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Teleportation in the presence of technical defects in transmission stations

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Abstract: The process of teleport information remotely in presence of an error in the implementation of the local operations as CNOT and Hadamard gate at the sending station is discussed. Through the control of the laboratory equipment the errors in the achieving the local operations could be resisted. The imperfect CNOT gates performance could be improved as the error's strength in implementing of the Hadamard gate increases. It is shown that the accuracy of the information transfer not only depends on the laboratory equipment, but also on nature of the information to be teleported.

Keywords: Teleportation; Technical Defects; Cooper pair box.

1. Introduction

Quantum teleportation is a process that could be used to rapidly transfer unknown state between two separate locations. It is one of more common method which has been investigated theoretically [1,2] and experimentally [3–5]. To achieve this protocol, one needs entangled pairs which represent the quantum channel between the sender and the receiver, local operations and measurements. Since entangled pairs are crucial resource for quantum communication, the preparation of maximally entangled states is a crucial task. There are several attempts have been carried out to generate entangled channels between different types of particles. Ideally, entangled atoms are used widely for this proposal because they would remain stable over long timescales. As an example, Wang and Schirmer [7] have generated a maximum entangled states between distant atoms by Lyapunov control. Generated entangled atoms in finite time between a pair of space-like separated atoms, is investigated by Leon and Sabin [8]. Lee and et. al. [9], have proposed a cavity-QED-based scalable scheme to an arbitrary number of atoms of generating entanglement between atoms. A different scheme to generate maximally entangled state, using trapped ions interacting with a resonant external laser and sideband tuned single mode of a cavity field has been proposed [10–13].

One of promising pairs in context of quantum information are the Cooper pairs boxes. Due to their potential in quantum information processing, there are several studies has been done investigated the properties of these particles [14,15]. One of the most important studies is quantum memories for theses pairs, where the speed and accuracy of storing information is investigated for these types of pairs [16]. Tordrup and Mo lmer have present a method for many-qubit quantum computing with a single molecular ensemble and a Cooper pair box [17]. Also, these particles have been employed to generate entangled state by interacting them with a single cavity mode [18]. Therefor, these pairs have been used in quantum teleportation as quantum channels [14,21].

To implement quantum information tasks, we need lab equipments with high efficiency and free from manufacturing defects. In reality, the process of these requirements is impossible. Also the quantum information processing is sensitive to any external noise [19]. As an example, quantum teleportation, coding, cryptography require a maximum entangled state to be achieved. experimentally, one can generate maximum entangled stated but due to the interactions with surrounding turn into partially entangled states [20]. Therefore the efficiency of performing these quantum information tasks decreases. For this reason, there are many efforts have been treated these tasks using par-

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tially entangled states [22,23]. Also, there are local operations have to be done to implement quantum information tasks. The most common local operations are the CNOT and Hadamard gates, where they are used in most quantum information and computation processing [24,25]. Due to the imperfect lab equipments and the noise environments the users may can not implement these gates perfectly. So, understanding achieving these tasks taking into account the noise operations is very important.

This leads us to the main aim of this article, where we use entangled state between a superconducting charged qubit and a single cavity mode as a quantum channel to perform quantum teleportation. In this strategy, we assume that the local operations are achieved partially perfect. The effect of the qubit and the field's parameters on the fidelity of transporting information between two users is investigated.

This paper is organized as follows: In Sec.2, an analytical solution to the suggested model is introduced. The model of the noise operations is described Sec.3. Employing the entangled state between a superconducting charged qubit and the cavity mode for transmitted unknown information remotely is discussed in Sec.4. Finally Sec.5, is devoted to discuss the results.

