Applied Mathematics & Information Sciences
An International Journal

http://dx.doi.org/10.12785/amis/070408

Optimization of Transverse Load Factor of Helical and Spur Gears Using Genetic Algorithm

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Received: 4 Jan. 2013, Revised: 27 Feb. 2013, Accepted: 25 Mar. 2013
Published online: 1 Jul. 2013

Abstract: In this paper work, it was discussed the model of meshing gears such that the transverse load factor does not change over time and along the line of contact in order to determine if there is some deviations from the assumed and to determine the extent of their changes. During the optimization all factors which determine transverse load factor, according to [1], [2], [3], [4], [5] and [6] were considered as relevant and as such varied using the genetic algorithm optimization process. Only the factors of the basic rack were pre-approved from [5] and as such are considered to be constant input parameters. It is presented new method for finding the optimal geometry compared to many other relevant factors based on a dynamic optimization of factors relevant to meshing of helical and spur gears that is performed in the form of the simulation of gear meshing along the line of contact. Optimization process of 12 input parameters is performed by genetic algorithm and in addition, many important parameters were computed by linear and non linear interpolation. Using this method, it appeared that the most affecting variable of changing the value of load transverse factor is helix angle β, but, despite of this, the profile shift coefficients x_1 and x_2 also affected to changing the value of load transverse factor. It is noted that for any number of teeth (from the range $18-54$) and any gear ratio (from the range $1-5$), this method achieves a value 1 of the load transverse factor, which therefore corresponds to uniform load distribution.

Keywords: hybrid algorithm, ISO standard, interpolation

1. Introduction

Load or torque transmission in mechanical systems is obtain by different kind of rotate transmission elements. One of possible ways is load transmission by tooth pairs (gears), which are take into the consideration here. Gear meshing during load transmission, in general case, characterized by non-uniform load distribution on teeth and teeth surfaces which are currently in mesh. Many different parameters are take influence on non-uniform load distribution like load intensity, kind of system actuator, machining grade (quality) of tooth contact surfaces, rotating speed, tooth profile geometry, etc. On the other hand, in teeth load calculations, influence of the above parameters on non-uniform load distribution are taken in consideration by different factors.

Metaheuristics are widely used tools in optimization. Among them the significant role play genetic and evolutionary algorithm [18], [19], [20], [21], [23]. There are different approaches of occurrence of non-uniform load distribution during gear teeth meshing with aims to reduce negative influence, increase efficiencies and period of exploitation of transmission systems elements.

In [15] authors were using some methods and expressions valid for every tool geometry, standard or not. In [17] an approximate equation for the addendum modification factors for gears with balanced specific sliding (which reduces wear and heavy scoring risks) is presented using simple analytical methods.

In [22] various optimization techniques are used in order to find a proper solution.

However, that model presented some discrepancies with experimental results because the changing rigidity of the pair of teeth along the path of contact produces a non-uniform load distribution, which implies that some load distribution factors are required to compute the contact stress.

In this paper work, it was presumed inverse that the model of meshing is such that the transverse load factor does not change over time and along the line of contact and that have the same value $K_{H\alpha} = K_{F\alpha} = 1$, for both double and single pair tooth-contact, in order to determine

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if there is still some deviations from the assumed and the extent of their changes. In addition, the rigidity of the pair of teeth was taken into consideration, as one very meaningful function, influenced by a lot of input data that are used for optimization and it is adopted that the gears are made from steel. Also, it is presented a new approach to calculate a best values of all relevant factors for meshing gears, so that the load is uniform at any point of the line of contact. During the optimization all the factors that determine transverse load factor were considered as relevant and as such varied until the and of the optimization process. Only the factors of the basic rack were pre-approved from [5] and as such are considered to be constant input parameters. It is presented new method for finding the optimum geometry compared to many other relevant factors based on a dynamic optimization of factors relevant to meshing of helical and spur gears that is performed in the form of the simulation of gear meshing along the line of contact. In order to optimize process of meshing gears, many formulas and procedures within ISO standards were used [4], [2], [3], [5], [1], [6] but, despite of this, the values of all specific variables were varied in order to find appropriate combination of geometry, stiffness, application factor and the accuracy grade for the best load transmission. Optimization process is performed by genetic algorithm and in addition, many important parameters were computed by other numerical methods as will be detailed discussed below.

