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A Comparative Study based on Bayes Estimation under Progressively Censored Rayleigh Data

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Abstract: A comparative study based on two different asymmetric loss functions presented in this article. Two-parameter Rayleigh model is consider here as the underline model for the present comparative study, that evaluate the properties of Bayes estimators under progressive Type-II right censored data.

Keywords: Bayes estimator, Progressive Type-II right censoring scheme, ISELF, LLF.

1 Introduction

The two (location-scale) parameter Rayleigh distribution has the probability density function and cumulative distribution function, given respectively, by

$$f(x;\theta,\sigma) = \frac{x-\sigma}{\theta^2} exp\left(-\frac{(x-\sigma)^2}{2\theta^2}\right); x > \sigma > 0, \theta > 0, \tag{1}$$

$$F(x;\theta,\sigma) = 1 - exp\left(-\frac{(x-\sigma)^2}{2\theta^2}\right); x > \sigma > 0, \theta > 0. \tag{2}$$

The considered model is useful in life testing experiments, which age with time as its failure rate is a linear function of time. The present distribution also plays an important role in communication engineering and electro-vacuum device.

The Rayleigh distribution is often used in physics related fields to model processes such as sound and light radiation, wave heights, wind speed, as well as in communication theory to describe hourly median and instantaneous peak power of received radio signals. It has been used to model the frequency of different wind speeds over a year at wind turbine sites and daily average wind speed.

In present paper, our focus is on presenting a comparative study of the Bayes estimation under two different asymmetric loss functions. Both known and unknown cases of scale parameter are considered here for estimation. For evaluation of performances of the proposed procedures, a simulation study carries out also.

A good deal of literature is available on Rayleigh model under different criterions. A little few of them are Sinha (1990), Bhattacharya & Tyagi (1990), Fernandez (2000), Hisada & Arizino (2002), Ali-Mousa & Al-Sagheer (2005), Wu et al. (2006), Kim & Han (2009), Prakash & Prasad (2010).

Soliman et al. (2010) presents some study about estimation and prediction of the inverse Rayleigh distribution based on lower record values. Dey & Maiti (2012) present some Bayes estimation for Rayleigh parameter under extended Jeffrey?s prior. Some Bayes estimation based on Rayleigh progressive Type-II censored data with binomial removals was

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discuss by Azimi & Yaghmaei (2013). Recently, Prakash (2013) presents some Bayes estimators for inverse Rayleigh model. Some Bayesian analysis for Rayleigh distribution also discussed recently by Ahmed et al. (2013).

2 The Progressive Type-II Right Censoring

The censoring arises when exact lifetimes are only partially known and it is useful in life testing experiments for time and cost restrictions. The progressive censoring appears to be a great importance in planned duration experiments in reliability studies. In many industrial experiments involving lifetimes of machines or units, experiments have to be terminated early and the number of failures must be limited for various reasons. In addition, some life tests require removal of functioning test specimens to collect degradation related information to failure time data. The samples that arise from such experiments are called censored samples.

The planning of experiments with the aim of reducing the total duration of experiment or the number of failures leads naturally to the Type-I & Type-II censoring scheme. The main disadvantage of Type-I & Type-II censoring schemes is that they do not allow removal of units at points other than the termination point of an experiment. Progressively Type-II censored sampling is an important method of obtaining data in lifetime studies. Live unties removed early on can be readily used in others tests, thereby saving cost to experimenter and a compromise can be achieved between time consumption and the observation of some extreme values.

The Progressive Type-II right censoring scheme is describes as follows:

Let us suppose an experiment in which n independent and identical units $X_1, X_2, ..., X_n$ are placed on a life test at the beginning time and first $m; (1 \leq m \leq n)$ failure times are observed. At the time of each failure occurring prior to the termination point, one or more surviving units are removed from the test. The experiment is terminated at the time of m^{th} failure, and all remaining surviving units are removed from the test.

