

Bit-plane Image Coding Scheme Based On Compressed Sensing

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Received: Jul 8, 2011; Revised Oct. 4, 2011; Accepted Oct. 6, 2011

Published online: 1 August 2012

Abstract: Based on compressed sensing, a new bit-plane image coding scheme was presented. Due to different importance for different image bit-planes, the new method is robust to bit error, and has the advantages of simple structure and easy software and hardware implementation. Because the values of the image bit-planes are 1 or zero, one order difference matrix was chosen as sparse transform matrix, and the simulation shows that it has more sparse presentations. For the general 8-bit images, it has 8 bit-planes, the eighth bit-plane is the Most Significant Bit-plane, so we can adopt more measure vectors for reconstruction image precision. At the same time, this kind of image codec scheme can meet many application demands. The method partitions an image into 8 bit-planes, and made the orthogonal transform using the one order difference matrix for each bit plane, and then formed multiple descriptions after using local Hadamard Matrix measurements of each bit plane. At decoding end, it reconstructed the original image approximately or exactly with the received bit streams by using the Orthogonal Match Pursuit (OMP) algorithms. The proposed method can construct more descriptions with lower complexity because the process of bit plane data measuring is simple and easy to hardware realize. Experiment results show that the proposed method can reconstruct image with different precision and it can easily generate more descriptions.

Keywords: bit-plane, random measurements, compressed sensing, image coding.

1. Introduction

Image compression is currently an active research area, as it offers the promise of making the storage or transmission of images more efficient. The aim of image compression [1] is to reduce the data size of image and then make the image stored or transmitted in an efficient form. Over the last two decades there has been significant research directed toward development of transform codes, with the discrete-cosine and wavelet transforms [2] constituting two important examples. The discrete cosine transform (DCT) is employed in the JPEG standard [3], with wavelets employed in the JPEG2000 standard [4]. Wavelet-based transform coding [5] explicitly exploits the structure [6] manifested in the wavelet coefficients of typical data. Specifically, for most natural data (signals and images) the wavelet coefficients are compressible, implying that a large fraction of the coefficients may be set to zero with minimal impact on the signal reconstruction accuracy. However, this is an inherently wasteful process (in terms of both sampling rate and computational complexity), since

one gathers and processes the entire image even though an exact representation is not required explicitly. This naturally suggests the question: can we sense compressible signals in a compressible way? In other words, can we sense only that part of the signal that will not be thrown away?

Over the past few years, a new framework called as compressive sampling (CS) has been developed for simultaneous sampling and compression. It builds upon the groundbreaking work by Candes et al. [7] and Donoho [8], who showed that under certain conditions, a signal can be precisely reconstructed from only a small set of measurements. The CS principle provides the potential of dramatic reduction of sampling rates, power consumption and computation complexity in digital data acquisitions. Due to its great practical potentials, it has stirred great excitement both in academia and industries in the past few years [9–11]. Most of the recent papers study two problems of CS. One is to find the optimal sampling ensembles and study the methods for fast implementation of the CS ensembles [12–14]. The other one is to develop fast and practical recon-

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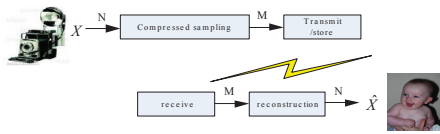


Figure 1 Image codec based on compressed sensing

reconstruction algorithms to recover the signal and suppress the noise introduced by CS [13–16]. However, most of existing works in CS remain at the theoretical study. In particular, they are not suitable for real-time sensing of natural image as the sampling process requires to access the entire target at once [17]. In addition, the reconstruction algorithms are generally very expensive.

In this paper, we apply CS to image representation and propose a new image representation scheme. Different from the previous works on compressive imaging [18, 19], which treat the whole image as a compressible signal, we decompose an image into 8 bit-planes. For the bit-plane compressed sensing algorithms of natural images proposed by this paper, the original image is divided into several bit planes according to pixel depth and each bit-plane is processed independently using the same transform operator and different measure operator. For the general 8-bit images, it has 8 bit-planes, the eighth bit-plane is the Most Significant Bit-plane, so we can adopt more measure vectors for reconstruction image precision. The main advantages of our proposed system include: (a) Measurement operator can be easily stored and implemented through a random under sampled filter bank; (b) bit plane-based measurement is more advantageous for the hardware application as the encoder does not need to send the sampled data until the whole image is measured; (c) Since each bit-plane is processed independently, the initial solution can be easily obtained and the reconstruction process can meet the different application demands, such as wireless Sensor Networks (WSN), remote sensing and compressed imaging.

