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Improved Exponential Ratio cum Exponential Dual to Ratio Estimator of Finite Population Mean in Presence of Non-Response

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Abstract: In this paper, we suggest an estimator of the population mean of the variable of interest y in the presence of non-response in two situations. We obtain expressions for the bias and mean square error. This study is supported by the theoretical and empirical results to show the performance of the proposed estimator over usual unbiased estimator and other existing estimators.

Keywords: Exponential estimator, Dual to ratio, Bias, Mean Square Error (MSE), Efficiency.

1 Introduction

In surveys concerning human populations, information in most cases is not obtained from all the units in the survey even after some call-backs. The failure to measure or to get information from some of the units in the selected sample is referred to as non-response. Non-respondents differ significantly from the respondents. An extensive description of the different types of non-response and their effects on surveys could be found in [2] and many other sampling literatures. [3] considered the problem of non response while estimating the population mean by taking a subsample from the non respondent group with the help of some extra efforts and an estimator was proposed by combining the information available from response and non-response groups. In estimating population parameters like the mean, total or ratio, sample survey experts sometimes use auxiliary information to improve precision of the estimates. When the population mean $\bar{X}$ of the auxiliary variable x is known and in presence of non-response, the problem of estimation of population mean $\bar{Y}$ of the study variable y has been discussed by [2], [7], [6] and [9]. In [3], questionnaires are mailed to all the respondents included in a sample and a list of non-respondents is prepared after the deadline is over. Then a sub sample is drawn from the set of non respondents and a direct interview is conducted with the selected respondents and the necessary information is collected.

Assume the population is divided into two groups, those who will not respond called non-response class. Let N_1 and N_2 be the number of units in the population that belong to the response class and the non response class respectively ($N_1 + N_2 = N$). Let n_1 be the number of units responding in a simple random sample of size n drawn from the population and let n_2 the number of units not responding in the sample. We may regard the sample of n_1 respondents as a simple random sample from the response class and the sample of n_2 as a simple random sample from the non-response class. Let k denote the size of the subsample from n_2 non-respondents to be interviewed and $f = \frac{n_2}{k}$; $f > 1$. Let $\bar{y_1}$ and $\bar{y}_{2k}$ denote

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the sample means of y character based on n_1 and k units respectively. Further we define:

$$W_1 = \frac{N_1}{N} \quad W_2 = \frac{N_2}{N} \quad \text{as their corresponding weights}$$
$$\bar{Y} = \frac{1}{N}\sum_{i=1}^{N} y_i, \quad \bar{Y}_2 = \frac{1}{N_2}\sum_{i=1}^{N_2} y_i, \quad \bar{X} = \frac{1}{N}\sum_{i=1}^{N} x_i, \quad \bar{X}_2 = \frac{1}{N_2}\sum_{i=1}^{N_2} x_i,$$
$$S_y^2 = \frac{1}{N-1}\sum_{i=1}^{N}(y_i-\bar{Y})^2, \quad S_{2y}^2 = \frac{1}{N_2-1}\sum_{i=1}^{N_2}(y_i-\bar{Y}_2)^2, \quad S_x^2 = \frac{1}{N-1}\sum_{i=1}^{N}(x_i-\bar{X})^2$$
$$S_{2x}^2 = \frac{1}{N_2-1}\sum_{i=1}^{N_2}(x_i-\bar{X}_2)^2, \quad S_{xy} = \frac{1}{N-1}\sum_{i=1}^{N}(y_i-\bar{Y})(x_i-\bar{X}),$$
$$S_{2xy} = \frac{1}{N_2-1}\sum_{i=1}^{N_2}(y_i-\bar{Y}_2)(x_i-\bar{X}_2)$$

[3] defined an unbiased estimator for estimating the population mean $\bar{Y}$ in the presence of non response as

$$\bar{y}^* = w_1\bar{y_1} + w_2\bar{y}_{2k}$$

where $w_1 = \frac{n_1}{n}$ and $w_2 = \frac{n_2}{n}$
The variance of $\bar{y}^*$ is given by

$$V(\bar{y}^*) = \lambda S_y^2 + \lambda' S_{2y}^2 \tag{1}$$

where $\lambda = \frac{1}{n} - \frac{1}{N}$, $\lambda' = \frac{W_2(f-1)}{n}$

Furthermore, in estimating the population parameters such as mean, total or ratio; it is well known that sample surveys experts sometimes use auxiliary information to improve the precision of the estimates. The auxiliary information can be used at the estimation stage to compensate for units from the sampling frame. In a household survey, for example, the household size can be used as an auxiliary variable for the estimation of, say, family expenditure. Information can be obtained completely on the family size during a household listing while there may be non response on the household expenditure.