Let $x_{(1)} \leq x_{(2)} \leq ... \leq x_{(m)}$ are the lifetimes of completely observed units to fail and $R_1, R_2, ..., R_m; (m \leq n)$ are the numbers of units withdrawn at these failure times. Here, $R_1, R_2, ..., R_m; (m \leq n)$ all are predefined integers follows the relation

$$\sum_{j=1}^{m} R_j + m = n.$$

At first failure time $x_{(1)}$, withdraw R_1 items randomly from remaining $n-1$ surviving units. Immediately after the second observed failure time $x_{(2)}, R_2$ items are withdrawn from remaining $n-2-R_1$ surviving units at random, and so on. The experiments continue until at m^{th} failure time $x_{(m)}$, the remaining items $R_m = n - m - \sum_{j=1}^{m-1} R_j$ are withdrawn. Here, $X_{1:m:n}^{(R_1,R_2,...,R_m)}, X_{2:m:n}^{(R_1,R_2,...,R_m)}, ..., X_{m:m:n}^{(R_1,R_2,...,R_m)}$ be m ordered failure times and $(R_1, R_2, ..., R_m)$ be the progressive censoring scheme (Balakrishnan & Aggarwala, 2000).

The resulting m ordered values, which are obtained as a consequence of this type of censoring, are appropriately referred to as progressively Type-II right censored order statistics.

Progressively Type-II right censoring scheme reduces to conventional Type-II censoring scheme if

$$R_i = 0 \, \forall \, i = 1, 2, ..., m-1 \Rightarrow R_m = n - m$$

and reduces to complete sample case if

$$R_i = 0 \, \forall \, i = 1, 2, ..., m \Rightarrow n = m.$$

Based on progressively Type-II censoring scheme, the joint probability density function of order statistic $X_{1:m:n}^{(R_1,R_2,...,R_m)}, X_{2:m:n}^{(R_1,R_2,...,R_m)}, ..., X_{m:m:n}^{(R_1,R_2,...,R_m)}$ is defined as

$$f_{(X_{1:m:n}, X_{2:m:n}, ..., X_{m:m:n})}(\theta, \sigma | \underline{x}) = K_m \prod_{i=1}^{m} f\left(x_{(i)}; \theta, \sigma\right) . \left(1 - F\left(x_{(i)}; \theta, \sigma\right)\right)^{R_i}; \tag{3}$$

here $f(.)$ and $F(.)$ are given respectively by (1) and (2) and K_m is a progressive normalizing constant defined as

$$K_m = n(n-R_1-1)(n-R_1-R_2-2)\ldots\left(n+1-\sum_{j=1}^{m-1}R_j-m\right).$$

The progressive Type-II censored sample is denoted by $\underline{x} \equiv \left(x_{(1)}, x_{(2)}, \ldots, x_{(m)}\right)$ and $(R_1, R_2, \ldots, R_m)$ being progressive censoring scheme for the considered Rayleigh model.

Substituting (1) and (2) in (3), the joint probability density function is obtain as:

$$f_{(X_{1:m:n}, X_{2:m:n}, \ldots, X_{m:m:n})}(\theta, \sigma|\underline{x}) = K_m H_m(\underline{x}, \sigma)\ \theta^{-2m} exp\left(-\frac{T_p(\underline{x}, \sigma)}{2\theta^2}\right); \tag{4}$$

where $H_m(\underline{x}, \sigma) = \prod_{i=1}^m \left(x_{(i)} - \sigma\right)$ and $T_p(\underline{x}, \sigma) = \sum_{i=1}^m (R_i+1)\left(x_{(i)}-\sigma\right)^2$.

It is noted here that when scale parameter is zero (i.e., $\sigma = 0$) ; $\left(x_{(i)}^2\left(2\theta^2\right)^{-1}\right)$ $\forall\, i = 1, 2, \ldots, n$, is distributed Exponential with mean two and the distribution of $\sum_{i=1}^m x_{(i)}^2 (R_i+1)$ is Gamma with shape parameter m and scale parameter two.

3 Bayes Estimation when Scale Parameter is Known

Assuming the scale parameter σ is known and location parameter θ is a random variable. A conjugate family of prior density for parameter θ is considered here as an inverted Gamma with probability density function

$$g_1(\theta) \propto \theta^{-2\alpha-1} exp\left(-\frac{1}{2\theta^2}\right); \alpha > 0, \theta > 0. \tag{5}$$

There is clearly no way in which one can say that one prior is better than other. It is more frequently the case that, we select to restrict attention to a given flexible family of priors, and we choose one from that family, which seems to match best with our personal beliefs. The prior (5) has advantages over many other distributions because of its analytical tractability, richness and easy interpretability.

