A pair (F,A) is called a soft set over X , where F is a mapping given by $F : A \to P(X)$. In other words, a soft set over X is a parameterized family of subsets of the universe X . For $e \in A$, $F(e)$ may be considered as the set of e-approximate elements of the soft set (F,A). Clearly, a soft set is not a set.

Definition. 2.3 [20]. Let I^X denotes the set of all fuzzy sets on X and $A \subseteq X$. A pair (f,A) is called a fuzzy soft set over X, where $f : X \to I^X$ is a function. That is, for each $a \in A$, $f(a) = f_a : X \to$ I is a fuzzy set on X .

Definition 2.4 [20]. For two fuzzy soft sets (f,A) and (g,B) over a common universe X, we say that (f,A) is a fuzzy soft subset of (g,B) if
(1) $A \subseteq B$ and
(2) for all $a \in A$, $f_a \leq g_a$; implies f_a is a fuzzy subset of g_a.
We denote it by $(f,A)\tilde{\leq}(g,A)$. (f,A) is said to be a fuzzy soft super set of (g,B), if (g,B) is a fuzzy soft subset of (f,A). We denote it by $(f,A)\tilde{\geq}(g,B)$.

Definition 2.5 [20]. Two fuzzy soft sets (f,A) and (g,B) over a common universe X are said to be fuzzy soft equal, if (f,A) is a fuzzy soft subset of (g,B) and (g,B) is a fuzzy soft subset of (f,A).

Definition 2.6 [20]. The union of two fuzzy soft sets of (f,A) and (g,B) over the common universe X is the fuzzy soft set (h,C), where $C = A \cup B$ and for all $c \in C$,

$$h_c = \begin{cases} f_c, & \text{if } c \in A - B \\ g_c, & \text{if } c \in B - A \\ f_c \vee g_c, & \text{if } c \in A \cap B \end{cases}$$

We write $(f,A)\tilde{\vee}(g,B) = (h,C)$.

Definition 2.7 [20]. The intersection (h,C) of two fuzzy soft sets (f,A) and (g,B) over a common universe X , denoted $(f,A)\tilde{\wedge}(g,B)$, is defined as $C = A \cap B$, and $h_c = f_c \wedge g_c$, for all $c \in C$.

Definition 2.8 [20]. A fuzzy soft set (f,A) over U is said to be a null fuzzy soft set and is denoted by $\tilde{\phi}$ if and only if, for each $e \in A$, $f_e = \tilde{0}$, where $\tilde{0}$ is the membership function of null fuzzy set over U, which takes value 0, for all $x \in U$.

Definition 2.9 [24]. A fuzzy soft set (f,A) over U is said to be an absolute fuzzy soft set and is denoted by $\tilde{U}$ if and only if, for each $e \in A$, $f_e = \tilde{1}$, where $\tilde{1}$ is the membership function of absolute fuzzy set over U, which takes value 1, for all $x \in U$.

Definition 2.10 [24]. The relative complement of a fuzzy soft set (f,A) is the fuzzy soft set (f^c,A), which is denoted by $(f,A)^c$, where $f^c : A \to F(U)$ is a fuzzy set-valued function. That is, for each $a \in A$, $f^c(A)$ is a fuzzy set in U, whose membership function is $f^c_a(x) = 1\tilde{-} f_a(x)$, for all $x \in U$. Here f^c_a is the membership function of $f^c(a)$.

Definition 2.11 [30]. The difference (h,C) of two fuzzy soft sets (f,A) and (g,B) over X, denoted by $(f,A)\tilde{\setminus}(g,B)$, is defined as $(f,A)\tilde{\setminus}(g,B) = (f,A)\tilde{\wedge}(f,B)^c$.

For our convenience, we will use the notation f_A for fuzzy soft set instead of (f,A).

Definition 2.12 [28]. Let τ be the collection of fuzzy soft sets over X, then τ is said to be a fuzzy soft topology on X if
(1) $\tilde{0}_A$, $\tilde{1}_A$ belong to τ.
(2) If $(f_A)_i \in \tau$, for all $i \in I$, then $\bigvee_{i\in I}(f_A)_i \in \tau$.
(3) $f_a, g_b \in \tau$ implies that $f_a \tilde{\bigwedge} g_b \tilde{\in} \tau$.
The triplet (X,τ,A) is called a fuzzy soft topological space over X. Every member of τ is called fuzzy soft open set. A fuzzy soft set is called fuzzy soft closed if and only if its complement is fuzzy soft open.

Theorem 2.13 [8]. Let τ be the collection of fuzzy soft sets over X. Then
(1) $\tilde{0}_A$, $\tilde{1}_A$ are fuzzy soft closed sets over X.
(2) The intersection of any number of fuzzy soft closed sets is a fuzzy soft closed set over X.
(3) The union of any two fuzzy soft closed sets is a fuzzy soft closed set over X.

Definition 2.14 [29]. Let (X,τ,A) be a fuzzy soft topological space over X and f_A be a fuzzy soft set over X. Then fuzzy soft interior of fuzzy soft set f_A over X is denoted by $(f_A)^\circ$ and is defined as the union of all fuzzy soft open sets contained in f_A. Thus $(f_A)^\circ$ is the largest fuzzy soft open set contained in f_A.

Definition 2.15 [29]. Let (X,τ,A) be a fuzzy soft topological space over X and f_A be a fuzzy soft set over X. Then the fuzzy soft closure of f_A , denoted by $\overline{f_A}$ is the intersection of all fuzzy soft closed super sets of f_A. Clearly $\overline{f_A}$ is the smallest fuzzy soft closed set over X which contains f_A.

Definition 2.16 [31]. Let (X,τ,A) be a fuzzy soft topological space over X and f_A be a fuzzy soft set over X. Then the fuzzy soft boundary of f_A, denoted by $\underline{f_A}$ and is defined as, $\underline{f_A} = \overline{f_A}\tilde{\bigwedge}\overline{(f_A)^c}$.

3 Fuzzy soft semi-open and Fuzzy soft semi-closed sets

In this section, we introduce and establish the topological structures of fuzzy soft semi-open sets and fuzzy soft semi-closed sets.

Definition 3.1. Let (X,τ,A) be a fuzzy soft topological space, where X is a nonempty set and τ is a family of fuzzy soft sets. A fuzzy soft set (F,A) (in short f_A) in fuzzy topological space (X,τ) is called fuzzy soft semi-open, if there exists a fuzzy soft open set g_A such that $g_A \leq f_A \leq \overline{g_A}$. The class of all fuzzy soft semi-open sets in X is denoted by FSSO(X).
A fuzzy soft set f_A in fuzzy soft topological space (X,τ,A) is fuzzy soft semi-closed if and only if its complement $(f_A)^c$ is fuzzy soft semi-open. The class of fuzzy soft semi-closed sets is denoted by FSSC(X).

The following theorem is straightforward.

Theorem 3.2. Every fuzzy soft open set is fuzzy soft semi-open in fuzzy soft topological space (X,τ,A).