2. Model description

We consider a single superconducting island connect to a superconducting electron reservoir. It could be considered as a two-level quantum system which is useful in building a block of quantum computations. The most relevant example of pairing is BCS superconductivity, in which attractive interactions cause electrons to perfectly anticorrelate in momentum and spin, forming Cooper pairs [28,29]

$$H_s = 4E_c(n-n_g)^2 - E_j\cos\phi, \tag{1}$$

where, $E_c = \frac{1}{2}e^2\left(C_J + C_g\right)$ is the charging energy, $E_J = \frac{1}{2}\frac{\hbar}{e}I_c$ is the Joesphson coupling energy, e is the charge of the electron, $n_g = \frac{1}{2}\frac{V_g}{e}C_g$ is the dimensionless gate charge, n is the number operators of excess cooper pair on the island and ϕ is the phase operator [28,29]. The Hamiltonian of the system (1) can be simplified, if the Josephon coupling energy, E_j is much smaller than the charging energy i.e $E_j << E_c$. In this case, the Hamiltonian of the system can be parameterized by the number of cooper pairs n on the island. If the temperature is low enough, the system can be reduced to a qubit.

$$H_s = -\frac{1}{2}B_z\sigma_z - \frac{1}{2}B_x\sigma_x, \tag{2}$$

where $B_z = -\left(2n-1\right)E_{cl}$, E_{cl} is the electric energy, $B_x = E_j$ and $\sigma_x, \sigma_y, \sigma_z$ are Pauli matrices. This cooper pair can be viewed as an atoms with large dipole moment coupled to microwave frequency photons in a quasi-one-dimensional transmission line cavity (a coplanar waveguide resonator). The combined Hamiltonian for qubit and transmission line cavity is given by [30,31],

$$\begin{aligned}\mathcal{H} &= \varpi a^\dagger a + \varpi_c\sigma_z \\ &\quad -\lambda(\mu - \nu\sigma_z + \sqrt{1-\nu^2}\sigma_x(a^\dagger + a)),\end{aligned} \tag{3}$$

where, ω is the cavity resonance frequency,
$\omega_c = \sqrt{E_j^2 + 16E_c^2\left(2n_g - 1\right)^2}$
is the transition frequency of the cooper pair qubit, $\lambda = \frac{\sqrt{C_j}}{C_{g+C_J}}\sqrt{\frac{1}{2}\frac{\varpi}{\hbar}e^2}$ is coupling strength of resonator to the cooper pair qubit, $\mu = 1 - n_g$, $\nu = \cos\theta$ and
$\theta = -\arctan\left(\frac{1}{E_c}\frac{E_j}{2n_g - 1}\right)$
is mixing angle.

Let us assume that the initial state of the system is prepared initially in $|\psi_s(0)\rangle = |e,n\rangle$. The time evolution of the initial system is given by

$$|\psi_s(t)\rangle = \mathcal{U}(t)\,|\psi_s(o)\rangle\,, \tag{4}$$

where, $u(t)$ is a unitary operator defined by,

$$\begin{aligned}U_{11}(t) &= \frac{1}{2}(1-\frac{\delta}{\eta_n})e^{i\eta_n t} + \frac{1}{2}(1+\frac{\delta}{\eta_n})e^{-i\eta_n t},\\ U_{12}(t) &= -\frac{\lambda}{2\eta_n}(e^{i\eta_n t} - e^{-i\eta_n t})a,\\ U_{22}(t) &= \frac{1}{2}(1+\frac{\delta}{\eta_n})e^{i\eta_n' t} + \frac{1}{2}(1-\frac{\delta}{\eta_n})e^{-i\eta_n' t},\end{aligned} \tag{5}$$

and $U_{21}(t) = U_{12}^*$, $\grave{\eta}_n = \lambda^2 a^\dagger a + \frac{\delta^2}{4}$, $\eta_n = \grave{\eta}_n + \lambda^2[a, a^\dagger]$ and $\delta = E_j - \varpi$ is the detuning parameter between the Josephson energy and the cavity field frequency. Now, by using (4) and (5), the density operator of the system ρ_s is given by,