Based on Bayes theorem, the posterior density is defined as

$$\pi(\theta|\underline{x}, \sigma) = \frac{f_{(X_{1:m:n}, X_{2:m:n}, \ldots, X_{m:m:n})}(\theta, \sigma|\underline{x}) . g_1(\theta)}{\int_\theta f_{(X_{1:m:n}, X_{2:m:n}, \ldots, X_{m:m:n})}(\theta, \sigma|\underline{x}) . g_1(\theta)\, d\theta}. \tag{6}$$

Using (4) and (5) in (6), the posterior density is obtain as

$$\pi(\theta|\underline{x}, \sigma) = \frac{K_m H_m(\underline{x}, \sigma)\, \theta^{-2m} exp\left(-\frac{T_p(\underline{x}, \sigma)}{2\theta^2}\right) . \theta^{-2\alpha-1} exp\left(-\frac{1}{2\theta^2}\right)}{\int_\theta K_m H_m(\underline{x}, \sigma)\, \theta^{-2m} exp\left(-\frac{T_p(\underline{x}, \sigma)}{2\theta^2}\right) . \theta^{-2\alpha-1} exp\left(-\frac{1}{2\theta^2}\right) d\theta}.$$

After simplification

$$\pi(\theta|\underline{x}, \sigma) = \eta^* exp\left(-\frac{\hat{T}_p(\underline{x}, \sigma)}{2\theta^2}\right)\theta^{-2(m+\alpha)-1} \tag{7}$$

where $\eta^* = \frac{\hat{T}_p(\underline{x}, \sigma)^{m+\alpha}}{\Gamma(m+\alpha) 2^{m+\alpha-1}}$ and $\hat{T}_p(\underline{x}, \sigma) = T_p(\underline{x}, \sigma) + 1$.

The selection of loss function may be crucial in Bayesian analysis. It has always been recognized that the most commonly used loss function, squared error loss function (SELF) is inappropriate in many situations. If SELF is taken as a measure of inaccuracy then the resulting risk is often too sensitive to assumptions about the behavior of tail of the probability distribution. To overcome this difficulty, a useful asymmetric loss function based on the squared error loss function is say as invariant squared error loss function (ISELF) and defined for any estimate $\hat{\theta}$ corresponding to the parameter θ as

$$L\left(\hat{\theta},\theta\right)=\left(\theta^{-1}\partial\right)^{2};\partial=\hat{\theta}-\theta. \tag{8}$$

The Bayes estimator corresponding to location parameter θ under ISELF is obtained as

$$\hat{\theta}_{I1}=\left[E\left(\theta^{-1}\right)\right]\left[E\left(\theta^{-2}\right)\right]^{-1}$$

$$=\left[\int_{\theta}\theta^{-1}\pi\left(\theta|\underline{x},\sigma\right)d\theta\right]\left[\int_{\theta}\theta^{-2}\pi\left(\theta|\underline{x},\sigma\right)d\theta\right]^{-1}$$

$$\Rightarrow\hat{\theta}_{I1}=\varphi_1\sqrt{\frac{\hat{T}_p\left(\underline{x},\sigma\right)}{2}};\varphi_1=\frac{\Gamma\left(m+\alpha+2^{-1}\right)}{\Gamma\left(m+\alpha+1\right)}. \tag{9}$$

When positive and negative errors have different consequences, the use of SELF or ISELF in Bayesian estimation may not be appropriate. In addition, in some estimation problems overestimation is more serious than the underestimation, or vice-versa. To deal with such cases, a useful and flexible class of asymmetric loss function (LINEX loss function (LLF)) is given as

$$L\left(\partial^{*}\right)=e^{a\partial^{*}}-a\partial^{*}-1;a\neq 0,\partial^{*}=\left(\theta^{-1}\partial\right).$$

The shape parameter of LLF is denoted by $'a'$. Negative (positive) value of $'a'$, gives more weight to overestimation (underestimation) and its magnitude reflect the degree of asymmetry. It is also seen that, for $a=1$, the function is quite asymmetric with overestimation being more costly than underestimation. For small values of $|a|$, the LLF is almost symmetric and is not far from SELF.