The following example shows that the converse of above theorem is not true in general:

Example 3.3. Let $X = \{h_1,h_2,h_3\}$, $A = \{e_1,e_2\}$ and $\tau = \{\tilde{0},\tilde{1},(f_A)_1,(f_A)_2,(f_A)_3,$
$(f_A)_4\}$ where $(f_A)_1$, $(f_A)_2$, $(f_A)_3$, $(f_A)_4$ are fuzzy soft sets over X, defined as follows:
$f_1(e_1)(h_1) = 0.5,\ f_1(e_1)(h_2) = 0.3,\ f_1(e_1)(h_3) = 0.2,$
$f_1(e_2)(h_1) = 0.3,\ f_1(e_2)(h_2) = 0.5,\ f_1(e_2)(h_3) = 0.2,$
$f_2(e_1)(h_1) = 1,\ f_2(e_1)(h_2) = 0,\ f_2(e_1)(h_3) = 0.5,$
$f_2(e_2)(h_1) = 0.5,\ f_2(e_2)(h_2) = 0.3,\ f_2(e_2)(h_3) = 1,$
$f_3(e_1)(h_1) = 0.5,\ f_3(e_1)(h_2) = 0,\ f_3(e_1)(h_3) = 0.2,$
$f_3(e_2)(h_1) = 0.3,\ f_3(e_2)(h_2) = 0.3,\ f_3(e_2)(h_3) = 0.2,$
$f_4(e_1)(h_1) = 1,\ f_4(e_1)(h_2) = 0.3,\ f_4(e_1)(h_3) = 0.5,$
$f_4(e_2)(h_1) = 0.5,\ f_4(e_2)(h_2) = 0.5,\ f_4(e_2)(h_3) = 1.$
Then τ is a fuzzy soft topology on X and hence (X,τ,A) is a fuzzy soft topological space over X.
Let us take fuzzy soft set g_A over X defined by
$g(e_1)(h_1) = 0.5, g(e_1)(h_2) = 0, g(e_1)(h_3) = 0.3,$
$g(e_2)(h_1) = 0.4, g(e_2)(h_2) = 0.4, g(e_2)(h_3) = 0.2.$
That is, $g_A = \{\{h_{0.5},h_0,h_{0.3}\},\{h_{0.4},h_{0.4},h_{0.2}\}\}$. Then there exists fuzzy soft open set $(f_A)_3$ such that
$(f_A)_3\tilde{\leq}g_A\tilde{\leq}\overline{(f_A)_3} = \{\{h_{0.5},h_{0.7},h_{0.8}\},\{h_{0.7},h_{0.5},h_{0.8}\}\}$.
Hence g_A is fuzzy soft semi-open set, but g_A is not fuzzy soft open set.

Proposition 3.4. Let f_A be fuzzy soft set in fuzzy soft topological space (X,τ,A). Then f_A is fuzzy soft semi-open if and only if $f_A\tilde{\leq}\overline{(f_A)^0}$.

Proof. Suppose that f_A is fuzzy soft semi-open then the there exists a fuzzy soft open set g_A such that $g_A \tilde{\leq} f_A \tilde{\leq} \overline{g_A}$. Now $g_A\tilde{\leq}(f_A)^0$ implies that $\overline{(g_A)}\tilde{\leq}\overline{(f_A)^0}$. Therefore $f_A \tilde{\leq} \overline{g_A} \tilde{\leq} \overline{(f_A)^0}$.
Conversely, suppose that $f_A \tilde{\leq} \overline{(f_A)^0}$. Take $g_A = (f_A)^0$, we have $g_A \tilde{\leq} f_A \tilde{\leq} \overline{g_A}$. This completes the proof.

Theorem 3.5. Let (X,τ,A) be a fuzzy soft topological space. Then an arbitrary union of fuzzy soft semi-open sets is a fuzzy soft semi-open set.

Proof. Let $\{(g_A)_\alpha : \alpha \in \Delta\}$ be a collection of fuzzy soft semi-open sets. Let $h_A = \tilde{\bigvee}_{\alpha\in I}(g_A)_\alpha$. Since each $(g_A)_\alpha$ is fuzzy soft semi-open, there exist a fuzzy soft open set $(f_A)_\alpha$ such that $(f_A)_\alpha \tilde{\leq} (g_A)_\alpha \tilde{\leq} \overline{(f_A)_\alpha}$ and so $\tilde{\bigvee}_{\alpha \in I}(f_A)_\alpha \tilde{\leq} \overline{\tilde{\bigvee}_{\alpha \in I}(g_A)_\alpha}$ $\tilde{\leq} \tilde{\bigvee}_{\alpha \in I}\overline{(f_A)_\alpha} \tilde{\leq} \overline{\tilde{\bigvee}_{\alpha \in I}(f_A)_\alpha}$. Let $f_A = \tilde{\bigvee}_{\alpha \in I}(f_A)_\alpha$. Then f_A is fuzzy soft open and $f_A \tilde{\leq} \tilde{\bigvee}_{\alpha \in I}(g_A)_\alpha \ \tilde{\leq} \overline{f_A}$. Hence $\tilde{\bigvee}_{\alpha \in I}(g_A)_\alpha$ is a fuzzy soft semi-open set. This completes the proof.

Proposition 3.6. Let f_A be fuzzy soft semi-open set in fuzzy soft topological space (X, τ, A). Let $f_A \tilde{\leq} h_A \tilde{\leq} \overline{f_A}$. Then h_A is fuzzy soft semi-open set in X.

Proof. Since f_A be fuzzy soft semi-open set in fuzzy soft topological space (X, τ, A). Thus there exists a fuzzy soft open set g_A such that $g_A \tilde{\leq} f_A \tilde{\leq} \overline{g_A}$. Now $g_A \tilde{\leq} h_A$ and $\overline{f_A} \tilde{\leq} \overline{g_A}$ implies that $h_A \tilde{\leq} \overline{g_A}$. Therefore $g_A \tilde{\leq} h_A \tilde{\leq} \overline{g_A}$. Hence h_A is fuzzy soft semi-open set in X. This completes the proof.

The following proposition is immediate from the definition of fuzzy soft semi-closed sets:

Proposition 3.7. Let f_A be fuzzy soft set in fuzzy soft topological space (X, τ, A). Then f_A is fuzzy soft semi-closed if and only if there exists a fuzzy soft closed set h_A such that $(h_A)^0 \tilde{\leq} f_A \tilde{\leq} h_A$.

The following proposition is obvious.

Proposition 3.8. Every fuzzy soft closed set is fuzzy soft semi-closed in fuzzy soft topological space (X, τ, A).

The following example shows that the converse of the above proposition is not true in general.