$$\begin{aligned}\rho_s(t) &= A\,|e,n\rangle\,\langle e,n| + B\,|e,n\rangle\,\langle g,n+1| \\ &\quad + C\,|g,n+1\rangle\,\langle e,n| + D\,|g,n+1\rangle\,\langle g,n+1|\,,\end{aligned} \tag{6}$$

where

$$\begin{aligned}A &= C_{n+1}^2 + \Delta^2 S_{n+1}^2,\\ B &= -i\sqrt{(n+1)}(C_{n+1} - i\Delta S_{n+1})S_n,\\ C &= B^*, \qquad D = (n+1)S_n^2,\end{aligned} \tag{7}$$

$$\begin{aligned}C_n &= \cos\Omega\tau\sqrt{(\frac{\delta}{2\lambda})^2 + n},\\ S_n &= \frac{2\lambda}{\sqrt{\Delta^2 + 4\lambda^2 n}}\sin\Omega\tau\sqrt{\Delta^2 + n},\end{aligned}$$

with $\Omega = \frac{\sqrt{C_j}}{C_j + C_g}$, $\tau = \sqrt{\frac{\omega}{2\hbar}}e$ and $\Delta\frac{\delta}{2\lambda}$.

Since we have got the density operator of the system, we can investigate all the classical and quantum phenomena associated with the quantum channel (6). In this context, we use this quantum channel to achieve the quantum teleportation to transmit unknown information between two users, Alice and Bob.

3. Imperfect Teleportation

In this section, a scheme to implement quantum teleportation is proposed. In this scheme, we assume that the local operations, CNOT and Hadamard gate, which are needed to achieve the quantum teleportations are non ideal. Assume that Alice is given unknown state defined by,

$$\rho = |\alpha|^2 |e\rangle \langle e| + \alpha\beta^* |e\rangle \langle g| + \beta\alpha^* |g\rangle \langle e| + |\beta|^2 |g\rangle \langle g| , \quad (8)$$

and she is asked to send it to Bob, who share with Alice an entangled state given by (6).

1. Due to the defect on the equipment Alice performs imperfectly the CNOT gate on her qubit and the given unknown qubit. After this operation the final state of the system is given by

$$\rho^{(1)}_{out} = pCNOT\rho^{(0)}_{in} CNOT + (1-p)\rho^{(0)}_{in} \quad (9)$$

where, with probability p the CNOT gate is performed perfectly and with probability $(1-p)$ the operation fails and $\rho^{(0)}_{in} = \rho_u \otimes \rho$.
2. If Alice applies the imperfect Hadamard gate on the output state , then the output state is given by,

$$\rho^{(2)}_{out} = qH\rho^{(1)}_{out} H + (1-q)\rho^{(1)}_{out} \quad (10)$$

where, the Hadamard gate is applied correctly with probability q and fils with probability $(1-q)$.
3. Alice measures her qubit and the unknown qubir randomly in one of the basis $|ee\rangle$, $|eg\rangle$, $|ee\rangle$ and $|gg\rangle$ and sends her results to Bob by using classical channel.
4. As soon as Bob receives the classical data from Alice,he applies a single qubit operation on his qubit depending on Alies's results. Therefor, if Alice measures in the basis $|gg\rangle$, Bob will obtain the state

$$\rho_{Bob} = A\eta_1 |n\rangle \langle n| - B\eta_2 |n\rangle \langle n+1| - C\eta_3 |n+1\rangle \langle n| + D\eta_4 |n+1\rangle \langle n+1| , \quad (11)$$

where,

$$\eta_1 = \frac{q^2(1-p)^2}{2} \left(|\alpha|^2 - \alpha\beta^* - \beta\alpha^* + |\beta|^2\right) + \left(\frac{q^2p^2}{2}|\beta|^2 + (1-p)^2(1-q)^2|\alpha|^2\right),$$

$$\eta_2 = \frac{\beta\alpha^*}{2}q^2p^2, \quad \eta_3 = \frac{\alpha\beta^*}{2}q^2p^2,$$

$$\eta_4 = |\alpha|^2 \left[\frac{1}{2}q^2p^2 + p^2(1-q)^2\right]. \quad (12)$$

The fidelity, F of the teleported state (8) is given by,

$$F = D|\alpha|^4\eta_4 + A|\beta|^2\eta_1 + |\alpha|^2|\beta|^2q^2p^2\Delta S_n S_{n+1}. \quad (13)$$