2. Load distribution model of helical and spur gears

Load transmission by gear pairs, as stated in the introduction, is followed by non-uniform load distribution in the meshing process. As a result of load transmission, on the teeth contact surface and root stresses are occurred. This stresses are main parameters in gear calculations, design procedures and period of exploitation.

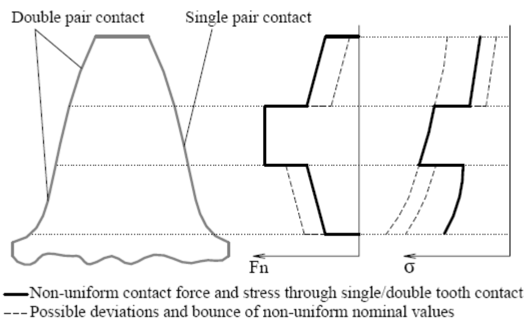

Figure 1: Gear contact model.

Due to gear parameters deviations of nominal values and depending on the number of teeth pair in contact, stress which occurred takes non-uniform load distributions and different values along the line of contact, as shown in Figure 1.

Maximal contact Eq. 1 and tooth-root stress Eq. 2 in load distribution, which are taken into consideration for further calculations, according to [2] and [3] are calculated as:

$$\sigma_H = Z\sigma_{H0}\sqrt{K_A K_V K_{H\beta} K_{H\alpha}} \leq \sigma_{HP} \qquad (1)$$

$$\sigma_F = \sigma_{F0} K_A K_V K_{F\beta} K_{F\alpha} \leq \sigma_{FP} \qquad (2)$$

In the above equations for contact stress calculation: $\Sigma_{H0/F0}$ is the nominal contact/tooth-root stress, which is the stress induced in error-free gearing by application of static nominal torque; Z is contact factor which converts contact stress at the pitch point to the contact stress at the inner point of tooth pair contact (different for pinion and wheel); K_A is the application factor, which take into account the load increment due to externally influenced variations of input or output torque; K_V is the dynamic factor, which take into account the load increment due to internal dynamic effects; $K_{H\beta}$ is the face load factor for contact stress; $K_{H\alpha}$ is the transverse load factor for contact stress; $K_{F\beta}$ is the face load factor for tooth-root stress; $K_{F\alpha}$ is the transverse load factor for tooth-root stress; $\sigma_{HP/FP}$ is the permissible contact/bending stress.

Transverse load factor of helical and spur gears, based on ISO standard [1], depends on many factors, and it is assumed that is variable along the line of contact. Models given by standardization are not in good agreement with experimental results because the changing meshing stiffness of the pair of teeth along the line of action produces a non-uniform load distribution, causing some load distribution factors to be required to compute bending and contact stresses [16].

These factors have characterized rate of non-uniform distribution of load and stress during the tooth meshing in case of above parameters deviations from nominal values. According to [1], these factors calculated by following equations:

$$K_{H\alpha} = K_{F\alpha} = \frac{\varepsilon_\gamma}{2}\left(0,9 + 0,4\frac{c_{\gamma\alpha}(f_{pb} - y_a)}{F_{tH}/b}\right), \qquad (3)$$

for gears with total contact ratio $\varepsilon_\gamma \leq 2$,

$$K_{H\alpha} = 0,9 + 0,4\sqrt{\frac{2(\varepsilon_\gamma - 1)}{\varepsilon_\gamma}}\frac{c_{\gamma\alpha}(f_{pb} - y_a)}{F_{tH}/b}, \qquad (4)$$

for gears with total contact ratio $\varepsilon_\gamma > 2$.

For gears with helix angle $\beta = 0$, the model is described with the equations 5 - 36.