Bayes estimator $\hat{\theta}_{L1}$ of location parameter θ under LLF is obtain by simplifying following equality

$$E\left\{\frac{1}{\theta}exp\left(-a\frac{\hat{\theta}_{L1}}{\theta}\right)\right\}=e^{a}E\left(\frac{1}{\theta}\right)$$

$$\Rightarrow\int_{\theta}\theta^{-1}exp\left(-a\frac{\hat{\theta}_{L1}}{\theta}\right)\pi\left(\theta|\underline{x},\sigma\right)d\theta=e^{a}\int_{\theta}\theta^{-1}\pi\left(\theta|\underline{x},\sigma\right)d\theta$$

$$\Rightarrow\int_{\theta}\frac{exp\left\{-a\frac{\hat{\theta}_{L1}}{\theta}-\frac{\hat{T}_p(\underline{x},\sigma)}{2\theta^2}\right\}}{\theta^{2(m+\alpha+1)}}d\theta=\frac{e^{a}}{2}\Gamma\left(m+\alpha+2^{-1}\right)\left(\frac{2}{\hat{T}_p\left(\underline{x},\sigma\right)}\right)^{\left(m+\alpha+2^{-1}\right)}. \tag{10}$$

A nice closed form of Bayes estimator $\hat{\theta}_{L1}$ does not exist. However, a numerical technique is applied here for obtaining the numerical value of the estimate by solving given equality.

4 Bayes Estimation when Scale Parameter is Unknown

When location and scale both parameters are consider as random variable, the joint probability density function under progressive Type - II censoring criterion is given by

$$f_{\left(X_{1:m:n},X_{2:m:n},\ldots,X_{m:m:n}\right)}\left(\theta,\sigma|\underline{x}\right)=K_m H_m\left(\underline{x},\sigma\right)\theta^{-2m}exp\left(-\frac{T_p\left(\underline{x},\sigma\right)}{2\theta^2}\right). \tag{11}$$

It is clear from (11) that, the function $H_m\left(\underline{x},\sigma\right)$ and $T_p\left(\underline{x},\sigma\right)$ both depends upon scale parameter σ. Hence, in present case when both parameters are consider being random variables, the joint prior density for parameter θ and σ is defined as

$$g\left(\theta,\sigma\right)=g_2\left(\theta|\sigma\right).g_3\left(\sigma\right). \tag{12}$$

Here $g_2(\theta|\sigma)$ and $g_3(\sigma)$ are the inverted gamma densities and defined as

$$g_2(\theta|\sigma) = \frac{\theta^{-2\sigma-1}e^{-1/2\theta^2}}{\Gamma(\sigma)\,2^{\sigma-1}};\theta>0,\sigma>0, \tag{13}$$

and

$$g_3(\sigma) = \frac{\sigma^{-2\beta-1}e^{-1/2\sigma^2}}{\Gamma(\beta)\,2^{\beta-1}};\sigma>0,\beta>0. \tag{14}$$

The joint posterior density function is now obtained as

$$\pi^*(\theta,\sigma|\underline{x}) = \frac{f_{(X_{1:m:n},X_{2:m:n},\ldots,X_{m:m:n})}(\theta,\sigma|\underline{x}).g(\theta,\sigma)}{\int_\sigma\int_\theta f_{(X_{1:m:n},X_{2:m:n},\ldots,X_{m:m:n})}(\theta,\sigma|\underline{x}).g(\theta,\sigma)\,d\theta d\sigma}$$
$$= \frac{\left(\theta^{-2(\sigma+m)-1}exp\left(-\frac{\hat{T}_p(\underline{x},\sigma)}{2\theta^2}\right)\right)\left(\frac{H_m(\underline{x},\sigma)}{\Gamma(\sigma)2^\sigma}\sigma^{-2\beta-1}exp\left(-\frac{1}{2\sigma^2}\right)\right)}{\int_\sigma\frac{H_m(\underline{x},\sigma)}{\Gamma(\sigma)2^\sigma}\sigma^{-2\beta-1}exp\left(-\frac{1}{2\sigma^2}\right)\int_\theta\theta^{-2(\sigma+m)-1}exp\left(-\frac{\hat{T}_p(\underline{x},\sigma)}{2\theta^2}\right)d\theta d\sigma}$$

$$\pi^*(\theta,\sigma|\underline{x}) = \bar{\bar{\sigma}}\left(\theta^{-2(\sigma+m)-1}exp\left(-\frac{\hat{T}_p(\underline{x},\sigma)}{2\theta^2}\right)\right)\left(\frac{H_m(\underline{x},\sigma)}{\Gamma(\sigma)\,2^\sigma}\sigma^{-2\beta-1}exp\left(-\frac{1}{2\sigma^2}\right)\right); \tag{15}$$

where $\bar{\bar{\sigma}} = \frac{1}{2^{m-1}\bar{\sigma}}$ and $\bar{\sigma} = \int_\sigma\frac{\Gamma(m+\sigma)}{\Gamma(\sigma)}\frac{H_m(\underline{x},\sigma)}{\left(\hat{T}_p(\underline{x},\sigma)\right)^{m+\sigma}}\sigma^{-2\beta-1}exp\left(-\frac{1}{2\sigma^2}\right)d\sigma$.