Fig.(1), shows the behavior of the fidelity, F, under the effect of noise CNOT operation while the Hadamard gate is applied perfectly. We investigate the effect of the correlation parameter λ between the Cooper pair and the cavity mode as a function of the capacities C_j and C_g, where $\lambda \propto \Omega = \frac{\sqrt{C_j}}{C_j+C_g}$. Let us assume that Alice is given unknown state defined by $\alpha = 0.2$ and $\beta = \sqrt{1-\alpha^2}$. In Fig.(1a), we investigate the time evolution of the fidelity, F, where small values of the superconducting charged qubit capacity, C_j and the gate capacity C_g are considered. It is clear that, for small values of p i.e., Alice fails to perform the CNOT gate correctly, the fidelity, $F \simeq 0.29$ at $t = 0.0$. This behavior is due to the second term of Eq. (13), where all the other terms are cancelled. As p increases i.e Alice is partially successes in achieving the CNOT gate, F increases gradually and reaches to its max-

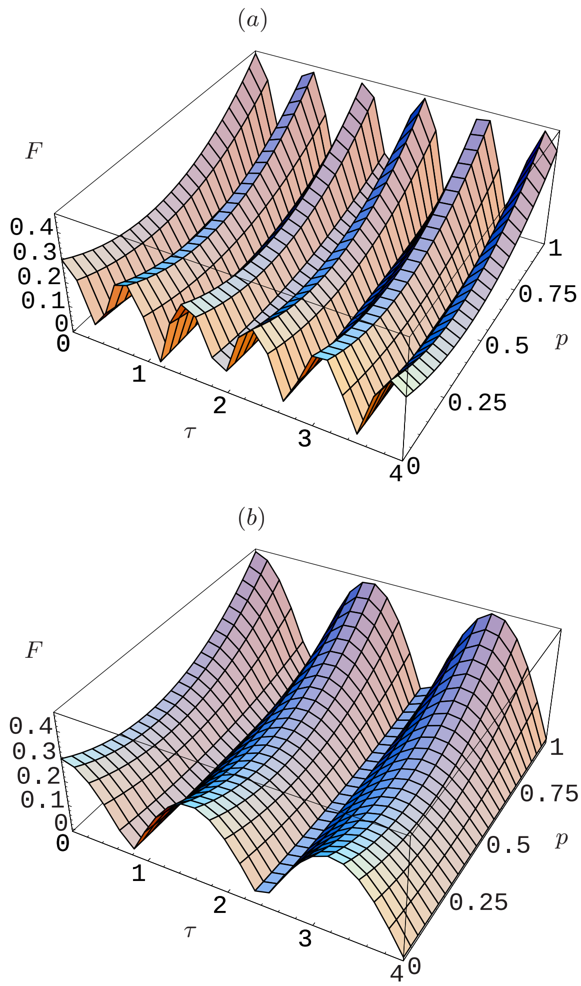

Figure 1 The fidelity,F of the transmitted unknown state (8), where $q = 1$ and $\delta = 0$ and the CNOT operation is implemented with probability $0 \leq p \leq 1$ (a) The unknown state is defined by $\alpha = 0.2$ and $\beta = \sqrt{1-\alpha^2}$ and $C_j = C_g = 0.01$(b) The same as Fig.(a) but $C_j = 0.5$ and $C_g = 0.01$.