The rest of this paper is organized as follows: section 2 briefly reviews the basics of compressed sensing. OMP algorithms are introduced in section 3. Bit-plane coding scheme based on compressed sensing are presented in Section 4. The performance of the methods is then assessed in Section 5. We conclude our paper and suggest some possible future directions in Section 6.

2. Compressed Sensing

Compressed sensing acquisition of data might have an important impact for the design of imaging devices where data acquisition is expensive. The Image codec system based on compressed sensing was shown in "Fig.1". CS theory asserts that it is possible to recover certain signals from far fewer samples than those used by traditional methods. To make this possible two premises must be met: sparsity,

which refers to the signals of interest, and incoherence, which refers to the random sensing vectors. Compressed sensing theory mainly includes sparse representation, measurement and reconstruction.

1. Sparse representations: Consider a length- N , real-valued, one-dimensional, discrete-time signal x indexed as $x(n)$, $n \in [1, 2, \dots, N]$. According to the signal processing theory, the signal x is a linear combination of the sparse basis $\Psi^T = [\psi_1, \psi_2, \dots, \psi_N]$, that is

$$f = \sum_{k=1}^N \psi_k \alpha_k = \Psi \alpha \quad (1)$$

Where $\alpha_k = \langle f, \psi_k \rangle$, α and f is $N \times 1$ column vector, and the sparse basis matrix Ψ is $N \times N$ with the basis vectors ψ_n as columns. Since ψ should be orthogonal, we can obtain α from f as

$$\alpha = \psi^* \quad (2)$$

The signal f is K -sparse if it is a linear combination of only K basis vectors; that is, only K of the α_k coefficients in (1) are nonzero and $(N - K)$ are zero. In general, most of the signal f have a sparse or nearly sparse representation α if we choose ψ as a DCT or wavelet matrix. For example, one can consider only keeping the largest K coefficients and discarding the $N - K$ small coefficients without much perceptual loss. Then the coefficient vector is sparse in a strict sense since all but a few of its entries are zero, which meets demand of compressed sensing theory.

The sample-then-compress is transform coding framework. The theory of compressed sensing exploits the signal sparsity, bypasses the sampling process and directly acquires a compressed form of the signal by measuring inner products between the signal and a set of functions. By doing so, measurements are no longer point samples, but rather random sums of samples taken across the entire signal.

2. Measurement: In CS, we do not measure or encode the K significant α_k directly. Rather, we measure and encode $M < N$ projections $y_m = \langle f, \varphi_m^T \rangle$ of the signal using the measure matrix $\Phi = [\varphi_1, \varphi_2, \dots, \varphi_M]$, where Φ_m^T denotes the transpose of Φ_m .

$$y = \phi f \quad (3)$$

Where y is an $M \times 1$ column vector, and the measurement basis matrix ϕ is $M \times N$. Then, by substituting f in equation (2) using equation (1), y can be written as

$$y = \phi f = \phi \psi \alpha = \Theta \alpha \quad (4)$$

Where $\Theta = \Phi \Psi$ is $M \times N$ matrix. The measurement process is not adaptive, meaning that Φ is fixed and does not depend on the signal x . To ensure the stability of CS, the measurement matrix Φ must be incoherent with the sparsifying basis Ψ . Surprisingly, the incoherence holds with high probability between an arbitrary basis and a randomly generated one, e.g., i.i.d. Gaussian or Bernoulli ± 1

vectors. The matrix $\phi(M \times N)$ is called the measurement matrix, and the number of measurements M is very important for CS reconstruction.

