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R. Ameri, et al
School of Mathematics, Statistics and Computer Science, College of Sciences, University of Tehran, rameri@ut.ac.ir

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Soft Ideals of *BCC*-algebras

R. Ameri[1,∗], *R.A. Borzooei*[2] *and R. Moradian*[3]

[1] School of Mathematics, Statistics and Computer Science, College of Sciences, University of Tehran, Teheran, Iran,
[2] Department of Mathematics, Shahid Beheshti University, Tehran, Iran
[3] Department of Mathematics, Payam Noor University, Tehran, Iran

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Abstract: The purpose of this paper is the study of algebraic properties of soft sets in *BCC*-algebras. In this regards we introduce and study soft ideals and idealistic soft *BCC*-algebras.

Keywords: Soft set, (Idealistic) Soft *BCC*-algebra, Soft ideal

1 Introduction

The concept of rough set was originally proposed by Pawlak [9],[10] as a formal tool for modeling and processing in complete information in information systems. It seems that the rough set approach is fundamentally important in artificial intelligence and cognitive sciences, especially in research areas such as machine learning, intelligent systems, inductive reasoning, pattern recognition, knowledge discovery, decision analysis and expert systems. Various problems in identification system involve characteristics which are essentially non-probabilistic in nature [11]. In response to this situation Zadeh [?] introduced fuzzy set theory as an alternative to probability theory. Uncertainty is an attribute of information in order to suggest a more general framework. The approach to uncertainty is outlined by Zadeh [13] to solve complicated problem in economics, engineering and environment. We can not successfully use classical methods because of various uncertainties typical for those problems. There are three theories: theory of fuzzy sets, theory of probability and the interval mathematics which we can consider as mathematical tools for dealing with uncertainties. Uncertainties can't be handled using traditional mathematical tools but may be dealt with using a wide range of exiting theories such as probability theory, theory of (intuitionistic) fuzzy sets, theory of vague sets, theory of interval mathematics and theory of rough sets. However, all of these theories have their own difficulties which are pointed out in [13]. Maji et al [6] and Molodtsov [8] suggest that one reason for these difficulties may be due to the inadequacy of the parametrization tool of the theory. To over come these difficulties, Molodtsov [8] introduced the concept of soft set as a new mathematical tool for dealing with uncertainties that is free from the difficulties that have troubled the usual theoretical approaches. Molodtsov pointed out several directions for the application of soft sets. At present, works on the soft set theory are progressing rapidly. Maji et al [6] described the application of soft set theory to a decision making problem. Maji et al [7] also studied several operations on the theory of soft sets. Chen et al [?] presented a new definition of soft set parametrization reduction and compared this definition to the related concept of attributers reduction in rough set theory. Aktas and Cogman [1] studied the basic concepts of soft set theory and compared soft sets to fuzzy and rough sets, providing examples to clarify their differences. They also discussed the notion of soft groups. In this paper, we deal with the algebraic structure of *BCC*-algebras by applying soft set theory. We discussed the algebraic properties of soft sets in *BCC*-algebras and introduced the notion of soft ideals and idealistic soft *BCC*-algebras. For there more we investigated relation between soft *BCC*-algebra and idealistic soft *BCC*-algebras. In follows we established the intersection, union, "AND"operation and "OR"operation of soft ideals and idealistic soft *BCC*-algebras.

* Corresponding author e-mail: rameri@ut.ac.ir

2 Preliminaries

In this section we gather some basic definitions and results on *BCC*-algebras and soft sets which we need to extending our paper. Recall that a *BCC-algebra* is an algebra $(X,*,0)$ of type $(2,0)$ satisfying the following axioms:

(C1)$((x*y)*(z*y))*(x*z)=0$,
(C2)$0*x=0$,
(C3)$x*0=x$,
(C4)$x*y=0$ and $y*x=0$ imply $x=y$,

for every $x,y,z\in X$. For any *BCC*-algebra X, the relation $\le$ defined by $x\le y$ if and only if $x*y=0$ is a partial order on X. In a *BCC*-algebra X, the following hold: (see [13]).