The marginal posterior density corresponding to the parameter θ is given as

$$\pi^{**}(\theta|\sigma,\underline{x}) = \int_\sigma\pi^*(\theta,\sigma|\underline{x})\,d\sigma. \tag{16}$$

Now, the Bayes estimator corresponding to location parameter θ under ISELF is obtain by solving following equality

$$\hat{\theta}_{I2} = \left[\int_\theta\theta^{-1}\pi^{**}(\theta|\underline{x},\sigma)\,d\theta\right]\left[\int_\theta\theta^{-2}\pi^{**}(\theta|\underline{x},\sigma)\,d\theta\right]^{-1}$$

$$\hat{\theta}_{I2} = \frac{\int_\theta\theta^{-2(m+1)}exp\left(-\frac{\hat{T}_p(\underline{x},\sigma)}{2\theta^2}\right)\int_\sigma\left(\frac{H_m(\underline{x},\sigma)}{\Gamma(\sigma)\,2^\sigma}\sigma^{-2\beta-1}\theta^{-2\sigma}exp\left(-\frac{1}{2\sigma^2}\right)\right)d\sigma d\theta}{\int_\theta\theta^{-2m-3}exp\left(-\frac{\hat{T}_p(\underline{x},\sigma)}{2\theta^2}\right)\int_\sigma\left(\frac{H_m(\underline{x},\sigma)}{\Gamma(\sigma)\,2^\sigma}\sigma^{-2\beta-1}\theta^{-2\sigma}exp\left(-\frac{1}{2\sigma^2}\right)\right)d\sigma d\theta} \tag{17}$$

Similarly, the Bayes estimator corresponding to LLF for parameter θ is obtain by solving following equality

$$\int_\theta\theta^{-1}e^{-a\hat{\theta}_{L2}/\theta}\,\pi^{**}(\theta|\underline{x},\sigma)\,d\theta = e^a\int_\theta\theta^{-1}\,\pi^{**}(\theta|\underline{x},\sigma)\,d\theta$$

$$\Rightarrow\int_\theta exp\left(-a\frac{\hat{\theta}_{L2}}{\theta}-\frac{\hat{T}_p(\underline{x},\sigma)}{2\theta^2}\right)\theta^{-2(m+1)}\int_\sigma\left(\frac{H_m(\underline{x},\sigma)}{\Gamma(\sigma)\,2^\sigma}\sigma^{-2\beta-1}\theta^{-2\sigma}exp\left(-\frac{1}{2\sigma^2}\right)\right)d\sigma d\theta$$

$$= e^a\int_\theta exp\left(-\frac{\hat{T}_p(\underline{x},\sigma)}{2\theta^2}\right)\theta^{-2(m+1)}\int_\sigma\left(\frac{H_m(\underline{x},\sigma)}{\Gamma(\sigma)\,2^\sigma}\sigma^{-2\beta-1}\theta^{-2\sigma}exp\left(-\frac{1}{2\sigma^2}\right)\right)d\sigma d\theta. \tag{18}$$

Again, it is clear from equations (17) and (18) that, there are no any possible close forms of the estimators. Hence, for obtaining the numerical values of the estimates we consider here a numerical technique.

Table 1: Censoring Scheme for Different Values of m

Case	m	$R_i \forall i = 1,2,\ldots,m$
1	10	1 2 1 0 0 1 2 0 0 0
2	10	1 0 0 3 0 0 1 0 0 1
3	20	1 0 2 0 0 1 0 2 0 0 0 1 0 0 0 1 0 0 1 0

Table 2: Risk Ratio Between $\hat{\theta}_{L1}$ and $\hat{\theta}_{I1}$ Under ISELF