[2] applied [3] technique to formulate a ratio estimator of the population mean $\bar{Y}$ when information is missing on both y and x. His proposed estimator with its bias and mean square error (MSE) is given as

$$\bar{y}_R^{**} = \frac{\bar{y}^*}{\bar{x}^*}\bar{X}$$
$$B(\bar{y}_R^{**}) = \frac{\lambda}{\bar{X}}\left(RS_x^2 - S_{yx}\right) + \frac{\lambda'}{\bar{X}}\left(RS_{2x}^2 - S_{2yx}\right)$$
$$MSE(\bar{y}_R^{**}) = \lambda S_d^2 + \lambda' S_{2d}^2 \tag{2}$$

where $\bar{x}^* = w_1\bar{x_1} + w_2\bar{x}_{2k}$, $\quad R = \frac{\bar{Y}}{\bar{X}}$, $\quad S_d^2 = S_y^2 - 2RS_{yx} + R^2S_x^2$,
$S_{2d}^2 = S_{2y}^2 - 2RS_{2yx} + R^2S_{2x}^2$

Similarly [7] suggested a ratio estimator based on the full response on the auxiliary variable x, whose population mean $\bar{X}$ is known. His proposed estimator with its bias and mean square error (MSE) is given as

$$\bar{y}_R^{*} = \frac{\bar{y}^*}{\bar{x}}\bar{X}$$
$$B(\bar{y}_R^{*}) = \frac{\lambda}{\bar{X}}\left(RS_x^2 - S_{yx}\right)$$
$$MSE(\bar{y}_R^{*}) = \lambda S_d^2 + \lambda' S_{2y}^2 \tag{3}$$

Using the transformation $\bar{x}_i^{\sigma} = (N\bar{X} - nx_i)/(N-n)$, $i = (1,2,3,...,N)$, [11] obtained dual to ratio estimator as

$$\bar{y}_{dR} = \bar{y}\left(\frac{\bar{x}^{\sigma}}{\bar{X}}\right)$$

where $\bar{x}^{\sigma} = (N\bar{X} - n\bar{x}) / (N - n)$

We study [11] in presence of non-response in two cases. When the non response occurs in the study variable y, the MSE is found to be

$$MSE\left(\bar{y}^{*}_{dR}\right) = \lambda S_y^2 + \lambda' S_{2y}^2 + g^2 \lambda R^2 S_x^2 - 2g\lambda R S_{yx} \tag{4}$$

When the non response occurs in both y and x, the MSE is found to be

$$MSE\left(\bar{y}^{**}_{dR}\right) = \lambda\left(S_y^2 - 2gRS_{yx} + g^2R^2S_x^2\right) + \lambda'\left(S_{2y}^2 - 2gRS_{2yx} + g^2R^2S_{2x}^2\right) \tag{5}$$

[1] introduced an exponential ratio-type and exponential product-type estimators for population mean as

$$\bar{y}_{eR} = \bar{y}exp\left[\frac{\bar{X} - \bar{x}}{\bar{X} + \bar{x}}\right]$$
$$\bar{y}_{eP} = \bar{y}exp\left[\frac{\bar{x} - \bar{X}}{\bar{x} + \bar{X}}\right]$$