$$z_2 = z_1 u \qquad (5)$$

$$S_{rn} = S_r m_n \tag{6}$$

$$\alpha_n = \frac{20\pi}{180} \tag{7}$$

$$\alpha_{p_n} = \frac{20\pi}{180} \tag{8}$$

$$\alpha = \alpha_n \tag{9}$$

$$\alpha_p = \alpha_{p_n} \tag{10}$$

$$\tan(\alpha_w)(x_{11}, x_{22}, z_1, z_2, \alpha) \\ = 2(x_{11} + x_{22})\frac{\tan(\alpha)}{(z_1 + z_2)} + \tan(\alpha) - \alpha \tag{11}$$

$$y_{factor} = \cos(\frac{(z_1 + z_2)}{2}) \\ \frac{\cos(\alpha)}{(\cos(\alpha_w) - 1)^{(-1)}} \tag{12}$$

$$Cth = (0.04723 + \frac{0.1551}{z_1} + \frac{0.25791}{z_2} - \\ -0.00635x_{11} - 0.11654\frac{x_{11}}{z_1} - 0.00193x_{22} - \\ 0.24188\frac{x_{11}}{z_2} + 0.00529x_{11}^2 + 0.00182x_{22}^2)^{-1} \tag{13}$$

$$C_r = 1 + (\frac{(log(odn))}{(5^{(\frac{S_{r1}}{5m_n})})}) \tag{14}$$

$$m = m_n \tag{15}$$

$$d_1 = mz_1 \tag{16}$$

$$f_{pb} = funct_{tableK_v(x_1, m_n, d_1)} \tag{17}$$

$$y_a = 0.075 f_{pb} \tag{18}$$

$$h_{fp} = 1.25m \tag{19}$$

$$C_b = (1 + 0.5(1.5 - \frac{h_{fp}}{m_n})) \\ (1 - 0.02(0.348888 - \alpha_{p_n})) \tag{20}$$

$$C_m = 0.8 \tag{21}$$

$$C = C_{th} C_m C_r C_b \\ if x_7 \geq 100 \tag{22}$$

$$C = C_{th} C_m C_r C_b x_7^{0.25} \\ if x_7 < 100 \tag{23}$$

$$h_{a1} = (1 + y_{factor} - x_{22})m \tag{24}$$

$$h_{a2} = (1 + y_{factor} - x_{11})m \tag{25}$$

$$a = (\frac{(z_1 + z_2)}{2} + y_{factor})m \tag{26}$$

$$d_2 = mz_2 \tag{27}$$

$$d_{b1} = d_1 \cos(\alpha) \tag{28}$$

$$d_{b2} = d_2 \cos(\alpha) \tag{29}$$

$$c_1 = 0.2 \tag{30}$$

$$c_2 = 0.2 \tag{31}$$

$$h = (2.25 + y_{factor} - (x_{11} + x_{22}))m_n \tag{32}$$

$$d_{a1} = d_1 + 2h_{a1} \tag{33}$$

$$d_{a2} = d_2 + 2h_{a2} \tag{34}$$

$$d_{f1} = d_{a1} - 2h \tag{35}$$

$$d_{f2} = d_{a2} - 2h \tag{36}$$

For gears with helix angle $\beta > 0$, the model is described with the equations 37 - 73.

$$z_2 = z_1 u \tag{37}$$

$$S_{rn} = S_r m_n \tag{38}$$

$$\alpha_n = \frac{20\pi}{180} \tag{39}$$

$$\alpha_{p_n} = \frac{20\pi}{180} \tag{40}$$

$$\alpha = a\tan(\frac{\tan(\alpha_n)}{\cos(beta)}) \tag{41}$$

$$\alpha_p = a\tan(\frac{\tan(\alpha_{p_n})}{\cos(beta)}) \quad (42)$$

$$\tan(\alpha_w)(x_{11},x_{22},z_1,z_2,\alpha) = 2(x_{11}+x_{22})\frac{\tan(\alpha)}{(z_1+z_2)}+\tan(\alpha)-\alpha \quad (43)$$

$$y_{factor} = \cos(\frac{(z_1+z_2)}{2})\frac{\cos(\alpha)}{(\cos(\alpha_w)-1)^(-1)} \quad (44)$$

$$Cth = (0.04723+\frac{0.1551}{z_1}+\frac{0.25791}{z_2}- -0.00635x_{11}-0.11654\frac{x_{11}}{z_1}-0.00193x_{22}- 0.24188\frac{x_{11}}{z_2}+0.00529x_{11}^2+0.00182x_{22}^2)^{-1} \quad (45)$$