$n = 20, a = 0.50$						
$m \downarrow$	$\alpha \downarrow \sigma \rightarrow$	0.50	1.00	2.50	5.00	10.00
10	-0.50	0.9657	0.9753	0.9844	0.9934	0.9941
	1.00	0.9431	0.9525	0.9614	0.9702	0.9709
	2.50	0.8889	0.8978	0.9062	0.9145	0.9152
	5.00	0.7616	0.7692	0.7764	0.7835	0.7841
	10.00	0.4788	0.4836	0.4881	0.4926	0.4930
10	-0.50	0.9355	0.9448	0.9536	0.9623	0.9630
	1.00	0.9136	0.9227	0.9313	0.9398	0.9405
	2.50	0.8611	0.8697	0.8778	0.8858	0.8864
	5.00	0.7378	0.7452	0.7522	0.7591	0.7596
	10.00	0.4638	0.4684	0.4728	0.4771	0.4774
20	-0.50	0.9063	0.9153	0.9239	0.9323	0.9330
	1.00	0.8851	0.8939	0.9023	0.9106	0.9113
	2.50	0.8342	0.8425	0.8504	0.8582	0.8588
	5.00	0.7147	0.7218	0.7285	0.7352	0.7357
	10.00	0.4493	0.4538	0.4580	0.4622	0.4625

Table 3: Risk Ratio Between $\hat{\theta}_{L1}$ and $\hat{\theta}_{I1}$ Under LLF

$n = 20, a = 0.50$						
$m \downarrow$	$\alpha \downarrow \sigma \rightarrow$	0.50	1.00	2.50	5.00	10.00
10	-0.50	0.9239	0.9331	0.9418	0.9504	0.9511
	1.00	0.9023	0.9113	0.9198	0.9282	0.9289
	2.50	0.8504	0.8590	0.8670	0.8749	0.8756
	5.00	0.7287	0.7359	0.7428	0.7496	0.7502
	10.00	0.4581	0.4627	0.467	0.4713	0.4717
10	-0.50	0.8763	0.8850	0.8933	0.9014	0.9021
	1.00	0.8558	0.8643	0.8724	0.8804	0.881
	2.50	0.8066	0.8147	0.8223	0.8298	0.8303
	5.00	0.6911	0.6981	0.7046	0.7111	0.7115
	10.00	0.4345	0.4388	0.4429	0.4469	0.4472
20	-0.50	0.8317	0.8399	0.8478	0.8555	0.8562
	1.00	0.8122	0.8203	0.828	0.8356	0.8363
	2.50	0.7655	0.7731	0.7804	0.7875	0.7881
	5.00	0.6559	0.6624	0.6685	0.6747	0.6751
	10.00	0.4123	0.4164	0.4203	0.4241	0.4244

5 Numerical Illustration

A simulation is use in order to compare the performance of the proposed Bayes estimators in terms of risk ratios under progressively Type-II censored sample. According to limitations of the computer time, we carry out this comparison by taking the sample sizes as $n = 20$ with three different censoring schemes wiz, $m = 10, 10, 20$ (Table 1).

Table 4: Risk Ratio Between $\hat{\theta}_{L2}$ and $\hat{\theta}_{I2}$ Under ISELF

$n = 20, a = 0.50$						
$m\downarrow$	$\alpha\downarrow\beta\rightarrow$	0.50	1.00	2.50	5.00	10.00
10	-0.50	0.6839	0.6908	0.6973	0.7036	0.7042
	1.00	0.6680	0.6747	0.6809	0.6871	0.6877
	2.50	0.6296	0.6359	0.6419	0.6478	0.6482
	5.00	0.5395	0.5449	0.5500	0.5550	0.5553
	10.00	0.3392	0.3425	0.3457	0.3489	0.3492
10	-0.50	0.6352	0.6414	0.6475	0.6534	0.6538
	1.00	0.6203	0.6265	0.6324	0.6382	0.6386
	2.50	0.5847	0.5906	0.5961	0.6015	0.6019
	5.00	0.5010	0.5060	0.5107	0.5154	0.5157
	10.00	0.3149	0.3180	0.3211	0.3239	0.3242
20	-0.50	0.5906	0.5964	0.6020	0.6075	0.6079
	1.00	0.5767	0.5825	0.5880	0.5933	0.5939
	2.50	0.5436	0.5490	0.5542	0.5592	0.5596
	5.00	0.4658	0.4704	0.4748	0.4791	0.4793
	10.00	0.2928	0.2957	0.2985	0.3012	0.3014

Table 5: Risk Ratio Between $\hat{\theta}_{L2}$ and $\hat{\theta_{I2}}$ Under LLF