(p1)$x\le x$,
(p2)$x*y\le x$,
(p3)$x\le y$ implies $x*z\le y*z$ and $z*y\le z*x$.

for all $x,y\in X$. A nonempty subset S of a *BCC*-algebra X is said to be a *subalgebra* of X if $x*y\in S$, when ever $x,y\in S$. A nonempty subset A of a *BCC*-algebra X is called an *ideal*, denoted by $A\trianglelefteq X$, if it satisfies:

(I1)$0\in A$,
(I2)$(x*y)*z\in A$ and $y\in A$ imply $x*z\in A$ for all $x,y,z\in X$.

Note that an ideal of a *BCC*-algebra X is a subalgebra of X. Molodtsov [8] defined the *soft set* in the following way: let U be an initial universe set and E be a set of parameters. Let $P(U)$ denotes the power set of U and $A\subset E$.

Definition 2.1 ([8]) A pair (p,A) is called a *soft set* over U, where p is a mapping given by $p:A\longrightarrow P(U)$. In other words, a soft set over U is a parameterized family of subsets of the universe U. For $a\in A$, $p(a)$ may be considered as the set of a-approximate elements of the soft set (p,A). Clearly, a soft set is not a set.

Definition 2.2 ([2])

(i)Let (p,A) and (q,B) be two soft sets over a common universe U. The intersection of (p,A) and (q,B) is defined to be the soft set (r,C) satisfying the following conditions:
(1) $C=A\cap B$,
(2) $(\forall e\in C)(r(e)=p(e)$ or $q(e)$, (as both are same set).
In this case, we write $(p,A)\tilde{\cap}(q,B)=(r,C)$.

(ii)Let $\{(p_i,A_i)|i\in I\}$ be a family of soft sets over a common universe U. The intersection $\cap_{i\in I}(p_i,A_i)$ is defined to be the soft set (r,C) satisfying the following conditions:
(1)$C=\cap A_i$,
(2)$(\forall e\in C)(r(e)=p_i(e)$ or $p_j(e),(i,j\in I)$, (as both are same set)).

In this case, we write $\tilde{\cap}(p_i,A_i)=(r,C)$.

Definition 2.3 ([6])

(i)Let (p,A) and (q,B) be two soft sets over a common universe U. The union of (p,A) and (q,B) is defined to be the soft set (r,C) satisfying the following conditions:
(1)$C=A\cup B$,
(2)for all $e\in C$,

$$r(e)=\begin{cases}p(e) & \text{if } e\in A\backslash B,\\ q(e) & \text{if } e\in B\backslash A,\\ p(e)\cup q(e) & \text{if } e\in A\cap B.\end{cases}$$

In this case, we write $(p,A)\tilde{\cup}(q,B)=(r,C)$.

(ii)Let $\{(p_i,A_i)|i\in I\}$ be a family of soft sets over a common universe U. The union $\cup_{i\in I}(p_i,A_i)$ is defined to be the soft set (r,C) satisfying the following conditions:
(1)$C=\cup A_i$,
(2)for all $e\in C$,

$$r(e)=\begin{cases}p_i(e) & \text{if } e\in A_i\backslash\cup_{i\neq j}A_j,\\ \cup p_i(e) & \text{if } e\in\cap A_i.\end{cases}$$

In this case, we write $\tilde{\cup}(p_i,A_i)=(r,C)$.

Definition 2.4 ([6]) If (p,A) and (q,B) are two soft sets over a common universe U, then "(p,A) AND (q,B)"denoted by $(p,A)\tilde{\wedge}(q,B)$ is defined by $(p,A)\tilde{\wedge}(q,B)=(r,A\times B)$, where $r(,\beta)=p()\cap q(\beta)$ for all $(,\beta)\in A\times B$.

Definition 2.5 ([6]) If (p,A) and (q,B) are two soft sets over a common universe U, then "(p,A) OR $(q,B)''$ denoted by $(p,A)\tilde{\vee}(q,B)$ is defined by $(p,A)\tilde{\vee}(q,B)=(r,A\times B)$, where $r(,\beta)=p()\cup q(\beta)$ for all $(,\beta)\in A\times B$.

Definition 2.6 ([6]) For two soft sets (p,A) and (q,B) over a common universe U, we say that (p,A) is a soft subset of (q,B), denoted by $(p,A)\tilde{\subset}(q,B)$, if it satisfies:

(i)$A\subset B$,
(ii)For every $a\in A$, $p(a)$ and $q(a)$ are identical approximations.