$$C_r = 1+(\frac{(\log(odn))}{(5^{(\frac{S_{r1}}{5m_n})})}); \quad (46)$$

$$m = \frac{m_n}{\cos\beta} \quad (47)$$

$$d_1 = mz_1 \quad (48)$$

$$f_{pb} = funct_{tableK_v(x_1,m_n,d_1)} \quad (49)$$

$$y_a = 0.075 f_{pb} \quad (50)$$

$$h_{fp} = 1.25m \quad (51)$$

$$C_b = (1+0.5(1.5-\frac{h_{fp}}{m_n}))(1-0.02(0.348888-\alpha_{p_n})) \quad (52)$$

$$C_m = 0.8 \quad (53)$$

$$C = C_{th}C_mC_rC_b\cos\beta \quad if x_7 \geq 100 \quad (54)$$

$$C = C_{th}C_mC_rC_b\cos\beta x_7^{0.25} \quad if x_7 < 100 \quad (55)$$

$$h_{a1} = (1+y_{factor}-x_{22})m \quad (56)$$

$$h_{a2} = (1+y_{factor}-x_{11})m \quad (57)$$

$$a = (\frac{(z_1+z_2)}{2}+y_{factor})m \quad (58)$$

$$d_2 = mz_2 \quad (59)$$

$$d_{b1} = d_1cos(\alpha) \quad (60)$$

$$d_{b2} = d_2cos(\alpha) \quad (61)$$

$$c_1 = 0.2 \quad (62)$$

$$c_2 = 0.2 \quad (63)$$

$$h = (2.25+y_{factor}-(x_{11}+x_{22}))m_n \quad (64)$$

$$d_{a1} = d_1+2h_{a1} \quad (65)$$

$$d_{a2} = d_2+2h_{a2} \quad (66)$$

$$d_{f1} = d_{a1}-2h \quad (67)$$

$$d_{f2} = d_{a2}-2h \quad (68)$$

$$\varepsilon_\beta = 0.9 \quad (69)$$

$$if \varepsilon_\alpha \geq 1.2 \quad if \beta < 0.5233 \quad c\gamma_\alpha = C(0.75\varepsilon_\alpha+0.25) \quad (70)$$

$$if \varepsilon_\alpha < 1.2 \quad if \beta \geq 0.5233 \quad c\gamma_\alpha = 0.9C(0.75\varepsilon_\alpha+0.25) \quad (71)$$

$$if \varepsilon_\alpha \geq 1.2 \quad if \beta \geq 0.5233 \quad c\gamma_\alpha = 0.9C(0.75\varepsilon_\alpha+0.25) \quad (72)$$

$$if \varepsilon_\alpha < 1.2 \quad if \beta < 0.5233 \quad c\gamma_\alpha = 0.9C(0.75\varepsilon_\alpha+0.25) \quad (73)$$

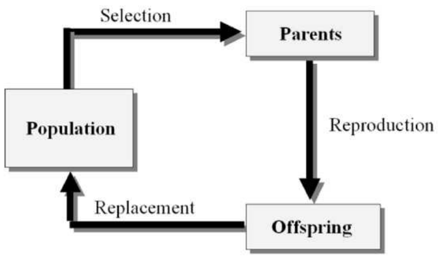

Figure 2: Flowchart of an evolutionary algorithm.

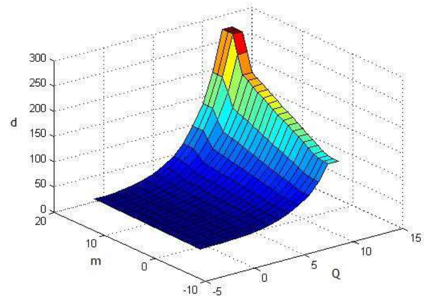

Figure 3: *Funcionality of accuracy grade, standard modulus and pitch diameter*

In opposite to above procedure for contact and tooth-root stress calculation, which include calculations of load factors in accordance with defined geometry of gears, in this paper inverse approach is taken. Optimal geometry is determine in a GA, by the requirement that the transverse load factor are equal or tends to one. This means that the load distribution tends to uniform.

3. Numerical methods

3.1. Genetic algorithm

Nature has a wonderful and powerful mechanism for optimization and problem solving through the process of evolution. The important components of EAs are genetic algorithms (GAs), genetic programming and evolutionary strategies [13]. The evolutionary algorithm can be applied to problems where heuristic solutions are not available or generally lead to unsatisfactory results. As a result, evolutionary algorithms have recently received increased interest, particularly with regard to the manner in which they may be applied for practical problem solving [14]. A simple flowchart of an evolutionary algorithm is given in Figure 2.