Example 3.9. Let us consider the fuzzy soft topological space (X, τ, A) over X as in Example 3.3. Take fuzzy soft set $l_A = (g_A)^c$ over X defined by
$l(e_1)(h_1) = 0.5$, $l(e_1)(h_2) = 1$, $l(e_1)(h_3) = 0.7$,
$l(e_2)(h_1) = 0.6$, $l(e_2)(h_2) = 0.6$, $l(e_2)(h_3) = 0.8$.
That is, $l_A = \{\{h_{0.5}, h_1, h_{0.7}\}, \{h_{0.6}, h_{0.6}, h_{0.8}\}\}$.
We observe that all the fuzzy soft closed sets in (X, τ, E) are
$\digamma = \{\tilde{0}, \tilde{1}, (k_A)_1, (k_A)_2, (k_A)_3, (k_A)_4\}$
where $(k_A)_1$, $(k_A)_2$, $(k_A)_3$, $(k_A)_4$ are fuzzy soft sets over X, defined as follows:
$k_1(e_1)(h_1) = 0.5$, $k_1(e_1)(h_2) = 0.7$, $k_1(e_1)(h_3) = 0.8$,
$k_1(e_2)(h_1) = 0.7$, $k_1(e_2)(h_2) = 0.5$, $k_1(e_2)(h_3) = 0.8$,
$k_2(e_1)(h_1) = 0$, $k_2(e_1)(h_2) = 1$, $k_2(e_1)(h_3) = 0.5$,
$k_2(e_2)(h_1) = 0.5$, $k_2(e_2)(h_2) = 0.7$, $k_2(e_2)(h_3) = 0$,
$k_3(e_1)(h_1) = 0.5$, $k_3(e_1)(h_2) = 1$, $k_3(e_1)(h_3) = 0.8$,
$k_3(e_2)(h_1) = 0.7$, $k_3(e_2)(h_2) = 0.7$, $k_3(e_2)(h_3) = 0.8$,
$k_4(e_1)(h_1) = 0$, $k_4(e_1)(h_2) = 0.7$, $k_4(e_1)(h_3) = 0.5$,
$k_4(e_2)(h_1) = 0.5$, $k_4(e_2)(h_2) = 0.5$, $k_4(e_2)(h_3) = 0$.
Clearly, $l_A \notin \digamma$. Hence l_A is fuzzy soft semi-closed but not a fuzzy soft closed sets.

Theorem 3.10. Let (X, τ, A) be a fuzzy soft topological space and f_A be fuzzy soft set in (X, τ, A). Then f_A is fuzzy soft semi-closed if and only if $(\overline{f_A})^0 \tilde{\leq} f_A$.

Proof. Suppose that f_A is fuzzy soft semi-closed, then by Proposition 3.7, there exists a fuzzy sot closed set h_A such that $(h_A)^0 \tilde{\leq} f_A \tilde{\leq} h_A$. Which implies that $\overline{f_A} \tilde{\leq} \overline{h_A} = h_A$ and hence $(\overline{f_A})^0 \tilde{\leq} (h_A)^0$. Thus $(\overline{f_A})^0 \tilde{\leq} (h_A)^0 \tilde{\leq} f_A$.
Conversely, suppose that f_A be fuzzy soft set in (X, τ, A) such that $(\overline{f_A})^0$
$\tilde{\leq} f_A$. Take $\overline{f_A} = h_A$. Then $(h_A)^0 \tilde{\leq} f_A \tilde{\leq} h_A$. This implies that f_A is fuzzy soft semi-closed set.

Theorem 3.11. Let (X, τ, A) be a fuzzy soft topological space and $\{(f_A)_\alpha : \alpha \in I\}$ be a collection of fuzzy soft semi-closed sets in (X, τ, A). Then the intersection $\tilde{\bigwedge}_{\alpha \in I}(f_A)_\alpha$ is fuzzy soft semi-closed in X.

Proof. Since each $\alpha \in I$, $(f_A)_\alpha$ is fuzzy soft semi-closed, then by Proposition 3.7, there exists fuzzy soft closed set $(h_A)_\alpha$ such that $((h_A)_\alpha)^0 \tilde{\leq} (f_A)_\alpha \tilde{\leq} (h_A)_\alpha$. Which implies that
$(\tilde{\bigwedge}_{\alpha \in I}((h_A)_\alpha)^0 \tilde{\leq} \tilde{\bigwedge}_{\alpha \in I}((h_A)_\alpha)^0 \tilde{\leq} \tilde{\bigwedge}_{\alpha \in I}(f_A)_\alpha \tilde{\leq} \tilde{\bigwedge}_{\alpha \in I}(h_A)_\alpha$.
Take $\tilde{\bigwedge}_{\alpha \in I}(h_A)_\alpha = h_A$. Then by Theorem 2.13, h_A is fuzzy soft closed and hence $\tilde{\bigwedge}_{\alpha \in I}(f_A)_\alpha$ is fuzzy soft semi-closed.

4 Fuzzy soft semi-interior, Fuzzy soft semi-closure and Fuzzy soft semi-boundary

In this section, we introduce and explore fuzzy soft semi-interior, fuzzy soft semi-closure and fuzzy soft semi-exterior. We also discuss the relationship between fuzzy soft semi-open(closed), fuzzy soft semi-interior(closure) and fuzzy soft open(closed)sets, fuzzy soft interior(closure). For our convenience, we will denote $F(X, A)$ as the family of all fuzzy soft sets over X in our sequel.

Definition 4.1. Let (X, τ, A) be a fuzzy soft topological space and $f_A \tilde{\in} F(X, A)$. The fuzzy soft semi-closure of f_A, denoted by $scl^{fs}(f_A)$ and is defined as the intersection of all fuzzy soft semi-closed supersets of f_A.
It is clear from the definition that $scl^{fs}(f_A)$ is the smallest fuzzy soft semi-closed set over X which contains f_A.

Example 4.2. Let us consider the fuzzy soft topological space (X, τ, A) over X and fuzzy soft closed sets as in Example 3.9. It is clear that for any fuzzy soft semi-closed set k_A, $scl^{fs}(k_A) = k_A$.

Definition 4.3. Let (X, τ, A) be a fuzzy soft topological space and $f_A \tilde{\in} F(X, A)$. The fuzzy soft semi-interior of f_A, denoted by $sint^{fs}(f_A)$ and is defined as the union of all fuzzy soft semi-open subsets of f_A.
It is clear from the definition that $sint^{fs}(f_A)$ is the largest fuzzy soft semi-open set over X contained in f_A.

Example 4.4. Let us consider the fuzzy soft topological space (X, τ, A) over X and fuzzy soft semi-open sets as in Example 3.3. It is clear that for any fuzzy soft semi-open set f_A, $sint^{fs}(f_A) = (f_A)$.

From the Theorem 3.2, Proposition 3.8 and Definitions 4.1, 4.3, we have:

Theorem 4.5. Let (X,τ,A) be a fuzzy soft topological space and $f_A\tilde{\in}F(X,A)$. Then $(f_A)^0\tilde{\leq}sint^{fs}(f_A)\tilde{\leq}(f_A)\tilde{\leq}scl^{fs}(f_A)\tilde{\leq}\overline{(f_A)}$.

The proof of the following properties of fuzzy soft semi-interior and fuzzy soft semi-closure follows as the proof of the corresponding properties of soft semi-interior and soft semi-closure in [17,18,19] and thus omitted.