3 Soft Ideals

In this section we define soft *BCC*-algebra, soft *BCC*-ideal and investigate the intersection and union of soft *BCC*-ideals. In what follows let X be a *BCC*-algebra.

Definition 3.1. Let S be a subalgebra of X. A subset I of X is called an ideal of X related to S (briefly, S-ideal of X), denoted by $I\triangleleft S$, if it satisfies:

(i)$0\in I$,
(ii)$(\forall y\in I)((x*y)*z\in I$ imply $x*z\in I)$ for all $x,z\in S$.

Note that if S is a subalgebra of X and I is a subset of X that contains S, then I is a S-ideal of X. Obviously, every ideal of X is a S-ideal of X for every subalgebra S of X, but the converse is not true in general as seen in the following example.

Example 3.2. Let $X=\{0,1,2,3,4\}$ be a *BCC*-algebra with the following Cayley table:

*	0	1	2	3	4
0	0	0	0	0	0
1	1	0	1	0	0
2	2	2	0	0	0
3	3	3	1	0	0
4	4	3	4	3	0

Then $S=\{0,1\}$ is a subalgebra of X and $I=\{0,1,2\}\lhd S$, but I is not an ideal of X, because $(3*2)*0=1\in I$ and $2\in I$ but $3*0=3\notin I$.

Definition 3.3. Let (p,A) be a soft set over X. Then (p,A) is called a *soft BCC-algebra* over X if $p(x)$ is a subalgebra of X for all $x\in A$.

Definition 3.4. Let (p,A) be a soft BCC-algebra over X. A soft set (q,I) over X is called a *soft ideal* of (p,A), denoted by $(q,I)\tilde{\lhd}(p,A)$, if it satisfies:

(i)$I\subset A$,
(ii)$\forall x\in I$, $q(x)\lhd p(x)$.

We illustrate this definition using the following examples.

Example 3.5. Let $X=\{0,a,b,c\}$ be a BCC-algebra with the following Cayley table:

*	0	a	b	c
0	0	0	0	0
a	a	0	0	a
b	b	b	0	b
c	c	c	c	0

Let (p,A) be a soft set over X, where $A=X$ and $p: AP(X)$ is a set-valued function defined by $p(x)=\{y\in X|y*x=0\}$ for all $x\in A$. Then $p(0)=\{0\}$, $p(a)=\{0,a\}$, $p(b)=\{0,a,b\}$ and $p(c)=\{0,c\}$ which are subalgebras of X. Hence (p,A) is a soft BCC-algebra over X.

Let (q,I) be a soft set over X, where $I=\{a,b\}$ and $q: IP(X)$ is a set valued function defined by $q(x)=\{y\in X|y*(y*x)\in\{0,a\}\}$ for all $x\in I$. Then $q(a)=\{0,a,b,c\}\lhd\{0,a\}=p(a)$ and $q(b)=\{0,a,c\}\lhd\{0,a,b\}=p(b)$. Hence, (q,I) is a soft ideal of (P,A).

Theorem 3.6. Let (p,A) be a soft BCC-algebra over X. For any soft sets (q_1,I_1) and (q_2,I_2) over X where $I_1\cap I_2\neq\phi$, we have:

$$(q_1,I_1)\tilde{\lhd}(p,A),(q_2,I_2)\tilde{\lhd}(p,A)(q_1,I_1)\tilde{\cap}(q_2,I_2)\tilde{\lhd}(p,A)$$

Proof. Using Definition 2.2, we can write:

$$(q_1,I_1)\tilde{\cap}(q_2,I_2)=(q,I),$$

where $I=I_1\cap I_2$ and $q(x)=q_1(x)$ or $q_2(x)$ for all $x\in I$. Obviously, $I\subset A$ and $q: Ip(X)$ is a mapping. Hence, (q,I) is a soft set over X. Since $(q_1,I_1)\tilde{\lhd}(p,A)$ and $(q_2,I_2)\tilde{\lhd}(p,A)$, we know that $q(x)=q_1(x)\lhd p(x)$ or $q(x)=q_2(x)\lhd p(x)$ for all $x\in I$. Hence

$$(q_1,I_1)\tilde{\cap}(q_2,I_2)=(q,I)\tilde{\lhd}(p,A).$$

This complete the Proof. □

Corollary 3.7. Let (p,A) be a soft BCC-algebra over X. For any soft sets (q,I) and (r,I) over X, we have:

$$(q,I)\tilde{\lhd}(p,A),(r,I)\tilde{\lhd}(p,A)(q,I)\tilde{\cap}(r,I)\tilde{\lhd}(p,A).$$

Proof. The proof is straightforward.