A GA represents an iterative process where each iteration is called a generation. A typical number of generations for a simple GA can range from 50 to over 500 [7]. The entire set of generations is called a run. At the end of a run, it is expected to find one or more highly fit chromosomes. The GA techniques have a solid theoretical foundation [8], [9], [10], [11]. That foundation is based on the Schema Theorem. John Holland introduced the notation of schema [8], which came from the Greek word meaning 'form'. A schema is a set of bit strings of ones, zeros and asterisks, where each asterisk can assume either value 1 or 0. The ones and zeros represent the fixed positions of a schema, while asterisks represent 'wild cards'. For example, the schema stands for a set of 4-bit strings. Each string in this set begins with 1 and ends with 0. These strings are called instances of the schema. [12].

3.2. Interpolation of three-dimensional data

In order to find a proper value of the transverse base pitch deviation f_{pb}, it was necessary to perform interpolation based on the accuracy grade, standard modulus and pitch diameter. Values of the appropriate base pitch deviation, for the ranges of mentioned three values are given in [4], and the three-dimensional functionality is given in the Figure 3. The accuracy grade and standard modulus are direct input values of the main function, and the pitch diameter is obtained by calculation. The interpolation is performed through separate program, and the values obtained for the transverse base pitch are dynamically transferred in the main program until the end of the genetic algorithm procedure.

3.3. Newton - Raphson numerical method for solving non-linear equation α_w

It is very difficult to find a root of a non-linear equation algebraically. Using some basic concepts of calculus, there are ways of numerically evaluating roots of complicated equations. In this purpose it is commonly used the Newton-Raphson method. The idea of the method is as follows: one starts with an initial guess which is reasonably close to the true root, then the function is approximated by its tangent line (which can be computed using the tools of calculus), and one computes the x-intercept of this tangent line (which is easily done with elementary algebra). This x-intercept will typically be a better approximation to the function's root than the original guess, and the method can be iterated. Suppose $f : [a,b] > R$ is a differentiable function defined on the interval $[a,b]$ with values in the real numbers R. The formula for converging on the root can be easily derived. Suppose we have some current approximation x_n. Then we can derive the formula for a better approximation, $x_n + 1$ by referring to the diagram on the right. We know

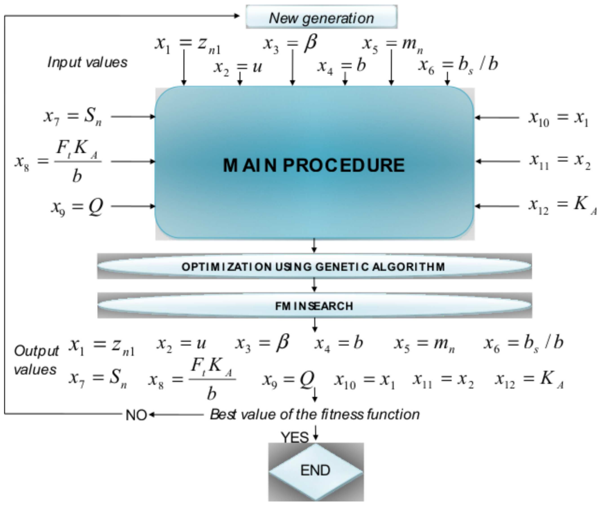

Figure 4: *Hybrid algorithm procedure*

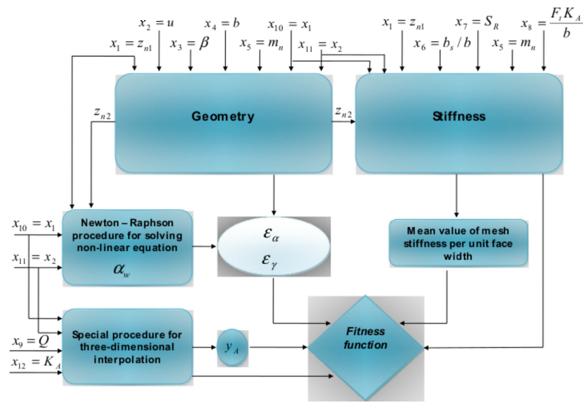

Figure 5: *Main procedure algorithm*

from the definition of the derivative at a given point that it is the slope of a tangent at that point...