[10] have studied [1] estimators in presence of non response, when non-response occurs on the study variable alone and on both the study and auxiliary variables as well. The MSE of [10] are respectively given as

$$MSE\left(\bar{y}^{*}_{eR}\right) = \lambda\left(S_y^2 + \frac{S_x^2R^2}{4} - RS_{yx}\right) + \lambda' S_{2y}^2 \tag{6}$$

$$MSE\left(\bar{y}^{**}_{eR}\right) = \lambda\left(S_y^2 + \frac{R^2S_x^2}{4} - RS_{xy}\right) + \lambda'\left(S_{2y}^2 + \frac{R^2S_{2x}^2}{4} - RS_{2yx}\right) \tag{7}$$

[8] suggested a ratio cum dual to ratio estimator in simple random sampling as

$$\hat{\bar{Y}}_{bk1} = \bar{y}\left[\alpha\frac{\bar{X}}{\bar{x}} + (1-\alpha)\frac{\bar{x}^{\sigma}}{\bar{X}}\right] \tag{8}$$

Motivated by [8] and [1], we have suggested an exponential ratio cum exponential dual to ratio estimator in SRSWOR in presence of non response. Numerical illustration will also be carried out to judge the merits of the suggested estimator.

2 The Suggested Estimator

In this section, utilizing information on the auxiliary variable x with known population mean $\bar{X}$, we suggest the following estimator for the population mean $\bar{Y}$ in two different situations, which are as follows:

2.1 Case I: Non-response only on y

We define the following estimator for $\bar{Y}$ in the presence of non response as

$$T_1 = \bar{y}^{*}\left[\alpha_1 exp\left(\frac{\bar{X} - \bar{x}}{\bar{X} + \bar{x}}\right) + (1-\alpha_1)\,exp\left(\frac{\bar{x}^{\sigma} - \bar{X}}{\bar{x}^{\sigma} + \bar{X}}\right)\right] \tag{9}$$

where $\bar{x}^{\sigma} = (N\bar{X} - nx) / (N - n)$ and α_1 is suitably chosen constant whose value will minimize the MSE of T_1.

To obtain the bias and MSE of T_1, we define:

$$\bar{y}^{*} = \bar{Y}(1 + e_0^{*}), \quad \bar{x} = \bar{X}(1 + e_1) \quad \text{such that} \quad E(e_0^{*}) = E(e_1) = 0$$

For simple random sampling without replacement, the following expectations can be obtained either directly or by the method discussed in [4] as

$$E(e_0^{*2}) = \frac{V(\bar{y}^{*})}{\bar{Y}^2} = \frac{1}{\bar{Y}^2}\left(\lambda S_y^2 + \lambda' S_{2y}^2\right), \quad E(e_1^2) = \frac{V(\bar{x})}{\bar{X}^2},$$
$$E(e_0^{*}e_1) = \frac{Cov(\bar{y}^{*}, \bar{x})}{\bar{Y}\bar{X}} = \frac{\lambda S_{yx}}{\bar{Y}\bar{X}}$$

Now expressing (9) in terms of e's, we have

$$T_1 = \bar{Y}(1+e_0^*)\left[\alpha_1 exp\left\{\frac{-e_1}{2}\left(1+\frac{e_1}{2}\right)^{-1}\right\}+(1-\alpha_1)\,exp\left\{\frac{-ge_1}{2}\left(1-\frac{ge_1}{2}\right)^{-1}\right\}\right] \tag{10}$$

where $g = \frac{n}{N-n}$

Assuming $|\frac{e_1}{2}| < 1$, $|\frac{ge_1}{2}| < 1$ so that $\left(1+\frac{e_1}{2}\right)^{-1}$ and $\left(1-\frac{ge_1}{2}\right)^{-1}$ are expandable in terms of e's. By expanding the right hand side of (10), multiplying out and neglecting terms involving power of e's greater than two, we have

$$T_1 - \bar{Y} = \bar{Y}\left[e_0^* - \frac{ge_1}{2} - \frac{g}{2}e_0^*e_1 - \frac{g^2}{8}e_1^2 + \alpha_1\left(\frac{g-1}{2}e_1 + \frac{g-1}{2}e_0^*e_1 + \frac{3+g^2}{8}e_1^2\right)\right] \tag{11}$$