Theorem 3.8. Let (p,A) be a soft BCC-algebra over X. For any soft sets (q,I) and (r,J) over X in which I and J are disjoint, we have

$$(q,I)\tilde{\lhd}(p,A),(r,J)\tilde{\lhd}(p,A)(q,I)\tilde{\cup}(r,J)\tilde{\lhd}(p,A).$$

Proof. Assume that $(q,I)\tilde{\lhd}(p,A)$ and $(r,J)\tilde{\lhd}(p,A)$. By means of Definition 2.3, we can write $(q,I)\tilde{\cup}(r,J)=(s,K)$, where $K=I\cup J$ and for every $x\in K$,

$$s(x)=\begin{cases} q(x) & \text{if } x\in I\backslash J,\\ r(x) & \text{if } x\in J\backslash I,\\ q(x)\cup r(x) & \text{if } x\in I\cap J.\end{cases}$$

Since $I\cap J=\phi$, either $x\in I\backslash J$ or $x\in J\backslash I$ for all $x\in K$. If $x\in I\backslash J$, then $s(x)=q(x)\lhd p(x)$, since $(q,I)\tilde{\lhd}(p,A)$. If $x\in J\backslash J$, then $s(x)=r(x)\lhd p(x)$, since $(r,J)\tilde{\lhd}(p,A)$. Thus, $s(x)\lhd p(x)$ for all $x\in K$ and so $(q,I)\tilde{\cup}(r,J)=(s,k)\tilde{\lhd}(p,A)$. Note that if I and J are not disjoint in Theorem 3.8, then Theorem 3.8 is not true in general as seen in the following example.

Example 3.9. Let $X=\{0,a,b,c,d\}$ be a BCC-algebra with the following Cayley table:

*	0	a	b	c	d
0	0	0	0	0	0
a	a	0	0	0	0
b	b	b	0	b	0
c	c	c	c	0	0
d	d	d	c	b	0

Let (p,A) be a soft set over X, where $A=X$ and $p: AP(X)$ is a set-valued function defined by $p(x)=\{y\in X|y*(y*x)\in\{0,b\}\}$ for all $x\in A$. Then $p(0)=X$, $p(a)=p(b)=\{0,b,c,d\}$ and $p(c)=p(d)=\{0,b\}$ which are subalgebras of X. Hence (p,A) is a soft BCC-algebra over X.

Let (q,I) be a soft set over X, where $I=\{b,c\}$ and $q(x)=\{y\in X|y*x=0\}\}$ for all $x\in I$. Then $q(b)=\{0,a,b\}\lhd\{0,b,c,d\}=p(b)$ and $q(c)=\{0,a,c\}\lhd\{0,b\}=p(c)$, and so (q,I) is soft ideal of (p,A). Let (r,J) be a soft set over X, where $J=\{b\}$ and $r(x)=\{y\in X|x*y=x\}-\{a\}$ for all $x\in J$. Then $r(b)=\{0,c\}\lhd\{0,b,c,d\}=p(b)$, and so (r,J) is a soft ideal of (p,A). But $(s,U)=(q,I)\cup(r,J)$ is not a soft ideal of (p,A) since $s(b)=q(b)\cup r(b)=\{0,a,b,c\}$ is not an $p(b)$-ideal because $(d*b)*0=c\in s(b),b\in s(b)$ but $d*0=d\notin s(b)$.

4 Idealistic soft BCC-algebra

Definition 4.1. Let (p,A) be a soft set over X. Then (p,A) is called an *idealistic soft BCC-algebra* over X if $p(x)$ is an ideal of X for all $x\in A$.

Example 4.2. Let $X = \{0,a,b,c\}$ be a *BCC*-algebra with the following Cayley table:

*	0	a	b	c
0	0	0	0	0
a	a	0	0	a
b	b	b	0	b
c	c	c	c	0

Let $A = X$ and let $p : AP(X)$ be a set-valued function defined by $p(x) = \{y \in X | y * (y * x) \in \{0,a\}\}$ for all $x \in A$. Then $p(0) = p(a) = X, p(b) = \{0,a,c\}$ and $p(c) = \{0,a,b\}$ which are ideals of X. Hence (p,A) is an idealistic soft *BCC*-algebra over X.