In this paper, Newton - Raphson numerical method is used to solve non-linear equation of the working pressure angle, which is depending on the number of teeth on pinion gear z_{n1}, the number of teeth on wheel gear z_{n2}, profile shift coefficient of pinion x_1 and profile shift coefficient of wheel gear x_2. All of these values are input values of the main procedure, and as they dynamically changing their values thanks to genetic algorithm procedure in order to find optimum values, it is even more complicated to calculate the appropriate value of the working pressure angle. Solving of this non-linear equation is performed in the separate program, and the values obtained for the angle are dynamically transferred in the main program until the end of the genetic algorithm procedure.

4. Main procedure

Hybrid algorithm of this procedure, has 12 direct input variables affecting the output function, as shown in Figure 4, where the main procedure is divided into several sub procedures and each procedure will be explained in detail (Figure 5).

Settings of genetic algorithm during the process are shown in Table 1. According to data from Table 1, simulation was iterated three times, with different hybrid genetic algorithm setup in order to find best convergence of the process. Criteria for stopping hybrid genetic algorithm process is reaching number of stall generations, while the stall time was infinitive. A special characteristic that leads to slow convergence of the process is migration in both directions, which means that the accepted (good) individuals from the nth subpopulation migrates into both

Table 1: Selected parameters for performing numerical operation of genetic algorithm

Name of parameter	*First iteration*	*Second iteration*	*Third iteration*
Population type	Double vector	Double vector	Double vector
Encoding	Binary	Binary	Binary
Scaling func.	Proportional	Rank	Top w. q. 0.4
Selection	Roulette	Stochastic uniform	Uniform
Elite count for reproduction	4	15	30
Crossover func.	Scattered	Single point	Two point
Population size	40	100	300
Mutation	Uniform	Adaptive feasible	Gaussian
Probability of mutation	Rate 0.1	-	Scale 1.0, Shrink 1.0
Max number of generations	10000	10000	10000
Migration direction	Forward	Forward	Both
Migration fraction fraction	0.2	0.2	0.2
Hybrid func.	fminsearch	fminsearch	fminsearch
Func. tolerance	$10^{(-15)}$	$10^{(-15)}$	$10^{(-15)}$
Stopping criteria	Maximal number of generations or number of stall generations (1000)		

$(n-1)$th and the $(n+1)$th subpopulation in order to achieve the balance between generations.

In this paper we considered different parameters which impact transverse load factor of spur and helical gears. These parameters are related to geometry, specific load distribution $\frac{Ft}{b}$, stiffness C', application factor K_A and accuracy grade Q. When we use term geometry, we mean optimization against the number of teeth on pinion gear z_{n1}, gear ratio u (which is giving us the number of teeth on wheel gear z_{n2}, multiplied by number of teeth on pinion gear), standard modulus m_n, face width b and helix angle β. All of these factors, together with pressure angle,