Theorem 4.6. Let (X,τ,A) be a fuzzy soft topological space over X and $f_A,g_A\tilde{\in}F(X,A)$. Then
(1) $sint^{fs}(\tilde{0})=scl^{fs}(\tilde{0})=\tilde{0}$ and $sint^{fs}(\tilde{1})=scl^{fs}(\tilde{1})=\tilde{1}$.
(2) f_A is a fuzzy soft semi-open (respt. fuzzy soft semi-closed) set if and only if $sint^{fs}(f_A)=f_A$(respt. $scl^{fs}(f_A)=f_A$).
(3) $sint^{fs}(sint^{fs}(f_A))=f_A$.
(4) $f_A\tilde{\leq}(g_A)$ implies $sint^{fs}(f_A)\tilde{\leq}sint^{fs}(g_A)$ and $scl^{fs}(f_A)\tilde{\leq}scl^{fs}(g_A)$.
(5) $sint^{fs}(f_A)\tilde{\bigwedge}sint^{fs}(g_A)=sint^{fs}(f_A\tilde{\bigwedge}g_A)$ and $scl^{fs}(f_A)\tilde{\bigwedge}scl^{fs}(g_A)\tilde{\geq}scl^{fs}(f_A\tilde{\bigwedge}g_A)$.
(6) $sint^{fs}(f_A)\tilde{\bigvee}sint^{fs}(g_A)\tilde{\leq}sint^{fs}(f_A\tilde{\bigvee}g_A)$ and $scl^{fs}(f_A)\tilde{\bigvee}scl^{fs}(g_A)=scl^{fs}(f_A\tilde{\bigvee}g_A)$.

In the following theorem, (3) and (4) give the relationship between fuzzy soft interior, fuzzy soft closure, fuzzy soft semi-interior and fuzzy soft semi-closure of fuzzy soft sets:

Theorem 4.7. Let (X,τ,A) be a fuzzy soft topological space over X and $f_A\tilde{\in}F(X,A)$. Then
(1) $(sint^{fs}(f_A))^c=scl^{fs}((f_A)^c)$.
(2) $(scl^{fs}(f_A))^c=sint^{fs}((f_A)^c)$.
(3) $sint^{fs}((f_A)^0)=(sint^{fs}(f_A))^0=(f_A)^0$.
(4) $scl^{fs}(\overline{f_A})=\overline{(scl^{fs}(f_A))}=\overline{f_A}$.

Proof.(1). $sint^{fs}(f_A)\tilde{\leq}f_A$ implies that $(f_A)^c\tilde{\leq}(sint^{fs}(f_A))^c$. Now by Theorem 4.6(2) and since $(sint^{fs}(f_A))^c$ is a fuzzy soft semi-closed set, we have $scl^{fs}((f_A)^c)\tilde{\leq}scl^{fs}((sint^{fs}(f_A))^c)=(sint^{fs}(f_A))^c$. For the reverse inclusion, by Theorem 4.5, $(f_A)^c\tilde{\leq}scl^{fs}((f_A)^c)$. This implies that $(scl^{fs}((f_A)^c))^c\tilde{\leq}((f_A)^c)^c=f_A$. $scl^{fs}((f_A)^c)$ being fuzzy soft semi-closed implies that $(scl^{fs}((f_A)^c))^c$ is fuzzy soft semi-open. Thus $(scl^{fs}((f_A)^c))^c\tilde{\leq}sint^{fs}(f_A)$ and hence $(sint^{fs}(f_A))^c\tilde{\leq}((scl^{fs}((f_A)^c))^c)^c=scl^{fs}((f_A)^c)$.
(2). The proof is same as (1).
(3). By Theorem 3.2, $(f_A)^0$ being fuzzy soft open implies it is fuzzy soft semi-open. Therefore, by Theorem 4.6(2), $sint^{fs}((f_A)^0)=(f_A)^0$. Now by Theorem 4.5, $(f_A)^0\tilde{\leq}sint^{fs}(f_A)=f_A$. This implies that $sint^{fs}((f_A)^0)=(f_A)^0$.
(4). By Proposition 3.6, $\overline{f_A}$ being fuzzy soft closed implies it is fuzzy soft semi-closed. Therefore, by Theorem 4.6(2), $scl^{fs}(\overline{f_A})=\overline{f_A}$. Now by Theorem 4.5, $f_A\tilde{\leq}scl^{fs}(f_A)\tilde{\leq}\overline{f_A}$. Thus $scl^{fs}(f_A)\tilde{\leq}\overline{scl^{fs}(f_A)}\tilde{\leq}scl^{fs}(f_A)$. This follows that $\overline{(scl^{fs}(f_A))}=\overline{f_A}$. Hence the proof.

Definition 4.8. Let (X,τ,A) be a fuzzy soft topological space over X and $f_A\tilde{\in}F(X,A)$ then fuzzy soft semi-exterior of f_A is denoted by $sext^{fs}(f_A)$ and is defined as $sext^{fs}(f_A)=sint^{fs}((f_A)^c)$.
We observe that $sext^{fs}(f_A)$ is the largest fuzzy soft semi-open set contained in $(f_A)^c$.

Example 4.9. Let us consider the fuzzy soft topological space (X,τ,A) over X as in Example 3.3. Take fuzzy soft set k_A over X defined by
$k(e_1)(h_1)=0.5$, $k(e_1)(h_2)=1$, $k(e_1)(h_3)=0.7$,
$k(e_2)(h_1)=0.6$, $k(e_2)(h_2)=0.6$, $k(e_2)(h_3)=0.8$.
That is, $k_E=\{\{h_{0.5},h_1,h_{0.7}\},\{h_{0.6},h_{0.6},h_{0.8}\}\}$ be fuzzy soft set in fuzzy soft topological space over X. Then $(k_A)^c=\{\{h_{0.5},h_0,h_{0.3}\},\{h_{0.4},h_{0.4},h_{0.2}\}\}$. Here we see that $sext^{fs}(k_A)=sint^{fs}((k_A)^c)=\{\{h_{0.5},h_0,h_{0.3}\},\{h_{0.4},h_{0.4},h_{0.2}\}\}$, since $(k_A)^c$ is fuzzy soft semi-open set which is shown in Example 3.3.

the following theorem directly follows from definition of fuzzy soft semi-exterior:

Theorem 4.10. Let (X,τ,A) be a fuzzy soft topological space over X and $f_A\tilde{\in}F(X,A)$. Then
(1)$sext^{fs}(f_A)=sint^{fs}((f_A)^c)$.
(2)$sext^{fs}(f_A\tilde{\bigvee}g_A)=sext^{fs}(f_A)\tilde{\bigwedge}sext^{fs}(g_A)$.
(3)$sext^{fs}(f_A)\tilde{\bigvee}sext^{fs}(g_A)\tilde{\leq}sext^{fs}(f_A\tilde{\bigwedge}g_A)$.

Now we define fuzzy soft semi-boundary in fuzzy soft topological spaces and establish relationship between fuzzy soft semi-clopen sets and fuzzy soft semi-boundary. Moreover, we construct the combined behavior between fuzzy soft semi-interior, fuzzy soft semi-exterior, fuzzy soft semi-closure and fuzzy soft semi-boundary.