Example 4.3. Let X be a *BCC*-algebra defined in Example 4.2, let $A = X$ and let $p : AP(X)$ be a set-valued function defined by $p(x) = \{y \in X | y*(y*x) \in \{0,x\}\}$ for all $x \in A$. Then (p,A) is not an idealistic soft *BCC*-algebra over X. Since $p(b) = \{0,b,c\}$ is not an ideal of X because of $(a*b)*0 = 0 \in p(b)$ and $b \in p(b)$ but $a*0 = a \notin p(b)$.

Theorem 4.4. Let (p,A) and (p,B) be soft sets over X where BAX. If (p,A) is an idealistic soft *BCC*-algebra over X, Then so is (p,B).

Proof. It is obvious.

Theorem 4.5. Let (p,A) and (q,B) be two idealistic soft *BCC*-algebra over X. If $A \cap B \neq \phi$, then the intersection $(p,A)\tilde{\cap}(q,B)$ is an idealistic soft *BCC*-algebra over X.

Proof. Using Definition 2.2, we can write $(p,A)\tilde{\cap}(q,B) = (r,C)$, where $C = A \cap B$ and $r(x) = p(x)$ or $q(x)$ for all $x \in C$. Note that $r : Cp(X)$ is a mapping, and therefore (r,C) is a soft set over X. Since (p,A) and (q,B) and idealistic soft *BCC*-algebra over X, it follows that $r(x) = p(x)$ is an ideal of X, or $r(x) = q(x)$ is an ideal of X for all $x \in C$. Hence $(r,C) = (p,A)\tilde{\cap}(q,B)$ is an idealistic soft *BCC*-algebra over X. The next corollaries immediately follow from Theorem 4.5.

Corollary 4.6. Let $\{(p_i,A_i)|i \in I\}$ be a family of idealistic soft *BCC*-algebra over X If $A_i \cap A_j \neq \phi; i \neq j$, then the intersection $\cap_{i\in I}(p_i,A_i)$ is an idealistic soft *BCC*-algebra over X.

Corollary 4.7. Let (p,A) and (q,A) be two idealistic soft *BCC*-algebra over X. Then their intersection $(p,A)\tilde{\cap}(q,A)$ is an idealistic soft *BCC*-algebra over X.

Theorem 4.8. Let (p,A) and (q,B) be two idealistic soft *BCC*-algebra over X. If A and B are disjoint, then the union $(p,A)\tilde{\cup}(q,B)$ is an idealistic soft *BCC*-algebra over X.

Proof. Using Definition 2.3, we can write $(p,A)\tilde{\cup}(q,B) = (r,C)$, where $C = A \cup B$ and for every $e \in C$,

$$r(e) = \begin{cases} p(e) & \text{if } e \in A\backslash B, \\ q(e) & \text{if } e \in B\backslash A, \\ p(e) \cup q(e) & \text{if } e \in A \cap B. \end{cases}$$

Since $A \cap B = \phi$, either $x \in A\backslash B$ or $x \in B\backslash A$ for all $x \in C$. If $x \in A\backslash B$, then $r(x) = p(x)$ is an ideal of X since (p,A) is an idealistic soft *BCC*-algebra over X. If $x \in B\backslash A$, then $r(x) = q(x)$ is an ideal of X since (q,B) is an idealistic soft *BCC*-algebra over X. Hence $(r,C) = (p,A)\tilde{\cup}(q,B)$ is an idealistic soft *BCC*-algebra over X. □

Corollary 4.9. Let $\{(p_i,A_i)|i \in I\}$ be a family of idealistic soft *BCC*-algebra over X If $A_i \cap A_j = \phi; i \neq j$, then the union $\cup_{i\in I}(p_i,A_i)$ is an idealistic soft *BCC*-algebra over X.

Theorem 4.10. If (p,A) and (q,B) are idealistic soft *BCC*-algebras over X, then $(p,A)\tilde{\wedge}(q,B)$ is an idealistic soft *BCC*-algebra over X.