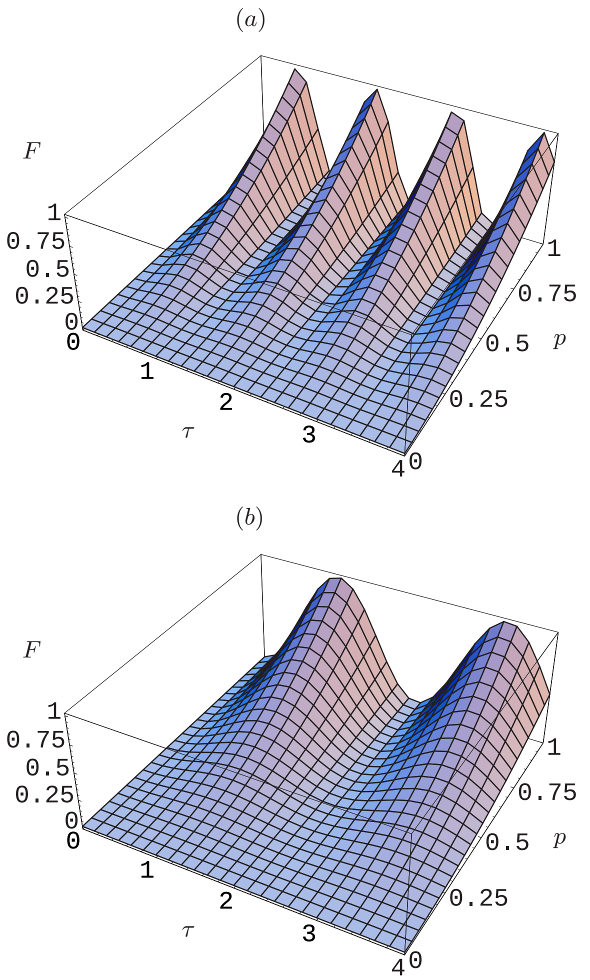

Figure 2 The same as Fig.(1), respectively but $\alpha = 1$.

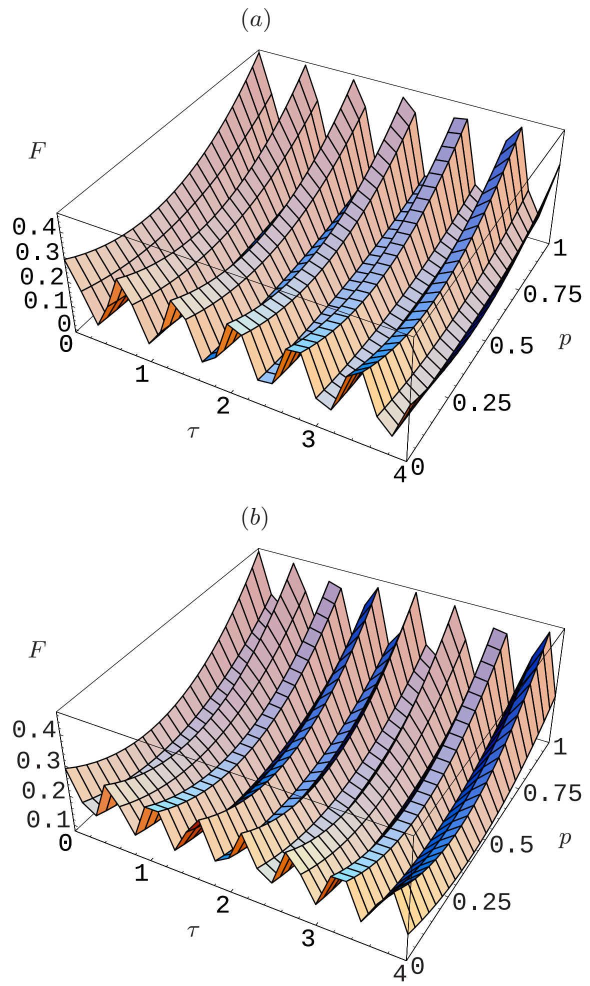

Figure 3 The same as Fig.(1a) but (a) $\delta = 0.15$ and (b) $\delta = 1.0$.

imum value $F = 0.4$. at $p = 1$. Fig. (1b), shows the dynamics of the fidelity for different initial values setting for the Ω, where we set $C_j = 0.5$ and $C_g = 0.01$. In this case, the fidelity increases as p increases, while the number of revivals decreases.