Definition 4.11. Let (X,τ,A) be a fuzzy soft topological space over X and $f_A\tilde{\in}F(X,A)$ then fuzzy soft semi-boundary of f_A is denoted by $sbd^{fs}(f_A)$ and is defined as $sbd^{fs}(f_A)=(sint^{fs}(f_A)\tilde{\bigvee}sext^{fs}(f_A))^c$.

Remark 4.12. From the above definition it follows directly that the fuzzy soft sets f_A and $(f_A)^c$ have same fuzzy soft semi-boundary.

The proof of the following properties of fuzzy soft semi-interior, fuzzy soft semi-closure and fuzzy soft semi-boundary follows as the proof of the corresponding properties of soft semi-interior , soft semi-closure and soft semi-boundary in [17-19] and is therefore omitted.

Theorem 4.13. Let (X,τ,A) be a fuzzy soft topological space over X and $f_A,g_A\tilde{\in}F(X,A)$. Then
(1)$scl^{fs}(f_A)=sint^{fs}(f_A)\tilde{\bigvee}sbd^{fs}(f_A)$.
(2) $sbd^{fs}(f_A)=scl^{fs}(f_A)\tilde{\bigwedge}scl^{fs}(f_A)^c=scl^{s}(f_A)\tilde{\setminus}sint^{fs}(f_A)$.
(3) $(sbd^{fs}(f_A))^c=sint^{fs}(f,A)\tilde{\bigvee}sint^{fs}((f_A)^c)=sint^{fs}(f_A)\tilde{\bigvee}sext^{fs}(f_A)$.
(4) $sint^{fs}(f_A)=f_A\tilde{\setminus}sbd^{fs}(f_A)=scl^{fs}(sint^{fs}(f_A))$.
(5) $sbd^{fs}(f_A)\tilde{\bigwedge}sint^{fs}(f_A)=\tilde{0}$.
(6) f_A is fuzzy soft semi-open (respt. fuzzy soft

semi-closed) if and only if $f_A \tilde{\bigwedge} sbd^{fs}(f_A) = \tilde{0}$
(respt. $sbd^{fs}(f_A) \tilde{\leq} f_A$).
(7) $sbd^{fs}((f_A)\tilde{\bigvee}(g_B)) \tilde{\leq} [sbd^{fs}(f_A)\tilde{\bigwedge} scl^{fs}((g_B)^c)]$
$\tilde{\bigvee}[sbd^{fs}(g_B)\tilde{\bigwedge} scl^{fs}((f_A)^c)]$.
(8) $sbd^{fs}[f_A\tilde{\bigwedge} g_B] \tilde{\leq} [sbd^{fs}(f_A)\tilde{\bigwedge} scl^{fs}(g_B)]\tilde{\bigvee}[sbd^{fs}(g_B)$
$\tilde{\bigwedge} scl^{fs}(f_A)]$.
(9) $sbd^{fs}(sbd^{fs}(sbd^{fs}(f_A))) = sbd^{fs}(sbd^{fs}(f_A))$.
(10) $sbd^{fs}(f_A) = \tilde{0}$ if and only if f_A is a fuzzy soft semi-closed set and a fuzzy soft semi-open set.
(11) $f_A \tilde{\setminus} sint^{fs}(g_A)) \tilde{\leq} (f_A)^{\circ} \tilde{\setminus} sint^{fs}(g_A)$.
(12) $sbd^{fs}(sint^{fs}(f_A)) \tilde{\leq} sbd^{fs}(f_A)$.
In the above theorem, (10) gives the relationship between fuzzy soft semi-clopen sets and fuzzy soft semi-boundary.

5 Fuzzy soft semi-continuous and Fuzzy soft semi-open mappings

Here we define and explore the characterizations and properties of fuzzy soft semi-continuous and fuzzy soft semi-open mappings in a fuzzy soft topological spaces. First we recall some definitions.
Definition 5.1 [24]. Let $F(X,A)$ and $F(Y,B)$ be families of fuzzy soft sets. $u : X \to Y$ and $p : A \to B$ be mappings. Then image and inverse image of a function $f_{pu} : F(X,A) \to F(Y,B)$ is defined as :
(1) Let f_A be a fuzzy soft set in $F(X,A)$. The image of f_A under f_{pu} , written as $f_{pu}(f_A)$, is a fuzzy soft set in $F(Y,B)$ such that for $\beta \in p(A) \subseteq B$, $\alpha \in p^{-1}(\beta) \cap A$ and $y \in Y$,

$$f_{pu}(f_A)(\beta)(y) = \begin{cases} \bigvee_{x \in u^{-1}(y)}(\bigvee_{\alpha}(f_A(\alpha)), & u^{-1}(y) \neq \phi, p^{-1}(\beta) \cap A \neq \phi \\ 0, & \text{otherwise} \end{cases},$$

for all $y \in B$. $f_{pu}(f_A)$ is known as a fuzzy soft image of a fuzzy soft set f_A.
(2) Let g_B be a fuzzy soft set in $F(Y,B)$. Then the fuzzy soft inverse image of g_B under f_{pu} , written as $f_{pu}^{-1}(g_B)$, is a fuzzy soft set in $F(X,A)$ such that

$$f_{pu}^{-1}(g_B)(\alpha)(x) = \begin{cases} g(p(\alpha))(u(x)), & p(\alpha) \in B \\ 0, & \text{otherwise} \end{cases},$$

for all $x \in A$. $f_{pu}^{-1}(g_B)$ is known as a fuzzy soft inverse image of a fuzzy soft set g_B.

The fuzzy soft function f_{pu} is called fuzzy soft surjective, if p and u are surjective. The fuzzy soft function f_{pu} is called fuzzy soft injective, if p and u are injective.

Definition 5. [24]. Let $f_{pu} : F(X,A) \to F(Y,B)$ be a mapping and $f_A, g_A \in F(X,A)$. Then we define the fuzzy soft union and fuzzy soft intersection of fuzzy soft images $f_{pu}(f_A)$ and $f_{pu}(g_A)$ as :
$(f_{pu}(f_A)\tilde{\bigvee} f_{pu}(g_A))(\beta)(y)$
$= f_{pu}(f_A)(\beta)(y)\tilde{\bigvee} f_{pu}(g_A)(\beta)(y)$
and
$(f_{pu}(f_A)\tilde{\bigwedge} f_{pu}(g_A))(\beta)(y)$
$= f_{pu}(f_A)(\beta)(y)\tilde{\bigwedge} f_{pu}(g_A)(\beta)(y)$, for $\beta \in B, y \in Y$.