3.Reconstruction: in Equation (4), there are N unknowns but only M equations, with $M \ll N$, the solutions for are infinite. Obviously, it is an ill-posed problem recovering α from y . the strong prior knowledge of sparsity in f gives us achieved using optimization by searching for the sparsest signal that agrees with the M observed measurements in y . the key observation is that the signal f is the solution to the L_0 minimization

$$\min_{\alpha} \|\alpha\|_{L_0}$$

subject to $y = \phi\psi\alpha(5)$

Unfortunately, solving the L_0 minimization is known to be an NP-complete problem[20]. As a consequence, there have been a large number of alter optimizations proposed in recent literature. Perhaps the most prominent of these is basis pursuit(BP)[21] which applies a convex relaxation to the L_0 problem resulting in an L_1 problem.

$$\min_{\alpha} \|\alpha\|_{L_1}$$

subject to $y = \phi\psi\alpha(6)$

According to the compressive sensing theory, if f is K -sparse and the condition $M \geq cK \log(N/K)$ is satisfied, we can exactly reconstruct the K -sparse vector with probability close to one by solving the following L_1 optimization

Compressed sensing theory requires one of two constraints for this L_1 recovery to be efficient: (1) Sparsity: the signal f should be sparse in the basis B . It means that f can be represented using only a small number $s \ll N$ of atoms from B . $\|\Psi f\|_0 \leq s$, The theory extends to signals that are well approximated with a signal that is s -sparse in B . (2) Incoherence: When sampling a signal, the coherence of the sensing signals with respect to the transform domain where the signal has a sparse representation takes a lot of relevance.

$$\mu(\Phi, \Psi) = \sqrt{N} \bullet \max_{1 \leq k, j \leq N} |\langle \varphi_k, \psi_j \rangle| \quad (7)$$

Where Φ and Ψ are the sensing and transform basis of \mathbb{R}^N , from linear algebra it follows that $\mu(\Phi, \Psi) \in [1, \sqrt{N}]$.

There were some iterative greedy algorithms for solving(6) such as the Primal-Dual Interior-Point Algorithm [22], Matching Pursuit (MP) algorithm [23], Orthogonal Matching Pursuit (OMP) algorithm[24], Gradient Projection for Sparse Reconstruction (GPSR) algorithm [25] etc.

3. Orthogonal matching pursuit

In this paper, OMP(Orthogonal matching pursuit) algorithm is used in the signal reconstruction process. OMP

is a so called greedy algorithm for sparse approximation. Tropp and Gilbert[24]showed that OMP can reliably reconstruct a signal acquired by compressed sensing. Suppose f is a K -sparse signal in \mathbb{R}^N , and let $\Phi \in \mathbb{R}^N$ be a measurement matrix with columns $\varphi_1, \varphi_2, \dots, \varphi_N$. Then the signal f can be represented by an M -dimensional measurement vector $y = \Phi f$. Since x has only K non-zero components, y can be regarded as a linear combination of K columns from Φ . The reconstruction of f can be recast as the problem of identifying the locations of these K columns. OMP solves this problem by going through an iteration process. At each iteration OMP selects the column of Φ which is mostly correlated with the residual of measurement y , and then it removes the contribution of this column to compute a new residual. It is hope that the locations of all K columns will be identified after K iterations. Through basis-pursuit[1] is the classical approach to recover signals acquired via compressed sensing, OMP gives us a very suitable alternative. the reason why one would prefer OMP over basis-pursuit is because it is faster and easier to implement, but its guarantees are not as strong as those of Basis Pursuit. The OMP algorithm can thus be described as follows:

Input: Measurement matrix Φ , measurement vector y , sparsity level K

Output: index set Λ , measurement estimate a_i , residual $r_i = y - a_i$.

procedure:

$r_0 = y, \Lambda_0 = \Phi, t = 1$

while

$t \leq k$

$\lambda_t = \operatorname{argmax}_{j=1,2,\dots,N} |\langle r_{t-1}, \varphi_j \rangle|$

$\Lambda_t = \Lambda_{t-1} \cup \{\lambda_t\}$

$\phi_t = [\phi_{t-1} \varphi_{\lambda_t}]$

$x_t = \operatorname{argmin}_x \|\phi_t x - y\|_2$

$a_t = \phi_t x_t, r_t = y - a$

end while

4. Bit-plane coding scheme

At this moment, the bit-plane (bp) concept is necessary to be defined. Bit-planes are each one of the different bits needed to express a number in binary. Since we consider the first bp is the highest bit, and bp n is the lowest bit for a number represented with n bits (Fig.2)it is possible to take profit of processing together the same bp of all the coefficients of a bit plane ,because by this way we can make difference between more and less important bits. the Most significant Bit-plane can captures most significant information,and has a higher sparsity than the others,so it requires much fewer measurements for perfect reconstruction and hence helps to improve reconstruction performance.