Proof. By use of Definition 2.4 we know that

$$(p,A)\tilde{\wedge}(q,B) = (r,A \times B),$$

where $r(x,y) = p(x) \cap q(y)$ for all $(x,y) \in A \times B$. Since $p(x)$ and $q(y)$ are ideals of X, the intersection $p(x) \cap q(y)$ is also an ideal of X. Hence $r(x,y)$ is an ideal of X for all $(x,y) \in A \times B$. Therefore, $(p,A)\tilde{\wedge}(q,B) = (r,A \times B)$ is an idealistic soft *BCC*-algebra over X. □

Definition 4.11. An idealistic soft *BCC*-algebra (p,A) is said to be trivial (resp, whole) if $(p(x) = \{0\}(resp, p(x) = X)$ for all $x \in A$.

Example 4.12. Let $X = \{0,a,b,c,d\}$ be a *BCC*-algebra with the following Cayley table:

*	0	a	b	c	d
0	0	0	0	0	0
a	a	0	0	0	0
b	b	b	0	b	0
c	c	c	c	0	0
d	d	d	d	d	0

Now, let $A = \{a,b\}$ and define $p : AP(X)$ be a set-valued function defined by $p(x) = \{y \in X | y*x \in \{0,b,c,d\}\}$, so we have $p(x) = X$ for all $x \in A$ and so (p,A) is a whole idealistic soft *BCC*-algebra.

Lemma 4.13.

(i)Let $f : XY$ be a mapping of *BCC*-algebras. For a soft set (p,A) over X, $(f(p),A)$ is a soft set over Y, where $f(p) : AP(Y)$ is defined by $f(p)(a) = f(p(a)) = \cup_{x\in p(a)} f(x)$ for all $a \in A$.

(ii)Let $f : XY$ be a mapping of *BCC*-algebras. For a soft set (q,B) over Y, $(f^{-1}(q),B)$ is a soft set over X, where $f^{-1}(q) : BP(X)$ is defined by $f^{-1}(q)(b) = \cup_{y\in q(b)} f^{-1}(y)$ for all $b \in B$.

Proof. It is easy and emitted.

Lemma 4.14.

(i)Let $f : XY$ be an onto homomorphism of *BCC*-algebras. If (p,A) is an idealistic soft *BCC*-algebra over X, then $(f(p),A)$ is an idealistic soft *BCC*-algebra over Y.

(ii)Let $f : XY$ be an onto homomorphism of *BCC*-algebras. If (q,B) is an idealistic soft *BCC*-algebra over Y, then $(f^{-1}(q),B)$ is an idealistic soft *BCC*-algebra over X.

Proof.

(i)For every $x \in A$, we have $f(p)(x) = f(p(x))$ is an ideal of Y, since $p(x)$ is an ideal of X and its onto homomorphic image is also an ideal of Y. Hence $(f(p),A)$ is an idealistic soft BCC-algebra over Y.

(ii)First we prove that if B be an ideal of Y, then $f^{-1}(B)$ is an ideal of X. Obviously we have $O \in f^{-1}(B)$. Now, let $x,y,z \in X$ be such that $(x*y)*z \in f^{-1}(B)$ and $y \in f^{-1}(B)$, so we have $f((x*y)*z) = (f(x)*f(y))*f(z) \in B$ and $f(y) \in B$. Since B is an ideal of Y, we have $f(x*z) = f(x)*f(z) \in B$ and so $(x*z) \in f^{-1}(B)$. Thus $f^{-1}(B)$ is an ideal of X. Now for every $b \in B$, since $q(b)$ is an ideal of Y, we have $f^{-1}(q)(b) = \cup_{y \in q(b)} f^{-1}(y)$ is an ideal of X. Thus, $(f^{-1}(q),B)$ is an idealistic soft BCC-algebra over X.

Theorem 4.15. Let $f : XY$ be an onto homomorphism of BCC-algebras and let (p,A) be an idealistic soft BCC-algebra over X.

(i)if $p(x) = \ker(f)$ for all $x \in A$, then $(f(p),A)$ is the trivial idealistic soft BCC-algebra over Y.

(ii)Suppose that (p,A) is whole, then $(f(p),A)$ is the whole idealistic soft BCC-algebra over Y.

Proof.