The structure of the unknown state plays an important role on the fidelity of the transport it between two locations. Consider that, Alice is given an unknown state with $\alpha = 1.$, while the other parameters are fixed as those in Fig. (1a). In this case the fidelity, $F = \frac{1}{2}p^2q^2S_n^2(n+1)$. Therefor at $p = 0$, the fidelity $F = 0$ and increases as p increases. The fidelity reaches its maximum values at the dimensionless time $\tau = \frac{\pi}{2\lambda}$, this behavior is seen in Fig. (2a). The dynamics of the fidelity for the same unknown state i.e., $\alpha = 1$ is shown in Fig. (2b) for different values of $C_j = 0.5$ and $C_g = 0.01$. The same behavior is seen as that depicted in Fig. (2a), but the number of revivals decreases.

In Fig. (3), we consider the effect of the dimensionless detuning parameter, $\Delta = \frac{\delta}{2\lambda}$ which measures the resonances between the cavity mode and the Cooper pair box . The dynamics of the fidelity, F for small value of the detuning, $\delta = \frac{1}{2}$ is shown in Fig. (3a). It is clear that the oscillations decreases this means that the minimum value of the fidelity increases. As one increases $\delta = 1$, the rate of decreasing the revivals decreases and consequently the minimum values of F increases as shown in Fig. (3b). However as the probability of performing the CNOT gate correctly, the fidelity of the teleported state increases.

Assume that Hadamard gate can be implemented with a probability $(q = 0.1)$. In this case the dynamics of F is depicted in Fig. (4a), where the values of the other parameter is the same as that for Fig. (1a). It is clear that, the parameters η_1 and η_4 control the dynamics of the fidelity F, where they have maximum values for small values of p. Therefore the fidelity becomes minimum for $p = 1.0$.

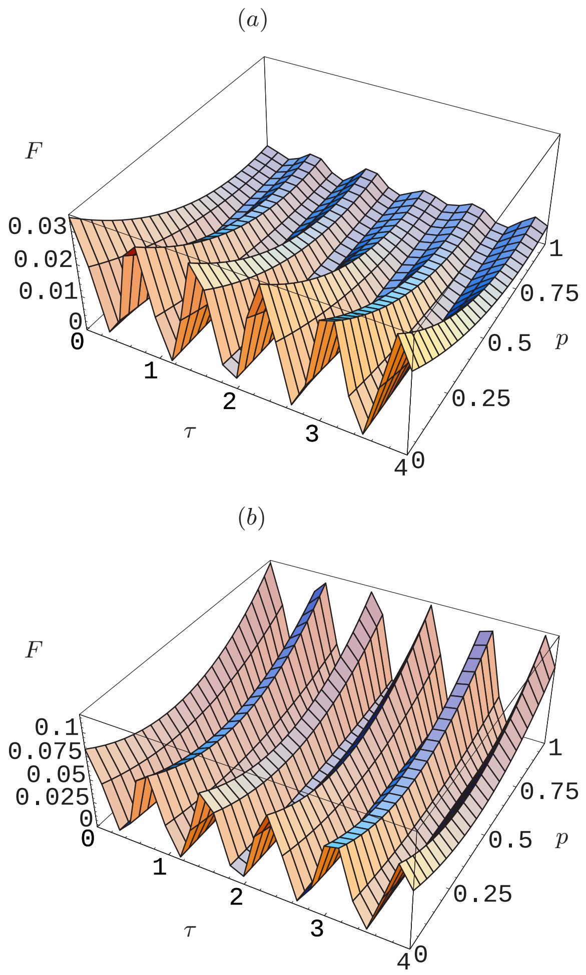

Figure 4 The same as Fig.(1a) but (a)$q = 0.1$ and (b) $q = 0.5$.