$n = 20, a = 0.50$						
$m\downarrow$	$\alpha\downarrow\beta\rightarrow$	0.50	1.00	2.50	5.00	10.00
10	-0.50	0.6016	0.6077	0.6134	0.6189	0.6195
	1.00	0.5876	0.5935	0.5990	0.6044	0.6049
	2.50	0.5538	0.5594	0.5647	0.5698	0.5702
	5.00	0.4746	0.4793	0.4838	0.4882	0.4885
	10.00	0.2984	0.3013	0.3041	0.3069	0.3072
10	-0.50	0.5588	0.5642	0.5696	0.5748	0.5751
	1.00	0.5457	0.5511	0.5563	0.5614	0.5617
	2.50	0.5143	0.5195	0.5244	0.5291	0.5295
	5.00	0.4407	0.4451	0.4492	0.4534	0.4536
	10.00	0.2770	0.2797	0.2825	0.2849	0.2852
20	-0.50	0.5195	0.5246	0.5296	0.5344	0.5347
	1.00	0.5073	0.5124	0.5172	0.5219	0.5224
	2.50	0.4782	0.4829	0.4875	0.4919	0.4923
	5.00	0.4097	0.4138	0.4177	0.4214	0.4216
	10.00	0.2576	0.2601	0.2626	0.2650	0.2651

5.1 When Scale Parameter is Known

The risk ratios between Bayes estimator under LLF and ISELF, are obtained by using following steps:

1. For given values of prior parameter $\alpha(= -0.50, 1.00, 2.50, 5.00, 10.00)$, a random value of the parameter θ is generated from prior density given by (5). It is remarkable here that the negative value of α makes the natural family of conjugate prior (5) into the non-conjugate (vague) prior. Hence, all the results are valid for both conjugate and non-conjugate family of priors.
2. Using the above generated values of θ obtained in Step (1), we generates a progressively Type-II censored sample, of size m for given values of censoring scheme $R_i; i = 1, 2, ..., m$, from the Rayleigh model, according to an algorithm proposed by Balakrishnan & Aggarwala (2000).
3. The results are based on $1,00,000$ simulation runs. For the selected values of $\sigma(= 0.50, 1.00, 2.50, 5.00, 10.00)$ and $a(= 0.50, 1.00, 1.50)$, a risk ratio between $\hat{\theta}_{L1}$ and $\hat{\theta}_{I1}$ are obtained and presented in Tables 2-3 under the loss criterion ISELF and LLF respectively.
4. It is seen from both the tables that, the risk ratios are smaller than the unity. This shows that the magnitude of risk with respect to LLF is smaller than compared to ISELF when other parameters values are considered to be fixed.

5. A decreasing trend have been seen for risk ratio when α increases in both cases. Similar properties also seen when censoring scheme m changed. The opposite trend for risk ratio have noted when σ increases.
6. All behaviors of risk ratios based on both loss functions are seen to be similar. Further, the risk ratios tend to be wider as the shape parameter of LLF, $'a'$ increases when other parametric values are consider to be fixed.

5.2 When Scale Parameter is Unknown

When both parameters are considered to be random variable, the risk ratio are obtained as follows:

1. For given values of prior parameter $\beta (= 0.50, 1.00, 2.50, 5.00, 10.00)$ a random value of parameter σ is generated from prior density given by (14).
2. Using (13) and generated values of σ, obtained the values of parameter θ.
3. Following the Steps discuss above and the considered parametric values, the risk ratio between $\hat{\theta}_{L2}$ and $\hat{\theta}_{I2}$ are obtained and presented in Tables 4 and 5, under ISELF and LLF loss criterion respectively.
4. Similar behaviors have been seen for unknown scale parameter case as compared to known scale parameter case. Further, it is observed that the magnitude of risk ratios are smaller than compared with previous one. However, the decrement in magnitude is robust.

6 Conclusion

A comparative study presented in this article for two-parameter Rayleigh model. Under progressive Type-II right censored data, evaluate the properties of Bayes estimators of location parameter. Invariant squared error loss function and LINEX loss function are used for the present comparative study. A simulation study has been carrying out for the analysis. It is observed that the risk ratio between the estimator obtained under LLF and ISELF is lesser than unity for all considered parametric values. This shows that the asymmetric loss function LLF minimizes more risk than compared to ISELF when other parametric values considered to be fixed. Hence, on behalf of the risk, one may use LLF instead of ISELF.

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