(i)Assume that $p(x) = \ker(f)$ for all $x \in A$, then $f(p)(x) = f(p(x)) = \{0_Y\}$ for all $x \in A$. Hence $(f(p),A)$ is the trivial idealistic soft BCC-algebra over Y by Lemma 5.12.

(ii)Suppose that (p,A) is whole. Then $p(x) = X$ for all $x \in A$, and so $f(p)(x) = f(p(x)) = f(X) = Y$ for all $x \in A$. It follows form Lemma 5.12 and Lemma 5.11 that $(f(p),A)$ is the whole idealistic soft BCC-algebra over Y.

5 Fuzzy ideal and fuzzy soft ideal

Definition 5.1. A fuzzy subset μ of a BCC-algebra X is said to be a fuzzy ideal of X if it satisfies:

(i)$\mu(0) \geq \mu(x)$ for all $x \in X$,

(ii)$\mu(x*z) \geq min\{\mu((x*y)*z), \mu(y)\}$ for all $x,y,z \in X$.

Definition 5.2. let X be a BCC-algebra and $F(X)$ be the se t of fuzzy set over X. A pair (p,A) is called a fuzzy soft set over BCC-algebra X, where p is a mapping given by:

$$p : A \to F(X)$$

In other word, for every $a \in A$, $p_a : X \to [0,1]$ is a fuzzy set over X. Note that for every fuzzy set μ, the set $\mu_t = \{x \in X | \mu(x) \geq t\}$ is called t-level relation over BCC-algebra X.

Definition 5.3. A fuzzy soft set (p,A) over BCC-algebra X is called fuzzy soft ideal, if for every $a \in A$, $p_a \in F(X)$ be a fuzzy ideal of X.

Theorem 5.4. Let $p : A \to F(X)$ be a fuzzy soft ideal over BCC-algebra X and $a \in A$. Then $p_a \in F(X)$ is a fuzzy ideal if and only if $(p_a)_t \neq \phi$ is an ideal of BCC-algebra X.

Proof. Let $p_a \in F(X)$ be a fuzzy ideal, we must prove that $(p_a)_t$ is an ideal of BCC-algebra X. Since $p_a(0) \geq p_a(x)$, obviously we have $0 \in (p_a)_t$. Now, let $x,y,z \in X$ be such that $(x*y)*z \in (p_a)_t$ and $y \in (p_a)_t$, then $p_a((x*y)*z) \geq t$ and $p_a(y) \geq t$. So we have:

$$p_a(x*z) \geq min\{p_a((x*y)*z), p_a(y)\} \geq t$$

Hence $(x*z) \in (p_a)_t$. Therefore $(p_a)_t$ is an ideal of BCC-algebra X. Conversely, suppose that $(p_a)_t \neq \phi$ is an ideal of X, we must prove that p_a is a fuzzy ideal of X. For any $x \in X$, since $x \in (p_a)_{p_a(x)} \neq \phi$, then $(p_a)_{p_a(x)}$ is a fuzzy ideal and so $0 \in (p_a)_{p_a(x)}$, that is $p_a(0) \geq p_a(x)$. Now, for any $x,y,z \in X$, we let $t = min\{p_a((x*y)*z), p_a(y)\}$. It follows that $(x*y)*z \in (p_a)_t$ and $y \in (p_a)_t$. Since, $(p_a)_t \neq \phi$ is an ideal of X, we have $(x*z) \in (p_a)_t$. Therefore we have:

$$p_a(x*z) \geq t = min\{p_a((x*y)*z), p_a(y)\}$$

This complete the proof.

We denote the set of soft ideal, fuzzy ideal and fuzzy soft ideal that constructed over BCC-algebra X by $SI(X)$, $FI(X)$ and $FSI(X)$, respectively.

Definition 5.5. Let X be a BCC-algebra and (p,A) be a soft BCC-algebra over X, we say that (p,A) satisfies the maximal condition, if each nonempty subset of $SI(p,A)$ contains least one maximal member with respect to the set theoretical inclusion $\subseteq$ and (p,A) satisfies the ascending chain condition, abbreviated by ACC, if there does not exist an infinite properly ascending chain $(q_1,I_1) \subseteq (q_2,I_2) \subseteq ...$ in $SI(p,A)$. In an entirely analogous way the minimal condition and the descending chain condition (abbreviated by DCC) are defined.