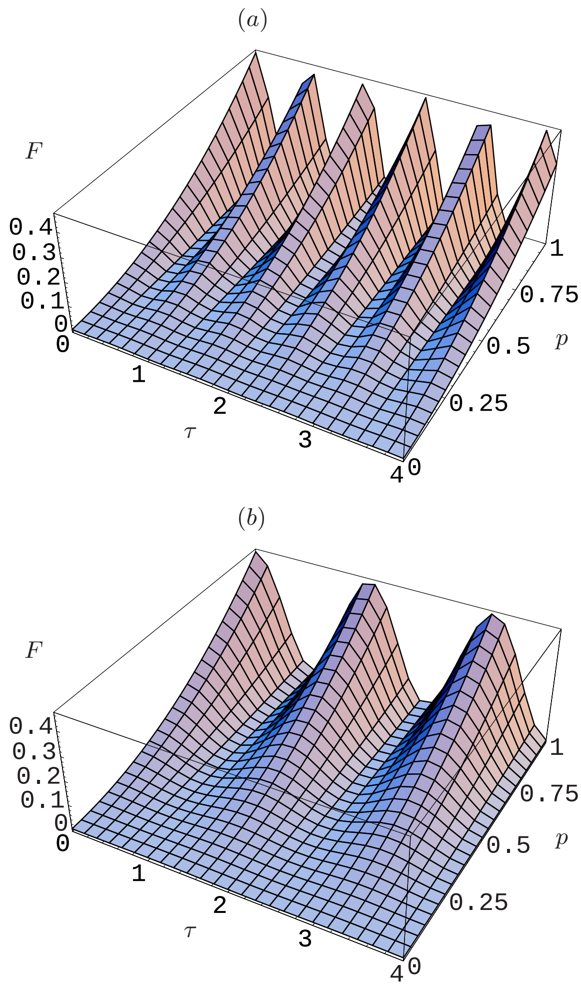

Figure 5 The fidelity of the transmitted unknown state F, where $p = 1$,$\delta = 0$ and the Hadamard gate is implemented with probability $0 \leq q \leq 1$ (a)$c_j = 0.1$ and $C_g = 0.01$ and (b)$c_j = 0.5$ and $C_g = 0.01$.

This means that CNOT operation is very fragile against any other noise. We notice that the general behavior of the accuracy of the transfer of information is the same behavior shown in Fig. (1a), but the fidelity of the teleported state is very small. However as one increases the probability of implementing Hadamard gate, the fidelity of the teleported state increases (see Fig. (4b)).

Let assume that Alice has implemented the CNOT gate efficiently i.e., with probability $p = 1$, while the Hadamard gate is achieved with probability $0.0 \leq q \leq 1.0$. Our first example is assumed with the same parameters as Fig. (1a) but $p = 1$, the behavior of the fidelity,F is shown in Fig. (5a). We can see that the fidelity reduces to become $F = \frac{q^2p^2}{2}\left(A|\beta|^4 + D|\alpha|^6\right)$. Therefor at $q = 0$, the fidelity vanishes and re-increases gradually as q increases. As Ω is increases, the fidelity increases and the number of oscillations decreases as shown in Fig. (5b).

Finally, we consider that the CNOT operation is performed correctly with a small probability $p = 0.1$, while the Hadamard is achieved correctly with probability q. In this case a surprising result is obtained, where the fidelity is improved at $q = 0$, where in this case the fidelity $F = D|\alpha|^6p^2 + A|\beta|^2|\alpha|^2(1-p)^2$. These results are depicted in Fig. (6b). On the other hand the average values of the fidelity decreases comparing with that depicted in Fig. (5a). In Fig. (6b), we set larger values of $p = 0.5$, the fidelity at $q = 0$ decreases because the value $(p-1)^2$ decreases as one increases p.