Taking expectations on both sides of (11), we get the bias of T_1 to the first degree of approximation as

$$B(T_1) = -\bar{Y}\left[\frac{\lambda g\rho_{yx}C_yC_x}{2} + \frac{\lambda g^2C_x^2}{8} - \alpha_1\left(\frac{\lambda(g-1)\rho_{yx}C_yC_x}{2} + \frac{\lambda(3+g^2)C_x^2}{8}\right)\right] \tag{12}$$

Squaring both sides of (11) and neglecting terms of e's involving power greater than two, we have

$$(T_1-\bar{Y})^2 = \bar{Y}^2\left(e_0^{*2} + \frac{g^2}{4}e_1^2 - ge_0^*e_1 + \alpha_1^2\frac{(g-1)^2}{4}e_1^2 + \alpha_1(g-1)e_0^*e_1 - \alpha_1\frac{g(g-1)}{2}e_1^2\right) \tag{13}$$

Taking expectations on both sides of (13), we get the MSE of T_1 to the first order of approximation as

$$\begin{aligned} MSE(T_1) = \lambda S_y^2 + \lambda' S_{2y}^2 + \lambda\frac{g^2}{4}R^2S_x^2 - g\lambda S_{yx}R + \alpha_1^2\frac{(g-1)^2}{4}\lambda R^2S_x^2 \\ +\alpha_1(g-1)\lambda RS_{yx} - \alpha_1\frac{g(g-1)}{2}\lambda R^2S_x^2 \end{aligned} \tag{14}$$

The minimum of T_1 in (14) is obtained for

$$\alpha_1 = \frac{g}{g-1} - \frac{2K_{yx}}{g-1} = \alpha^* \quad \text{(say)} \tag{15}$$

where $K_{yx} = \rho_{yx}\frac{C_y}{C_x}$

Substituting the value of (15) in (9) yields asymptotically optimum estimator for $\bar{Y}$ as

$$T_{1(opt)} = \bar{y}^*\left[\alpha^* exp\left(\frac{\bar{X}-\bar{x}}{\bar{X}+\bar{x}}\right) + (1-\alpha^*)\,exp\left(\frac{\bar{x^{o}}-\bar{X}}{\bar{x^{o}}+\bar{X}}\right)\right]$$

Putting (15) in (14), we get the MSE of $T_{1(opt)}$ as

$$\begin{aligned} MSE(T_{1(opt)}) &= \lambda S_y^2 + \lambda' S_{2y}^2 - \lambda R^2K_{yx}^2S_x^2 \\ &= \lambda S_y^2(1-\rho_{yx}^2) + \lambda' S_{2y}^2 \end{aligned} \tag{16}$$

which is the same as the variance of the linear regression estimator $\bar{y}_{lr} = \bar{y}^* + b(\bar{X}-\bar{x})$, where b is the sample regression coefficient of y on x.

Remarks:

1. When $\alpha_1 = 1$, the proposed estimator reduces to exponential ratio estimator $\bar{y}_{eR}^*$. The bias and MSE of $\bar{y}_{eR}^*$ is obtained by putting $\alpha_1 = 1$ in (12) and (14) as follows

$$\begin{aligned} B(\bar{y}_{eR}^*) &= \lambda\bar{Y}\left(\frac{3}{8}C_x^2 - \frac{\rho_{yx}C_yC_x}{2}\right) \\ MSE(\bar{y}_{eR}^*) &= \lambda\left(S_y^2 + \frac{R^2}{4}S_x^2 - RS_{yx}\right) + \lambda' S_{2y}^2 \end{aligned} \tag{17}$$

2. When $\alpha=0$, the proposed estimator reduces to exponential dual to ratio estimator $\bar{y}^{*}_{edR}$. The bias and MSE of $\bar{y}^{*}_{edR}$ is obtained by putting $\alpha_1=0$ in (12) and (14) as follows

$$B\left(\bar{y}^{*}_{edR}\right)=-\lambda\bar{Y}\left(\frac{g^2C_x^2}{4}+\frac{g\rho_{yx}C_yC_x}{2}\right)$$
$$MSE\left(\bar{y}^{*}_{edR}\right)=\lambda S_y^2+\lambda' S_{2y}^2+\lambda R^2\frac{g^2}{4}S_x^2-g\lambda RS_{yx} \tag{18}$$