Definition 5.3 [24]. Let $f_{pu} : F(X,A) \to F(Y,B)$ be a mapping and $f_B, g_B \in F(Y,B)$. Then we define the fuzzy soft union and fuzzy soft intersection of fuzzy soft inverse images $f_{pu}^{-1}(f_B)$ and $f_{pu}^{-1}(g_B)$ as :
$(f_{pu}^{-1}(f_B)\tilde{\bigvee} f_{pu}^{-1}(g_B))(\alpha)(x)$
$= f_{pu}^{-1}(f_B)(\alpha)(y)\tilde{\bigvee} f_{pu}(g_B)(\alpha)(x)$
and
$(f_{pu}^{-1}(f_B)\tilde{\bigwedge} f_{pu}^{-1}(g_B))(\alpha)(x)$
$= f_{pu}^{-1}(f_B)(\alpha)(x)\tilde{\bigwedge} f_{pu}^{-1}(g_B)(\alpha)(x)$, for $\alpha \in A, x \in X$.

Theorem 5.4 [24]. Let $f_A, g_A \in F(X,A)$. Then for a function $f_{pu} : F(X,A) \to F(Y,B)$, the following statements are true.
(1) $f_{pu}(\tilde{0}_A) = \tilde{0}_B$.
(2) $f_{pu}(\tilde{1}_A) = \tilde{1}_B$.
(3) $f_{pu}(f_A\tilde{\bigvee} g_A) = f_{pu}(f_A)\tilde{\bigvee} f_{pu}(g_A)$.
In general
$f_{pu}(\tilde{\bigvee}_i((f_A)_i) = \tilde{\bigvee}_i(f_{pu}(f_A)_i)$.
(4) $f_{pu}(f_A\tilde{\bigwedge} g_A) \tilde{\leq} f_{pu}(f_A)\tilde{\bigwedge} f_{pu}(g_B)$.
In general
$f_{pu}(\tilde{\bigwedge}_i(f_A)_i) \tilde{\leq} \tilde{\bigwedge}_i(f_{pu}(f_A)_i)$.
(5) If $f_A \tilde{\leq} g_A$, then $f_{pu}(f_A) \tilde{\leq} f_{pu}(g_A)$.

Theorem 5.5 [24]. Let $f_B, g_B \in FSS(X)_B$. Then for a function $f_{pu} : F(X,A) \to F(Y,B)$, the following statements are true.
(1) $f_{pu}^{-1}(\tilde{0_B}) = \tilde{0_A}$.
(2) $f_{pu}^{-1}(\tilde{1}_B) = \tilde{1}_A$.
(3) $f_{pu}^{-1}(f_B\tilde{\bigvee} g_B) = f_{pu}^{-1}(f_B)\tilde{\bigvee} f_{pu}^{-1}(g_B)$.
In general
$f_{pu}^{-1}(\tilde{\bigvee}_i(f_B)_i) = \tilde{\bigvee}_i(f_{pu}^{-1}(f_B)_i)$.
(4) $f_{pu}^{-1}(f_B\tilde{\bigwedge} g_B) \tilde{=} f_{pu}^{-1}(f_B)\tilde{\bigwedge} f_{pu}^{-1}(g_B)$.
In general
$f_{pu}^{-1}(\tilde{\bigwedge}_i(f_B)_i) \tilde{=} \tilde{\bigwedge}_i(f_{pu}^{-1}(f_B)_i)$.
(5) If $f_B \tilde{\leq} g_B$, then $f_{pu}^{-1}(f_B) \tilde{\leq} f_{pu}^{-1}(g_B)$.

The proof of the following theorem directly follows form the definitions of fuzzy soft images and inverse fuzzy soft images of fuzzy soft sets.
Theorem 5.6 [16]. Let $f_{pu} : F(X,A) \to F(Y,B)$ be a function. Then for any fuzzy soft sets (g,B) in $F(Y,B)$ and f_A in $F(X,A)$, we have:
(1) $f_{pu}^{-1}(g_B)^c = (f_{pu}^{-1}(g_B))^c$.
(2) $f_{pu}(f_{pu}^{-1}(g_B)) \tilde{\leq} g_B$.
(3) $f_A \tilde{\leq} f_{pu}^{-1}(f_{pu}(f_A))$.
Note that, if f_{pu} is soft surjective, the equality holds in (2) and if f_{pu} is soft injective, the equality holds in (3).

Now we define:
Definition 5.7. Let (X, τ_1, A) and (Y, τ_2, B) be two fuzzy

soft topological spaces.
(1) A fuzzy soft mapping $f_{pu}: F(X,A) \to F(Y,B)$ is said to be fuzzy soft semi-continuous, if for any fuzzy soft open set g_B in (Y,τ_2,B), $f_{pu}^{-1}(g_B)$ is fuzzy soft semi-open in (X,τ_1,A).
(2) A fuzzy soft mapping $f_{pu}: F(X,A) \to F(Y,B)$ is said to be a fuzzy soft semi-open, if for any fuzzy soft open set f_A in (X,τ_1,A), $f_{pu}(f_A)$ is fuzzy soft semi-open in (Y,τ_2,B).

Remark 5.8. Every fuzzy soft continuous function is fuzzy soft semi-continuous, since every fuzzy soft open set is fuzzy soft semi-open in fuzzy soft topological space.

The proof of the following theorem follows from Definition 5.7 and Theorem 2 [16].
Theorem 5.9. Let (X,τ_1,A) and (Y,τ_2,B) be two fuzzy soft topological spaces. Let $f_{pu}: F(X,A) \to F(Y,B)$ be a fuzzy soft mapping. Then the following are equivalent:
(1) f_{pu} is fuzzy soft semi-continuous.
(2) For any fuzzy soft closed set g_B in Y, $f_{pu}^{-1}(g_B)$ is fuzzy soft semi-closed in X.

Lemma 5.10. Let f_A be a fuzzy soft semi-open set in fuzzy soft topological space (X,τ,A) and $g_A \in F(X,A)$ with $f_A \tilde{\leq} g_A$, then $f_A \tilde{\leq} \overline{(g_A)^0}$.
Proof. Since f_A be fuzzy soft semi-open set, Proposition 3.4 implies that $f_A \tilde{\leq} \overline{(f_A)^0}$. Also $f_A \tilde{\leq} g_A$ implies that $(f_A)^0 \tilde{\leq} (g_A)^0$(by Theorem 3.11[29]). Therefore $\overline{(f_A)^0} \tilde{\leq} \overline{(g_A)^0}$ (by Theorem 3.9[29]). Which implies that $f_A \tilde{\leq} \overline{(g_A)^0}$.

Theorem 5.11. Let (X,τ_1,A) and (Y,τ_2,B) be two fuzzy soft topological spaces. A fuzzy soft mapping $f_{pu}: F(X,A) \to F(Y,B)$ is said to be fuzzy soft semi-open iff for any fuzzy soft set $f_A \in F(X,A)$, we have $f_{pu}((f_A)^0) \tilde{\leq} \overline{(f_{pu}(f_A))^0}$.
Proof. $(\Rightarrow)$ Suppose that f_{pu} is fuzzy soft semi-open then $f_{pu}((f_A)^0)$ is fuzzy soft semi-open set. Also $f_{pu}((f_A)^0) \tilde{\leq} f_{pu}(f_A)$. Thus by Lemma 5.10,
$f_{pu}((f_A)^0) \tilde{\leq} \overline{(f_{pu}(f_A))^0}$.
$(\Leftarrow)$ Suppose g_A be a fuzzy soft open set in (X,τ_1,A). Then
$(f_{pu}(g_A))^0 \tilde{\leq} f_{pu}(g_A) \tilde{\leq} f_{pu}((g_A)^0) \tilde{\leq} \overline{(f_{pu}(g_A))^0}$ implies that $f_{pu}(g_A)$ is fuzzy soft semi-open. Thus f_{pu} is fuzzy soft semi-open mapping.