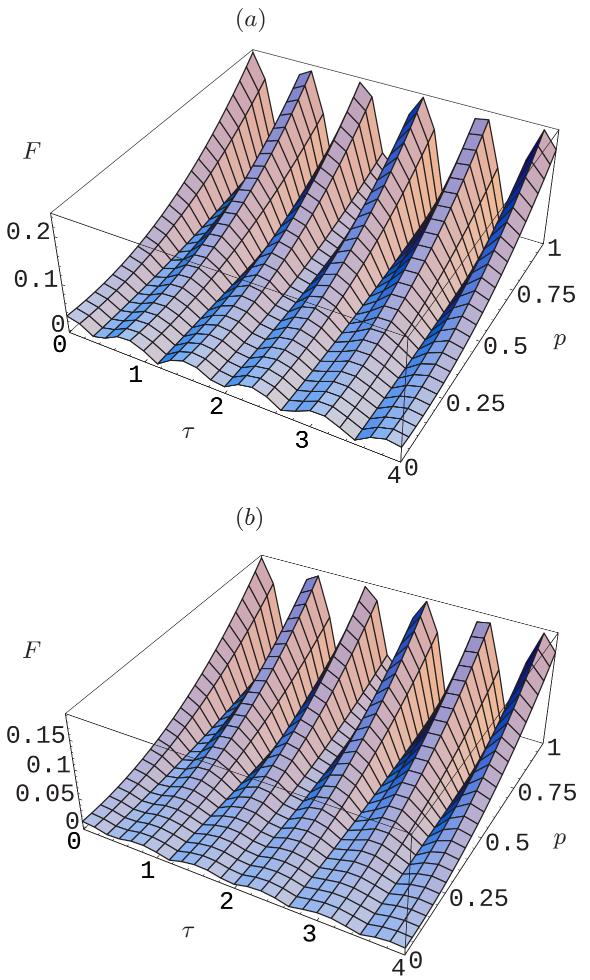

Figure 6 The same as Fig.(5a) but (a) $p = 0.1$ and (b)$p = 0.5$.

4. Conclusions

In this contribution, the problem of quantum teleportation in the presences of noise quantum operations is treated. An entangled state between a superconducting qubit and a single cavity mode is used as quantum channel. The effect of the noise CNOT, Hadamard gate and the channel parameters on the dynamics of the teleported state is investigated.

We have shown that, for small values of the coupling constant, which is a function of the cooper pair capacities, the fidelity of the teleported state increases as one increases the strength of the noise CNOT operation. However, only the number of revivals increases as one increases the coupling constant. On the other hand, for larger values of the detuning parameter, the fidelity oscillates so fast, but its lower bound increases for larger value of the detuning. The effect of the noise CNOT gate is investigated in the presences of the noise Hadamard gate. We showed that the fidelity decreases very vast for small values of the strength of noise Hadamard gate, while as one increases this strength, the fidelity is much better. These results show that, the imperfect CNOT operation is very sensitive to any additional noise (Hadamard), where the fidelity decreases gradually. However, the additional noise gate represented in (Hadamard) gate, improves the fidelity of information transfer. On the other hand, for perfect CNOT operation, the fidelity increases the strength of the Hadamard gate increases.

The effect of structure of the teleported state on the fidelity is investigated. For pure state, the fidelity increases as one increases the strength of the CNOT operation for small values of the coupling constant. Also, for larger values of the coupling constant, the number of oscillations decreases. On the other hand, if the teleported state is a mixed state, the average of the fidelity is much smaller than that depicted for pure state.

Finally, this study could be useful in building quantum computer, particulary by using the superconducting charged qubit. This study gives us an idea about the rate of information transfer, and then the pace of the calculations.

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