2.2 Case II: Non-response on both y and x

We define the estimator for the population mean $\bar{Y}$ assuming that there is non-response on y and x as

$$T_2=\bar{y}^{*}\left[\alpha_2 exp\left(\frac{\bar{X}-\bar{x}^{*}}{\bar{X}+\bar{x}^{*}}\right)+(1-\alpha_2)\,exp\left(\frac{\bar{x^{\sigma}}-\bar{X}}{\bar{x^{\sigma}}+\bar{X}}\right)\right] \tag{19}$$

To obtain the bias of T_2, we have $\bar{x}^{*}=\bar{X}\left(1+e_1^{*}\right)$ such that $E\left(e_1^{*}\right)=0$ and

$$E\left({e_1^{*}}^2\right)=\frac{V\left(\bar{x}^{*}\right)}{\bar{X}^2}=\frac{1}{\bar{X}^2}\left(\lambda S_x^2+\lambda' S_{2x}^2\right),\quad E\left(e_0^{*}e_1^{*}\right)=\frac{Cov\left(\bar{y}^{*},\bar{x}^{*}\right)}{\bar{Y}\bar{X}}=\lambda S_{yx}+\lambda' S_{2yx}$$

Following the procedure as in case I, we find the bias and MSE of T_2 as

$$\begin{aligned}B\left(T_2\right)=&-\bar{Y}E\left[-\frac{g}{2}\left(\lambda\rho_{yx}C_yC_x+\lambda'\rho_{2yx}C_{2y}C_{2x}\right)+\frac{g^2}{8}\left(\lambda C_x^2+\lambda' C_{2x}^2\right)\right.\\&\left.+\alpha_2\left\{\frac{g-1}{2}\left(\lambda\rho_{yx}C_YC_X+\lambda'\rho_{2yx}C_{2y}C_{2x}\right)+\frac{3+g^2}{8}\left(\lambda C_x^2+\lambda' C_{2x}^2\right)\right\}\right]\end{aligned} \tag{20}$$

$$\begin{aligned}MSE\left(T_2\right)=&\left[\left(\lambda S_y^2+\lambda' S_{2y}^2\right)+\frac{R^2g^2}{4}\left(\lambda S_x^2+\lambda S_{2x}^2\right)-gR\left(\lambda S_{yx}+\lambda' S_{2yx}\right)\right.\\&+\alpha_2^2\frac{R^2(g-1)}{4}\left(\lambda S_x^2+\lambda' S_{2x}^2\right)+\alpha_2(g-1)\left(\lambda S_{yx}+\lambda' S_{2yx}\right)\\&\left.-\alpha_2\frac{g(g-1)}{2}\left(\lambda S_x^2+\lambda' S_{2x}^2\right)\right]\end{aligned} \tag{21}$$

The minimum of T_2 in (21) is obtained for

$$\alpha_2=\frac{g}{g-1}-\frac{2}{(g-1)R}\frac{\left(\lambda S_{yx}+\lambda' S_{2yx}\right)}{\lambda S_x^2+\lambda' S_{2x}^2}=\alpha^{**}\qquad\text{(say)}$$

Substituting the value of α_2 in (21), we have the minimum MSE of T_2 as

$$MSE\left(T_{2(opt)}\right)=\lambda S_y^2+\lambda' S_{2y}^2-\frac{\left(\lambda S_{yx}+\lambda' S_{2yx}\right)^2}{\lambda S_x^2+\lambda' S_{2x}^2} \tag{22}$$

Remarks:

1. When $\alpha=1$, the proposed estimator reduces to exponential ratio estimator $\bar{y}^{**}_{eR}$. The bias and MSE of $\bar{y}^{**}_{eR}$ is obtained by putting $\alpha_2=1$ in (20) and (21) as follows

$$B\left(\bar{y}^{**}_{eR}\right)=\lambda\bar{Y}\left(\frac{3}{8}C_x^2-\frac{\rho_{yx}C_yC_x}{2}\right)+\lambda'\bar{Y}\left(\frac{3}{8}C_{2x}^2-\frac{\rho_{2yx}C_{2x}C_{2y}}{2}\right)$$
$$MSE\left(\bar{y}^{**}_{eR}\right)=\lambda\left(S_y^2+\frac{R^2}{4}S_x^2-RS_{yx}\right)+\lambda'\left(S_{2y}^2+\frac{R^2}{4}S_{2x}^2-RS_{2yx}\right) \tag{23}$$