At present, for Bit-plane image coding method, generally the first step in the encoder process is to apply wavelet transform to obtain the image in a transform domain in which is expected to be sparse. The transform image is

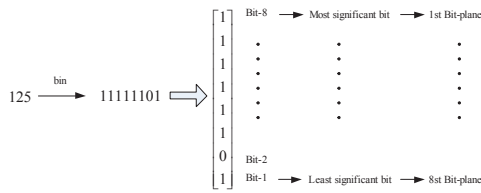


Figure 2 Bit-plane decomposition

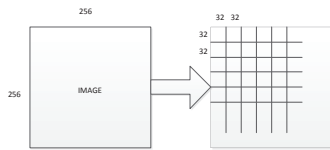


Figure 3 Image size division

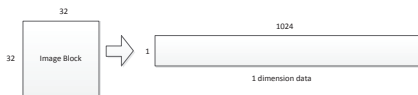


Figure 4 Two dimension image data transfer to one dimension data

divided in smaller blocks in the second step, and then bit-plane processing is finished in the third step [26]. For these algorithms, the word lengths of wavelet transform coefficients affect the complex of the algorithms. In this paper, the original image can be divided 8 bit-planes firstly, and then the each bit-plane will be processing using the orthonormal transform base. The presented algorithms are easy to design and hardware implement because the word length of the original image pixel value is constant.

In this paper, OMP was used for the image reconstruct. but OMP algorithm are not particularly suitable for two dimension image processing, the image data is transformed into one dimensional data. For the 256×256 image "lenna", We divide it into 32×32 small block (Fig.3), so we can take every 32×32 block as a length $N = 1024$ column vector x (Fig.4). OMP was used in the reconstruction step.

For every block, the bit plane CS method structure shown in Fig. 3. according to the important of different bit plane, CS codec can be flexible designed, so the scheme can meet the demand of different applications. In the presented method, the cs code part will be implemented as the structure of Fig.4. Considering that the value of Bit-plane pixel are "1" or "0", one order difference matrix was chosen as sparse transform matrix. The one order difference matrix was shown in Fig. 5. when the "lenna" (256×256) was used the tested image, Using the D matrix, the sparse representation of the 32×32 block of the "lenna" 8th bit plane can be ac-

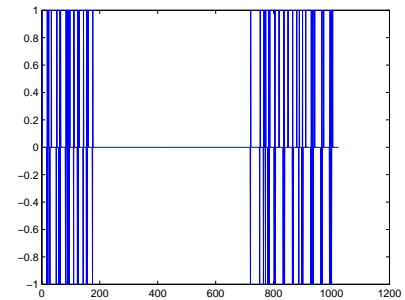


Figure 5 One dimension sparse representation for the 32*32 image block of the 8th bit plane

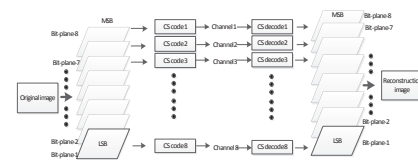


Figure 6 Bit-plane image codec scheme based on compressed sensing

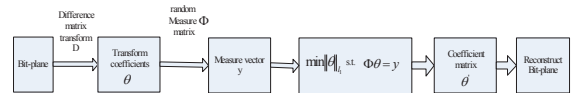


Figure 7 Coding process of bit plane based on Compressed sensing

$$D = \begin{bmatrix} 1 & 0 & 0 & \dots & \dots & 0 & 0 \\ -1 & 1 & 0 & \dots & \dots & 0 & 0 \\ 0 & -1 & 1 & \dots & \dots & 0 & 0 \\ \vdots & & & & & & \vdots \\ \dots & \dots & \dots & \dots & \dots & \dots & \dots \\ \vdots & & & & & & \vdots \\ 0 & 0 & 0 & \dots & \dots & 0 & -1 & 1 \\ 0 & 0 & 0 & \dots & \dots & 0 & 0 & -1 \end{bmatrix}$$

Figure 8 One order difference matrix

quired (Fig.6). for the image block, sparsity K is 890. after the sparse transform was finished, K is 81.