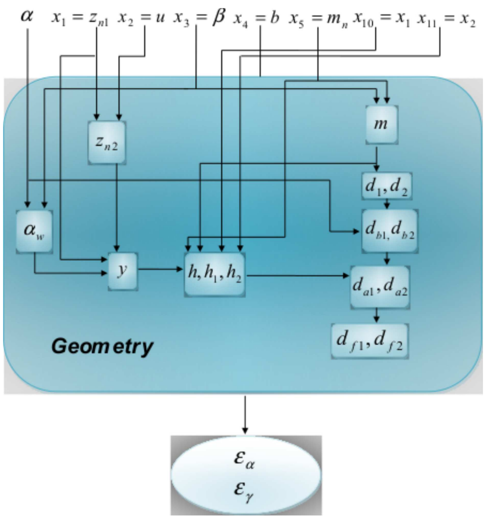

Figure 6: *Geometry algorithm*

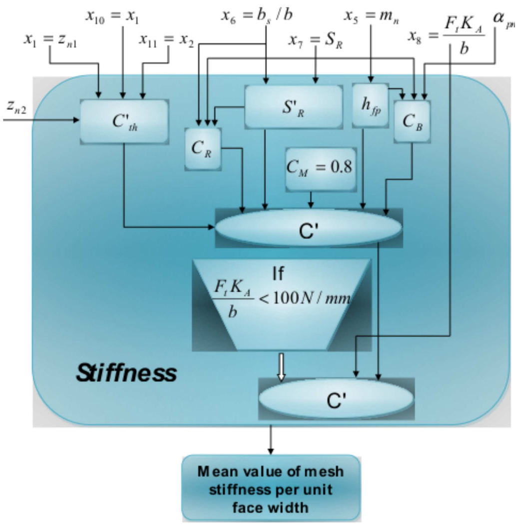

Figure 7: *Stiffness algorithm*

normal pressure angle, transverse pressure angle and pressure angle at the pitch cylinder, directly impact to calculation of pitch diameters, addendum diameters, base diameters and root diameters. To be more specific, in this paper, calculation of geometry, in first order implies, selection of the best solutions for the number of pinion gear, gear ratio, helix angle, standard modulus, face width, and profile shift coefficient using genetic algorithm, as six of even twelve inputs and than, in second order calculation of the pitch diameters $d_{1,2}$, base diameters $d_{b1,b2}$, root diameters $d_{f1,f2}$, and addendum diameters $d_{a1,a2}$, of both, pinion and wheel, respectively. The detailed algorithm process used for the calculation geometry is shown in the Figure 6. Other, when we use term stiffness, we mean optimization against the basic rack factor C_B, correction factor C_M, gear blank factor C_R, theoretical single stiffness C'_{th} and helix angle β. For cases where specific load is taking values less than $100\frac{N}{mm}$, specific load is also one of the factors which impact to optimization of the stiffness. In optimizing the basic rack factor C_B, it is taken into account standard modulus m_n, normal pressure angle of the basic rack α_{pn} and addendum of basic rack h_{fp}. In this optimization, the gear blank factor C_R is presented as function of gear rim thickness S_R, and ratio of central web thickness and gear width (b_s/b). c'_{th} is appropriate to solid disc gears and to the specified standard basic rack tooth profile. C'_{th} for a helical gear is the theoretical single stiffness relevant to the appropriate virtual spur gear [1]. According to [1], in this paper, $C'th$ is taken into consideration as function of the number of teeth on pinion gear z_{n1}, the number of teeth on wheel gear z_{n2}, profile shift coefficient of pinion x_1 and profile shift coefficient of wheel gear x_2. Therefore, it leads to the conclusion that specific load distribution is one of the very significant input values for the optimization. After finding the best value for the stiffness, stiffness can only be taken into calculation through formula of the mean value of mesh stiffness per unit face width $C_{\gamma\alpha}$, which is used for factors K_v, K_H and K_F and therefore, it is necessary to calculate the value of total contact ratio, ε_γ. The detailed algorithm process used for the calculation geometry is shown in the Figure 7. Apart to the geometry, the great influence to total contact ratio has working pressure angle, which value is calculated by special numerical program as it was discussed in section for numerical methods.

Finally, after calculation (optimization) of geometry, stiffness and specific load distribution the last and the most important calculation is the calculation of the values of the transverse load factors, $K_{H\alpha}$ for surface stress and $K_{F\alpha}$ for tooth root stress, account for the effect of the non-uniform distribution of transverse load between several pairs of simultaneously contacting gear teeth. According to [1], transverse load factors are presented as functions of the total contact ratio, mean value of mesh stiffness per unit face width, transverse base pitch deviation f_{pb} (the values may be used for calculations in accordance with ISO 6336, using tolerances complying with [4]), running-in allowance for a gear pair y_a and tangential load in a transverse plane, F_{tH}. Transverse base pitch deviation is adopted using interpolation between three values: the accuracy grade, standard modulus and pitch diameter by special numerical program as it was discussed in section for numerical methods. Tangential load in a transverse plane, F_{tH} is a function of tangential load, and application factor K_A, dynamic factor K_V, face load factor $K_{H\beta}$. The differences between the helical and spur gears are taken into account through a loop in the simulation, which takes into account the value of the helix angle selected in the optimization (optimal value).

5. Results

As shown in Table 1, three iterations of the same simulation were performed with different simulations of a genetic algorithm to determine the best possible convergence to a minimal solution. Therefore, genetic algorithm, reached the final results in different generations as it is shown in Table 2. Final results for each variable are shown in the Table 3.