2. When $\alpha = 0$, the proposed estimator reduces to exponential dual to ratio estimator $\bar{y}^{**}_{edR}$. The bias and MSE of $\bar{y}^{**}_{edR}$ is is obtained by putting $\alpha_2 = 0$ in (20) and (21) as follows

$$B\left(\bar{y}^{**}_{edR}\right) = -\bar{Y}\left[\frac{g}{2}\left(\lambda\rho_{yx}C_yC_x + \lambda'\rho_{2yx}C_{2y}C_{2x}\right) + \frac{g^2}{8}\left(\lambda C_x^2 + \lambda' C_{2x}^2\right)\right]$$

$$MSE\left(\bar{y}^{**}_{edR}\right) = \left(\lambda S_y^2 + \lambda' S_{2y}^2\right) + \frac{g^2R^2}{4}\left(\lambda S_x^2 + \lambda' S_{2x}^2\right) - gR\left(\lambda S_{yx} + \lambda' S_{2yx}\right) \quad (24)$$

3 Efficiency Comparisons

We will now investigate the efficiency of the suggested estimators $T_{1(opt)}$ over $\bar{y}^*$, $\bar{y}^*_{eR}$, $\bar{y}^*_{dR}$, $\bar{y}^*_{edR}$ & $\bar{y}^*_R$ and $T_{2(opt)}$ over $\bar{y}^*$, $\bar{y}^{**}_{eR}$, $\bar{y}^{**}_{dR}$, $\bar{y}^{**}_{edR}$ & $\bar{y}^{**}_R$ respectively.

3.1 Case I

3.1.1 Comparison with $\bar{y}^*$

From (1) and (16), we have

$$V\left(\bar{y}^*\right) - MSE\left(T_{1(opt)}\right) = R^2\lambda K_{yx}^2 S_x^2 > 0 \quad (25)$$

3.1.2 Comparison with $\bar{y}^*_R$

From (3) and (16), we have

$$MSE\left(\bar{y}^*_R\right) - MSE\left(T_{1(opt)}\right) = \lambda\left(RS_x + RK_{yx}S_x\right)^2 > 0 \quad (26)$$

3.1.3 Comparison with $\bar{y}^*_{dR}$

From (4) and (16), we have

$$MSE\left(\bar{y}^*_{dR}\right) - MSE\left(T_{1(opt)}\right) = \lambda\left(\frac{gRS_x}{4} + RK_{yx}S_x\right)^2 > 0 \quad (27)$$

3.1.4 Comparison with $\bar{y}^*_{eR}$

From (17) and (16), we have

$$MSE\left(\bar{y}^*_{eR}\right) - MSE\left(T_{1(opt)}\right) = \frac{\lambda R^2 S_x^2}{4}\left(\frac{1}{4} + K_{yx}\right)^2 > 0 \quad (28)$$

3.1.5 Comparison with $\bar{y}^*_{edR}$

From (18) and (16), we have

$$MSE\left(\bar{y}^*_{edR}\right) - MSE\left(T_{1(opt)}\right) = \frac{R^2\lambda g^2}{4}S_x^2 - gR\lambda S_{yx} + R^2\lambda K_{yx}^2 S_x^2 > 0 \quad (29)$$

if $S_{yx} < 0$

3.2 Case II

3.2.1 Comparison with $\overline{y}^*$

From (1) and (22), we have

$$V\left(\overline{y}^*\right) - MSE\left(T_{2(opt)}\right) = \frac{\left(\lambda S_{yx} + \lambda' S_{2yx}\right)^2}{\lambda S_x^2 + \lambda' S_{2x}^2} > 0 \tag{30}$$