Because the hadamard matrix is an nn matrix with entries 1, considering demand of compressed sensing theory, the local hadamard matrix was chose as the measure matrix for meeting the demand of hardware implementation of compressed sensing. For the bit-plane image coding scheme, the researchers can design own CS codec with using different measure matrix and sparse transform basis.

5. Experimental results

In order to demonstrated the performance of the algorithms presented by us, the lenna(256*256) image will chosen as tested image. We consider eight-bit grayscale images and denote the least significant bit-plane as the 1st bit-plane, the most significant bit-plane the 8th bit-plane. In the commonly used grayscale images the study shows binary 0s and 1s are almost equally distributed in the lower bit-planes. The bias between 0s and 1s starts gradually increasing in the higher bit-planes. This kind of bias indicates redundancy, implying that we can compress bits in a particular bit-plane or more than one bit-plane.

The 256×256 image "lenna" can be divided 256 the column vector, and each column vector can be seen as 256×1 one dimension data, so the column vector can be reconstructed using the OMP algorithms. the number of measurements M for the measurement matrix was choose as followed rules:

$$M \geq 4K \tag{8}$$

Regardless of the signal's dimension N . In order to evaluate image reconstruction quality, Peak Signal to Noise Ration(PSNR) are defined as follows:

$$MSE = \frac{1}{MN} \sum_{x=1}^M \sum_{y=1}^N [f(x, y) - \hat{f}(x, y)]^2 \tag{9}$$

$$PSNR = 10 \log_{10}(255 * 255 / MSE) \tag{10}$$

Adopted the method presented in this paper, the original and reconstruction image of the 8th bit plane of the lenna image were shown in Fig 9. For the presented method, the reconstruction image were shown in Fig. 10 and Fig. 11 when the measurement value M are 180 and 130, and PSNR were 48.8456 and 36.3046 respectively. The 7th bit plane of the lenna image were shown in Fig.12. The same operator was finished, and the reconstruction results were shown in Fig.13 and Fig.14. PSNR were 38.5599 and 35.3834 when the measurement value M are 200 and 180. Because the bit-plane image coding scheme are main research content in this paper, the OMP algorithms used for this paper can not been optimized by the authors, so the experimental results can been further improved.

From the simulation results, we know that the image reconstruction precision can be adjusted with the measurement value M. moreover, because the value are 0 or 1 in each bit plane, the process of bit plane data measuring is simple and easy to hardware realize through correctly adopting measurement and transform matrix.

According to the significant of the different bit plane, the measurement M can use the different value, so the proposed method can construct more descriptions with lower complexity. When the different bit plane used different M value, the reconstruction image was shown in Fig.15, and PSNR is 28.8811.



Figure 9 8th bit-plane of the lenna



Figure 10 8th bit-plane reconstruction for M=180



Figure 11 8th bit-plane reconstruction for M=130

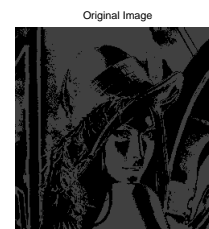


Figure 12 7th bit-plane of the lenna

6. Conclusion

We know today that most of existing works in CS remain at the theoretical study. In particular, they are not suitable



Figure 13 7th bit-plane reconstruction for $M=200$

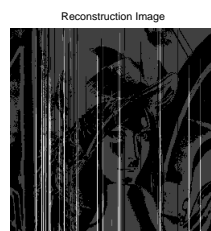


Figure 14 7th bit-plane reconstruction for $M=130$



Figure 15 Reconstruction lenna using different M in different bit plane

for hardware implementation. In addition, the reconstruction algorithms are generally very expensive. The method proposed in this paper take advantage of property of the bit plane, and can formulated many description for different application. Using the one order difference matrix, the bit plane can acquire more sparse presentations, and can be reconstructed for different precision. For the image codec scheme, most of the sensing process finished integer operator, so it is easy to hardware implementation. When the OMP algorithms used by this paper were optimized, we can acqurie better experimental results whcih demonstrate the image codec scheme. In the future, the fast CS hardware implementation algorithms based on the bit plane will be studied.

Acknowledgement

The authors acknowledge the financial support by Zhejiang Provincial Natural Science Foundation of China, project No.Y1110632. The author is grateful to the anonymous referee for a careful checking of the details and for helpful comments that improved this paper.

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