The following theorem can be proved in a similar fashion.
Theorem 5.12. Let (X,τ_1,A) and (Y,τ_2,B) be two fuzzy soft topological spaces. A fuzzy soft mapping $f_{pu}: F(X,A) \to F(Y,B)$ is said to be fuzzy soft semi-open iff for any fuzzy soft set $g_B \in F(Y,B)$, $(f_{pu}^{-1}(g_B))^0 \tilde{\leq} \overline{f_{pu}^{-1}((g_B)^0)}$.

Theorem 5.13. Let (X,τ_1,A) and (Y,τ_2,B) be two fuzzy soft topological spaces. Let $f_{pu}: F(X,A) \to F(Y,B)$ be a fuzzy soft mapping. Then the following are equivalent:
(1) f_{pu} is fuzzy soft semi-continuous.
(2) For any $g_B \tilde{\in} F(Y,B)$, $sbd^{fs}(f_{pu}^{-1}(g_B)) \tilde{\leq} f_{pu}^{-1}(\underline{(g_B)})$.
(3) For any $f_A \tilde{\in} F(X,A)$, $f_{pu}(scl^{fs}(f_A)) \tilde{\leq} \overline{f_{pu}(f_A)}$.
proof. $(1) \Rightarrow (2)$. Suppose that f_{pu} is fuzzy soft semi-continuous and g_B be any fuzzy soft set over Y. Now

$$\begin{aligned} sbd^{fs}(f_{pu}^{-1}(g_B)) &\tilde{=} scl^{fs}(f_{pu}^{-1}(g_B)) \tilde{\bigwedge} scl^{fs}(f_{pu}^{-1}(g_B)^c) \\ &\tilde{\leq} scl^{fs}(f_{pu}^{-1}\overline{(g_B)}) \tilde{\bigwedge} scl^{fs}(f_{pu}^{-1}\overline{(g_B^c)}) \\ &= f_{pu}^{-1}(\overline{(g_B)}) \tilde{\bigwedge} f_{pu}^{-1}(\overline{(g_B^c)}) \\ &= f_{pu}^{-1}(\overline{(g_B)} \tilde{\bigwedge} \overline{(g_B^c)}) \\ &= f_{pu}^{-1}(\underline{(g_B)}) \end{aligned}$$

$(2) \Rightarrow (1)$. Let g_B be fuzzy soft closed set in Y. To prove (1), we just show that $f_{pu}^{-1}(g_B)$is fuzzy soft semi-closed in X. By (2), we have
$sbd^{fs}(f_{pu}^{-1}(g_B)) \tilde{\leq} f_{pu}^{-1}(\underline{(g_B)}) \tilde{\leq} f_{pu}^{-1}(g_B)$ implies that $sbd^{fs}(f_{pu}^{-1}(g_B)) \tilde{\leq} f_{pu}^{-1}(g_B)$. This shows that $f_{pu}^{-1}(g_B)$ is fuzzy soft semi-closed in X.
$(1) \Rightarrow (3)$. Suppose $f_A \tilde{\in} F(X,A)$. Since $\overline{f_A}$ is fuzzy soft closed in Y and f_{pu} is fuzzy soft semi-continuous, then $f_{pu}^{-1}(\overline{f_A})$ is fuzzy soft semi-closed such that $f_A \tilde{\leq} f_{pu}^{-1}(\overline{f_A})$. So $scl^{fs}(f_A) \tilde{\leq} scl^{fs}(f_{pu}^{-1}(\overline{f_A})) = f_{pu}^{-1}(\overline{f_{pu}(f_A)})$ implies that $scl^{fs}(f_A) \tilde{\leq} f_{pu}^{-1}(\overline{f_{pu}(f_A)})$. Hence $f_{pu}(scl^{fs}(f_A)) \tilde{\leq} \overline{f_{pu}(f_A)}$.
$(3) \Rightarrow (1)$. We use Theorem 5.9 to prove (1). Let g_B be fuzzy soft closed set in Y, we show that $f_{pu}^{-1}(g_B)$ is fuzzy soft semi-closed in X. Since
$f_{pu}(scl^{fs}(f_{pu}^{-1}(g_B))) \tilde{\leq} \overline{f_{pu}(f_{pu}^{-1}(g_B))} \tilde{\leq} \overline{(g_B)} = g_B$. This implies that
$scl^{fs}(f_{pu}^{-1}(g_B)) \tilde{\leq} f_{pu}^{-1}(f_{pu}(scl^{fs}(f_{pu}^{-1}(g_B)))) \tilde{\leq} f_{pu}^{-1}(g_B)$ or $scl^{fs}(f_{pu}^{-1}(g_B)) \tilde{\leq} f_{pu}^{-1}(g_B)$. Hence $f_{pu}^{-1}(g_B)$ is fuzzy soft semi-closed in X.

Theorem 5.14. Let (X,τ_1,A) and (Y,τ_2,B) be two fuzzy soft topological spaces. Let $f_{pu}: F(X,A) \to F(Y,B)$ be a fuzzy soft mapping. Then the following are equivalent:
(1) f_{pu} is fuzzy soft semi-continuous.
(2) For any $g_B \tilde{\in} F(Y,B)$, $scl^{fs}(f_{pu}^{-1}(g_B)) \tilde{\leq} f_{pu}^{-1}(\overline{g_B})$.
Proof. $(1) \Rightarrow (2)$. Suppose that (1) holds, then by above Theorem 5.13, we get $f_{pu}(scl^{fs}(f_A)) \tilde{\leq} \overline{f_{pu}(f_A)}$. Consider $g_B \tilde{\in} F(Y,B)$. Take $f_A = f_{pu}^{-1}(g_B)$, then $f_{pu}(scl^{fs}(f_{pu}^{-1}(g_B))) \tilde{\leq} \overline{f_{pu}(f_{pu}^{-1}(g_B))} \tilde{\leq} \overline{g_B}$. Hence $scl^{fs}(f_{pu}^{-1}(g_B)) \tilde{\leq} f_{pu}^{-1}(\overline{g_B})$.
$(2) \Rightarrow (1)$. For fuzzy soft set f_A in X. Take $g_B = f_{pu}(f_A)$. Then $scl^{fs}(f_A) \tilde{\leq} scl^{fs}(f_{pu}^{-1}(g_B)) \tilde{\leq} f_{pu}^{-1}(\overline{g_B})$. Hence $f_{pu}(scl^{fs}(f_A)) \tilde{\leq} \overline{f_{pu}(f_A)}$. Thus by above Theorem 5.13, f_{pu} is fuzzy soft semi-continuous. This completes the proof.