3.2.2 Comparison with $\overline{y}_R^{**}$

From (2) and (22), we have

$$MSE\left(\overline{y}_R^{**}\right) - MSE\left(T_{2(opt)}\right) = \left[\lambda\left(RS_x^2 - S_{2yx}\right) + \lambda'\left(RS_{2x}^2 - S_{2yx}\right)\right]^2 > 0 \tag{31}$$

3.2.3 Comparison with $\overline{y}_{dR}^{**}$

From (5) and (22), we have

$$MSE\left(\overline{y}_{dR}^{**}\right) - MSE\left(T_{2(opt)}\right) = \left[\lambda\left(gRS_x^2 + S_{yx}\right) + \lambda'\left(gRS_{2x}^2 + S_{2yx}\right)\right]^2 > 0 \tag{32}$$

3.2.4 Comparison with $\overline{y}_{eR}^{**}$

From (23) and (22), we have

$$MSE\left(\overline{y}_{eR}^{**}\right) - MSE\left(T_{2(opt)}\right) = \left[\lambda\left(\frac{R}{2}S_x^2 - S_{yx}\right) + \lambda'\left(\frac{R}{2}S_{2x}^2 - S_{2yx}\right)\right]^2 > 0 \tag{33}$$

3.2.5 Comparison with $\overline{y}_{edR}^{**}$

From (24) and (22), we have

$$MSE\left(\overline{y}_{edR}^{**}\right) - MSE\left(T_{2(opt)}\right) = \left[\lambda\left(\frac{Rg}{2}S_x^2 - S_{yx}\right) + \lambda'\left(\frac{Rg}{2}S_{2x}^2 - S_{2yx}\right)\right]^2 > 0 \tag{34}$$

3.2.6 Comparison with $\overline{y}_{lr}^{**}$

The MSE of linear regression when non-response occur both on y and x is give as

$$MSE\left(\overline{y}_{lr}^{**}\right) = \left(\frac{1-f}{n}\right)S_y^2\left(1-\rho_{yx}^2\right) + \frac{W_2(k-1)}{n}S_{2y}^2 + \frac{W_2(k-1)}{n}\rho_{yx}\frac{S_y}{S_x}\left[\rho_{yx}\frac{S_y}{S_x}S_{2x}^2 - 2\rho_{2yx}S_{2y}S_{2x}\right] \tag{35}$$

From (35) and (22), we have

$$MSE\left(\overline{y}_{lr}^{**}\right) - MSE\left(T_{2(opt)}\right) = \frac{\left(\lambda S_{yx} + \lambda' S_{2yx}\right)^2}{\lambda S_x^2 + \lambda' S_{2x}^2} - \left(\frac{1-f}{n}\right)S_y^2\rho_{yx}^2 + \frac{W_2(k-1)}{n}\rho_{yx}\frac{S_y}{S_x}\left[\rho_{yx}\frac{S_y}{S_x}S_{2x}^2 - 2\rho_{2yx}S_{2y}S_{2x}\right] > 0 \tag{36}$$

if $\rho_{yx}\frac{S_y}{S_x}S_{2x} > 2\rho_{2yx}S_{2y}$ and $\left(\lambda S_{yx} + \lambda' S_{2yx}\right)^2 > \left(\frac{1-f}{n}\right)S_y^2\rho_{yx}^2\left(\lambda S_x^2 + \lambda' S_{2x}^2\right)$

4 Empirical Study

In this section, we have used the data of [5] to examine the performance of the different estimators. The data summary is presented below:
Population I- Source: [5]
x : Chest circumference (in cm) of the children
y : Weight (in kg) of the children
$N = 95,\ N_1 = 71,\ N_2 = 24,\ n = 35,\ \bar{Y} = 19.5,\ \bar{X} = 55.86,\ S_y^2 = 9.2416,\ S_{2y}^2 = 5.547,\ S_x^2 = 10.7158,\ S_{2x}^2 = 6.3001,$ $S_{yx} = 8.4587, S_{2yx} = 4.3095, \rho_{yx} = 0.85, \rho_{2yx} = 0.729$