Table 2: Genetic algorithm solver simulation properties

	First iteration	*Second iteration*	*Third iteration*
Stopped in	1001	1001	1001
Final time of process	127 sec	130 sec	104 sec
Convergence	Yes	Yes	Yes
Stopp. criteria	Stall generations	Stall generations	Stall generations
Stall gen.	900	903	904
Stall time	21 sec	23 sec	20 sec

Optimization terminated: average change in the fitness value less than options

Convergence obtained in the first, second and third iteration is given in the Figures 8-a, 8-b, 8-c, respectively.

Table 3: Final results

Variable	Name	*First iteration*	*Second iteration*	*Third iteration*
x_1	z_{n1}	44	36	27
x_2	u	3.5	3	4
x_3	b_s/b	$0,34$	1.048	1.109
x_4	S_R	1	2.9	3
x_5	β	21.5°	28°	30°
x_6	m_n	10	3	15
x_7	$\frac{F_t K_A}{b}$	1260 N/mm^2	1410 N/mm^2	1472 N/mm^2
x_8	b	69 mm	131 mm	54 mm
x_9	Q	1	3	1
x_{10}	K_A	1	1.6	1
x_{11}	x_1	0.905	0.946	0.951
x_{12}	x_2	0.806	0.811	0.795
$f(x)$	$K_{H\alpha}$	1	1	1

Optimization terminated: average change in the fitness value less than options

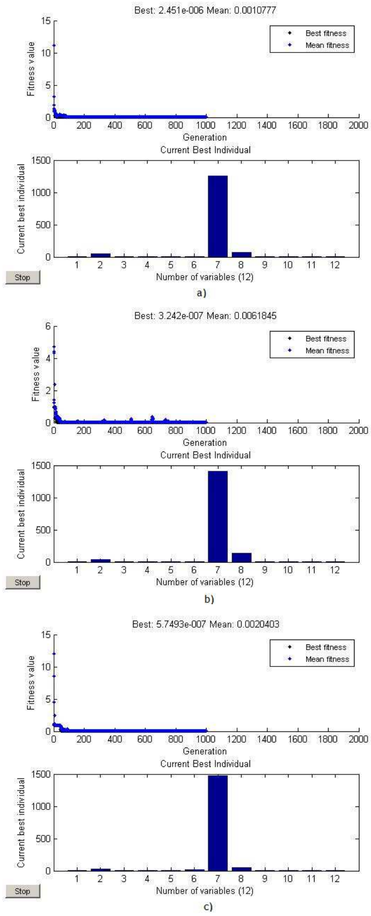

Figure 8: Convergence in generations

6. Conclusion

In this study, optimization of load transverse factor as a function of 12 variables was done using genetic algorithm. Although the load transverse factor could take any possible value, it is always converged to the value 0.5, which also affect to distribution of load and making it uniform, while the influential variables on the load transverse factor are taking corresponding values. Using this method, it appeared that the most affecting variable of changing the value of load transverse factor is helix angle β. Helix angle can take any value in the range of standard values from $0^{\circ} - 30^{\circ}$, but generally was taken the values between $20^{\circ} - 30^{\circ}$ to make the converges of load transverse factor to 0.5.

The profile shift coefficients x_1 and x_2 also affected to changing the value of load transverse factor and for achieving optimal value of the load transverse factor, it must be strongly respected conditions: $x_1 \geq x_2$; $-0.5 \leq x_1 + x_2 \leq 2.0$; according to [1]. Higher difference between values of profile shift coefficients of pinion and wheel leads to a value of 0.5 for load transverse factor.

The similar situation is with the specific load distribution: at low values of specific load distribution, load transverse factor converges to value 1, but at higher values of specific load distribution, load transverse factor is converging to value 0.5.

It is noted that for any number of teeth (from the range $18-54$) and any gear ratio (from the range $1-5$), this method achieves a value 0.5 of the load transverse factor, which therefore corresponds to uniform load distribution.

The idea of this method is to work closer simulation of actual timing coupled pairs, and thereby changing the various influential factors, it measures the change of the load transverse factor, and in the same time, determining the extent to which specific factors influence the change of load transverse factor. The method was performed according to ISO standards [1], [4], [5], [6].

Acknowledgement

The author's acknowledge the financial support to the Serbian Ministry of Education and Science, project No. III44006.
The author is grateful to the anonymous referee for a careful checking of the details and for helpful comments that improved this paper.

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