The proofs of the following lemmas are straightforward and thus omitted.
Lemma 5.15. Let f_A be fuzzy soft set and g_A be fuzzy soft semi-closed set such that $f_A \tilde{\leq} g_A$. Then $sbd^{fs}(f_A) \tilde{\leq} g_A$.

Lemma 5.16. Let f_A be fuzzy soft set and g_A be fuzzy soft open set such that $f_A \tilde{\bigwedge} g_A \tilde{=} \tilde{0}$. Then $f_A \tilde{\bigwedge} \overline{g_A} \tilde{=} \tilde{0}$.

The following proposition follows from the above Lemma 5.16:

Proposition 5.17. Let f_A be fuzzy soft set and g_A be fuzzy soft open set such that $f_A \tilde{\bigwedge} g_A \tilde{=} \tilde{0}$. Then $f_A \tilde{\bigwedge} \underline{(g_A)} \tilde{=} \tilde{0}$.

Theorem 5.18. Let (X, τ_1, A) and (Y, τ_2, B) be two fuzzy soft topological spaces. Let $f_{pu} : F(X,A) \to F(Y,B)$ be a bijective fuzzy soft mapping. Then the following are equivalent:
(1) f_{pu} is fuzzy soft semi-open.
(2) For any $g_B \tilde{\in} F(Y,B)$, $f_{pu}^{-1}(sbd^{fs}(g_B)) \tilde{\leq} \underline{f_{pu}^{-1}(g_B)}$.

Proof. $(1) \Rightarrow (2)$. Suppose that (1) holds and $g_B \tilde{\in} F(Y,B)$. Take

$$h_A \tilde{=} (\underline{f_{pu}^{-1}(g_b)})^c. \qquad (*)$$

Then h_A is fuzzy soft open. Therefore $f_{pu}(h_A)$ is fuzzy soft semi-open in $F(Y,B)$ implies that $(f_{pu}(h_A))^c$ is fuzzy soft semi-closed set in Y. Also f_{pu} is bijective,(*) implies that $g_B \tilde{\leq} (f_{pu}(h_A))^c$. Now Lemma 5.15 follows that $f_{pu}^{-1}(sbd^{fs}(g_B)) \tilde{\leq} f_{pu}^{-1}((f_{pu}(h_A))^c) \tilde{\leq} (h_A)^c \tilde{=} ((\underline{f_{pu}^{-1}(g_B)})^c)^c \tilde{=} \underline{f_{pu}^{-1}(g_B)}$. Hence we have $f_{pu}^{-1}(sbd^{fs}(g_B)) \tilde{\leq} \underline{f_{pu}^{-1}(g_B)}$.
$(2) \Rightarrow (1)$. Suppose that k_A be fuzzy soft open set in X. Take $g_B \tilde{=} (f_{pu}(k_A))^c$. Thus $g_B \tilde{\bigwedge} f_{pu}(k_A) \tilde{=} \tilde{0}$ implies $k_A \tilde{\bigwedge} f_{pu}^{-1}(g_B) \tilde{=} \tilde{0}$. Proposition 5.17 implies that $k_A \tilde{\bigwedge} \underline{f_{pu}^{-1}(g_B)} \tilde{=} \tilde{0}$. So, $f_{pu}^{-1}(sbd^{fs}(g_B)) \tilde{\leq} \underline{f_{pu}^{-1}(g_B)}$ implies that $k_A \tilde{\bigwedge} f_{pu}^{-1}(sbd^{fs}(g_B)) \tilde{=} \tilde{0}$. Therefore $f_{pu}(k_A \tilde{\bigwedge} f_{pu}^{-1}(sbd^{fs}(g_B))) \tilde{=} f_{pu}(k_A) \tilde{\bigwedge} sbd^{fs}(g_B)$ follows that $sbd^{fs}(g_B) \tilde{\leq} (f_{pu}(k_A))^c \tilde{=} g_B$. This shows that g_B is fuzzy soft semi-closed. Hence $f_{pu}(k_A)$ is fuzzy soft semi-open. Hence the proof.

Theorem 5.19. Let (X, τ_1, A) and (Y, τ_2, B) be two fuzzy soft topological spaces. Let $f_{pu} : F(X,A) \to F(Y,B)$ be a fuzzy soft mapping. Then the following are equivalent:
(1) f_{pu} is fuzzy soft semi-open.
(2) For any $f_A \tilde{\in} F(X,A)$, $f_{pu}((f_A)^0) \tilde{\leq} \overline{((f_{pu}(f_A))^0)}$.

Proof. $(1) \Rightarrow (2)$. By (1) and since $f_{pu}((f_A)^0) \tilde{\leq} f_{pu}(f_A)$, then $f_{pu}((f_A)^0)$ is fuzzy soft semi-open. Thus by Lemma 5.10, we have $f_{pu}((f_A)^0) \tilde{\leq} \overline{((f_{pu}(f_A))^0)}$.
$(2) \Rightarrow (1)$. Let $k_A \tilde{\in} F(X,A)$, then $f_{pu}(k_A) \tilde{=} f_{pu}((k_A)^0) \tilde{\leq} \overline{((f_{pu}(k_A))^0)}$. This implies that $f_{pu}(k_A)$ is a fuzzy soft semi-open set. Hence f_{pu} is fuzzy soft semi-open . Hence the proof.

Conclusion

Theory of fuzzy topology which generalizes the basic concepts of classical topology has been found to be very useful in solving many practical problems. Many researchers worked to fuzzify the different notions such as Du. et. al fuzzified the very successful 9-intersection Egenhofer model for depicting topological relations in Geographic Information Systems(GIS).
Researchers also showed that notion of fuzzy topology might be relevant to quantum particles physics and quantum gravity in connection with string theory and e^{∞} theory.
Soft set theory is very important during the study towards possible applications in classical and non classical logic. In recent years, many researchers worked on the findings of structures of soft sets theory initiated by Molodtsov and applied to many problems having uncertainties. It is worth mentioning that soft topological spaces based on soft set theory which is a collection of information granules is the mathematical formulation of approximate reasoning about information systems. The researchers have contributed toward the fuzzification of soft set theory. In the present work, we continued to investigate the properties of fuzzy soft semi-open sets and fuzzy soft semi-closed sets in fuzzy soft topological spaces. We defined fuzzy soft semi-interior, fuzzy soft semi-closure, fuzzy soft semi-exterior and fuzzy soft semi-boundary in fuzzy soft topological spaces. We discussed the characterizations of fuzzy soft semi-closed and fuzzy soft semi-open sets via fuzzy soft semi-interior, fuzzy soft semi-exterior, fuzzy soft semi-closure and fuzzy soft semi-boundary and have established several interesting properties. Moreover we defined and discussed the fuzzy soft semi-continuous and fuzzy soft semi-open functions. We hope that our findings will help to enhance and promote the further study on fuzzy soft topology proceed towards for the practical life application.

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