Here, we have computed the percent relative efficiencies (PRE) of the different estimators $\bar{y}_{eR}^*$, $\bar{y}_{dR}^*$, $\bar{y}_{edR}^*$, $\bar{y}_R^*$, $T_{1(opt)}$ and $\bar{y}_{eR}^{**}$, $\bar{y}_{dR}^{**}$, $\bar{y}_{edR}^{**}$, $\bar{y}_R^{**}$, $T_{2(opt)}$ w.r.t. $\bar{y}^*$, for different values of f and W_2 and are presented in the following tables:

Table 1: PRE of the different estimators $\bar{y}_{eR}^*$, $\bar{y}_{dR}^*$, $\bar{y}_{edR}^*$, $\bar{y}_R^*$ and $T_{1(opt)}$ with respect to $\bar{y}^*$

W_2	f	$\bar{y}^*$	$\bar{y}_{eR}^*$	$\bar{y}_{dR}^*$	$\bar{y}_{edR}^*$	$\bar{y}_R^*$	$T_{1(opt)}$
0.1	1.5	100	143.455	151.492	125.411	199.173	336.915
	2.00	100	146.768	154.484	129.261	199.239	319.451
	2.50	100	149.715	157.128	132.732	199.295	305.939
	3.00	100	152.353	159.482	135.879	199.343	295.172
0.20	1.50	100	146.768	154.484	129.261	199.239	319.451
	2.00	100	152.353	159.482	135.879	199.343	295.172
	2.50	100	156.877	163.492	141.365	199.422	279.096
	3.00	100	160.616	166.779	145.98	199.173	267.666
0.30	1.50	100	149.715	112.476	112.476	199.295	305.939
	2.00	100	156.877	122.185	122.185	199.422	279.096
	2.50	100	162.253	129.955	129.955	199.511	263.106
	3.00	100	166.437	136.314	136.314	199.576	252.494

Table 2: PRE of the different estimators $\bar{y}_{eR}^{**}$, $\bar{y}_{dR}^{**}$, $\bar{y}_{edR}^{**}$, $\bar{y}_R^{**}$ and $T_{2(opt)}$ with respect to $\bar{y}^*$,

W_2	f	$\bar{y}^*$	$\bar{y}_{eR}^{**}$	$\bar{y}_{dR}^{**}$	$\bar{y}_{edR}^{**}$	$\bar{y}_R^{**}$	$\bar{y}_{lr}^{**}$	$T_{2(opt)}$
0.1	1.50	100	145.599	144.441	119.844	206.379	364.570	364.597
	2.00	100	150.955	150.102	118.7942	212.926	368.184	368.294
	2.50	100	155.839	155.288	117.913	218.840	371.320	371.551
	3.00	100	160.311	160.057	117.165	224.211	374.068	374.444
0.20	1.50	100	150.955	150.102	118.794	212.926	368.184	368.294
	2.00	100	160.311	160.057	117.165	224.211	374.668	374.444
	2.50	100	168.214	168.528	115.959	233.595	378.655	379.366
	3.00	100	174.976	175.824	115.032	241.520	364.570	383.406
0.30	1.50	100	155.839	112.476	100	218.840	371.320	371.551
	2.00	100	168.214	122.185	100	233.595	378.655	379.366
	2.50	100	178.005	129.955	100	245.038	383.907	385.169
	3.00	100	185.944	136.314	100	254.172	387.854	389.666

5 Conclusion

We conclude from table 1 that when there is non response only on y, the suggested estimator at its optimum $T_{1(opt)}$ is better than the usual unbiased estimator $\bar{y}^*$; and the estimators $\bar{y}_{eR}^*$, $\bar{y}_{dR}^*$, $\bar{y}_{edR}^*$, $\bar{y}_R^*$ and equally efficient as linear regression estimator and table 2 when there is non response on y and x both that the suggested estimator at its optimum $T_{2(opt)}$ is better than the usual unbiased estimator $\bar{y}^*$; and the estimators $\bar{y}_{eR}^{**}$, $\bar{y}_{dR}^{**}$, $\bar{y}_{edR}^{**}$, $\bar{y}_R^{**}$, $\bar{y}_{lr}^{**}$.

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