Theorem 5.6. Let X be a BCC-algebra and (p,A) be a soft BCC-algebra over X. Then

(i)(p,A) satisfies the maximal condition if and only if (p,A) satisfies ACC.

(ii)(p,A) satisfies the minimal condition if and only if (p,A) satisfies DCC.

Proof.(i) suppose (p,A) satisfies the maximal condition and $(q_1,I_1) \subseteq (q_2,I_2) \subseteq ...$ is an ascending chain in $SI(X)$. Then the set $\{(q_i,I_i) : i = 1,2,...\}$ has maximal member (q_n,I_n). Consequently, $(q_i,I_i) = (q_n,I_n)$ for all $i \geq n$, this says (p,A) satisfies ACC. Conversely, suppose (p,A) satisfies ACC and E is any nonempty subset of $SI(X)$. If E has no maximal member, each member of E precedes another member of E, which permits the construction of an infinite chain $(q_1,I_1) \subseteq (q_2,I_2) \subseteq ...$ in E, where $(q_i,I_i) \neq (q_j,I_j)$ whenever $i \neq j$, a contradiction. Hence (p,A) satisfies the maximal condition. Likewise for (ii), the reader should supply the details.

6 R-soft Sets

Definition 6.1. Let X, Y be two sets and $B \subseteq Y$. Let (T,X) be a soft set over Y $(T : X \to P^*(Y))$, then the lower inverse and upper inverse of B under T are defined by:

$$T^{-1}(B) = \{x \in X | T(x) \cap B \neq \phi\};$$

$$T^{+}(B) = \{x \in X | T(x) \subseteq B\}.$$

Proposition 6.2. Let X, Y be two sets and (T,X) be a soft set over Y. If A and B are nonempty subsets of Y, then the following hold:

(1)$T^{-1}(A \cup B) = T^{-1}(A) \cup T^{-1}(B)$;
(2)$T^{+}(A \cap B) = T^{+}(A) \cap T^{+}(B)$;
(3)$A \subseteq B$ implies $T^{+}(A) \subseteq T^{+}(B)$;
(4)$A \subseteq B$ implies $T^{-1}(A) \subseteq T^{-1}(B)$;
(5)$T^{+}(A) \cup T^{+}(B) \subseteq T^{+}(A \cup B)$;
(6)$T^{-1}(A \cap B) \subseteq T^{-1}(A) \cap T^{-1}(B)$;

Proof. The proof is easy and emitted.

Now, using the lower and upper inverse, we define a binary relation on subsets of Y as follow:

$$A \approx B \Leftrightarrow T^{-1}(A) = T^{-1}(B) \text{ and } T^{+}(A) = T^{+}(B).$$

Obviously $\approx$ is an equivalence relation which induces a partition $P^{*}(Y)/\approx$ of $P^{*}(Y)$. An equivalence class of $\approx$ is called a R-soft set. Therefore a R-soft set is a family of subsets of Y as follow:

$$\langle A_1, A_2 \rangle = \{B \in P^{*}(Y) | T^{+}(B) = A_1, T^{-1}(B) = A_2\}.$$

The intersection $\sqcap$, union $\sqcup$ and complement $\neg$ are defined as follow:

$$\langle A_1, A_2 \rangle \sqcap \langle B_1, B_2 \rangle = \langle A_1 \cap B_1, A_2 \cap B_2 \rangle,$$

$$\langle A_1, A_2 \rangle \sqcup \langle B_1, B_2 \rangle = \langle A_1 \cup B_1, A_2 \cup B_2 \rangle,$$

$$\neg \langle A_1, A_2 \rangle = \langle \neg A_1, \neg A_2 \rangle.$$

Theorem 6.3. The induced system $(P^{*}(Y)/\approx, \sqcap, \sqcup)$ is a complete distributive lattice.

Proof. The proof is straightforward.

7 Conclusions

Soft sets are deeply related to fuzzy sets and rough sets. We applied soft sets to *BCC*-algebra and discussed the algebraic properties of soft sets in *BCC*-algebras. We introduced the notion of soft ideals and idealistic soft *BCC*-algebras, and gave several examples. Then the relation between soft *BCC*-algebras and idealistic soft *BCC*-algebras are investigated.Also we found the intersection, union, “AND”operation, and “OR”operation of soft ideals and idealistic soft *BCC*-algebras.

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