Journal of Statistics Applications & Probability
An International Journal

http://dx.doi.org/10.12785/jsap/040213

Modified Generalized Chain Ratio in Regression Type Estimator for Population Mean using Two Phase Sampling in the Presence of Non-Response

*B. B. Khare and Habib ur Rehman**

Department of Statistics, Banaras Hindu University, Varanasi, India

Received: 4 Feb. 2015, Revised: 16 Jun. 2015, Accepted: 19 Jun. 2015
Published online: 1 Jul. 2015

Abstract: Modified generalized chain ratio in regression type estimator for population mean using two phase sampling in the presence of non-response has been proposed and their properties have been studied. Relative efficiency of the proposed estimator is obtained in the case of fixed first phase sample, second phase sample and sub sample fraction. A comparison of the proposed estimator has been carried out with the relevant estimators. The performances of the proposed estimator in comparison to the relevant estimators have been made with the help of an empirical study.

Keywords: Non-response; Mean square error; two phase sampling; Auxiliary characters.

1 Introduction

The information on the auxiliary character provides a very important contribution in the field of sample surveys. In the case when the population mean of the auxiliary character x is known, Cocharn [2], Rao [3,4,5] and Khare and Srivastava [6,7] have proposed ratio, product and regression type estimators in the presence of non-response. In the case when the population mean of an auxiliary character is not known, the two phase sampling ratio, product and regression type estimators have suggested by Khare and Srivastava [8,9] ,Khare et al. [10],Khare and Kumar [11] and Khare and Srivastava [12].

In such case, Chand [13] proposed chain ratio type estimator for the population mean of the study character. Kiregyra [14,15] proposed chain ratio to regression, ratio in regression and regression in regression estimators for the population mean of the study character. Further, some class of estimators for population mean of the study character have been proposed by Srivastava et al. [16], Sahoo et al. [17], Singh et al. [18] and Dash & Mishra [19].

In the case when the population mean of the main auxiliary character is not known but the population mean of an additional auxiliary character is known, which is cheaper than the main auxiliary character but less correlated to the study character than the main auxiliary character,Khare and Kumar [20] and Khare et al.[21] have proposed chain regression type estimators and generalized chain estimators for the population mean in the presence of non-response.

In the present paper, we have proposed modified generalized chain ratio in regression type estimator for the population mean using two phase sampling in the presence of non-response. We have obtained the expressions for mean square error of the proposed estimator for the fixed first phase sample , second phase sample and the optimum value of constants. A comparative study of the variance of the proposed estimator is made with the relevant estimators. An empirical study has been given to show the performance of the proposed estimator for fixed sample sizes.

* Corresponding author e-mail: hrmbd007@gmail.com

2 The Estimators

Let $\bar{Y},\bar{X}$ and $\bar{Z}$ denote the population mean of study character y , auxiliary character x and additional auxiliary character z having j^{th} value Y_j,X_j and Z_j; $j=1,2,...,N$. Population of size N is supposed to be divided in N_1 responding units and N_2 non respondents units. According to Hansen and Hurwitz [1], a sample of size n is drawn from the population of size N by using simple random sampling without replacement (SRSWOR) method of sampling and it has been observed that n_1 units respond and n_2 units do not respond. Again by making extra effort, a sub-sample of size $r(=n_2k^{-1})$ is drawn from n_2 non-responding units by using SRSWOR method of sampling and collect information on r units for study character y . Hence the estimator for $\bar{Y}$ based on n_1+r observations on y is given by Hansen and Hurwitz[1] as follow:

$$\bar{y}^* = \frac{n_1}{n}\bar{y}_1 + \frac{n_2}{n}\bar{y}_2' \tag{1}$$

where $\bar{y}_1$ and $\bar{y}_1'$ are the sample means of study character y based on n_1 and r units respectively. Using the values of x corresponding to the incomplete information on y, the estimator for population mean $\bar{X}$ of auxiliary character x is defined as follow:

$$\bar{x}^* = \frac{n_1}{n}\bar{x}_1 + \frac{n_2}{n}\bar{x}_2' \tag{2}$$

Using the values of z corresponding to the incomplete information on y, the estimator for population mean $\bar{Z}$ of additional auxiliary character z is defined as follow:

$$\bar{z}^* = \frac{n_1}{n}\bar{z}_1 + \frac{n_2}{n}\bar{z}_2' \tag{3}$$

Variance of the estimators $\bar{y}^*,\bar{x}^*$ and $\bar{z}^*$ are given by

$$V(\bar{y}^*) = \frac{f}{n}S_y^2 + \frac{W_2(k-1)}{n}S_{y(2)}^2 \tag{4}$$

$$V(\bar{x}^*) = \frac{f}{n}S_x^2 + \frac{W_2(k-1)}{n}S_{x(2)}^2 \tag{5}$$

$$V(\bar{z}^*) = \frac{f}{n}S_z^2 + \frac{W_2(k-1)}{n}S_{z(2)}^2 \tag{6}$$

where $f=1-\frac{n}{N}, W_2=\frac{N_2}{n}, (S_y^2,S_{y(2)}),(S_x^2,S_{x(2)})$ and $(S_z^2,S_{z(2)})$ are population mean squares of y , x and z for entire population and non-responding part of population.
In the case when population mean N is not known, a first phase sample of size $n\prime(<N)$ is taken from the population of size N by using SRSWOR scheme of sampling and the population mean $\bar{X}$ is estimated by first phase sample mean $\bar{x}'$ based on n' units. In this case, the conventional / alternative generalized two phase sampling estimators (T_1,T_2) for population mean $\bar{Y}$ in the presence of non-response have been proposed by Khare and Srivastava [12], which are given as follows:

$$T_1 = \bar{y}^*\left(\frac{\bar{x}^*}{\bar{x}'}\right)^\alpha \tag{7}$$

$$T_2 = \bar{y}^*\left(\frac{\bar{x}}{\bar{x}'}\right)^\beta \tag{8}$$

where $\bar{x}' = \frac{1}{n'}\sum_{j=1}^{n'} x_j, \bar{x} = \frac{1}{n}\sum_{j=1}^{n} x_j$ and (α,β) are constants.
In the situation when population mean $\bar{X}$ of the auxiliary character x is unknown, but $\bar{Z}$, the population mean of additional auxiliary character z (closely related to x) is known, which may be cheaper and less correlated to the study character y in comparison to main auxiliary character x, a first phase sample of size $n'<N$ is taken from the population of size N by using SRSWOR scheme of sampling and the information of auxiliary character x and additional auxiliary character z based on n' units is collected. Using the sample means $\bar{x}'$ and $\bar{z}'$ based on n' units and the known additional population mean $\bar{Z}$, the population mean $\bar{X}$ is estimated by $\hat{\bar{X}}_r = \bar{x}'\bar{Z}/\bar{z}'$. The estimator $\hat{\bar{X}}_r$ is more precise than sample mean $\bar{x}'$ if $\rho_{xz} > C_z/2C_x$, where C_z,C_x and ρ_{xz} are the coefficients of variation of z and x and the correlation coefficient between x and z .
In this situation, the conventional (T_3) and alternative (T_4) generalized chain estimators for $\bar{Y}$ using information on auxiliary character x and the additional auxiliary character z in the presence of non-response have been proposed by Khare et al. [12], which are given as follows:

$$T_3 = \bar{y}^* \left(\frac{\bar{x}^*}{\bar{x}'}\right)^{\alpha_1} \left(\frac{\bar{z}'}{\bar{Z}}\right)^{\alpha_2} \tag{9}$$

$$T_4 = \bar{y}^* \left(\frac{\bar{x}}{\bar{x}'}\right)^{\beta_1} \left(\frac{\bar{z}'}{\bar{Z}}\right)^{\beta_2} \tag{10}$$

where $\alpha_1, \alpha_2, \beta_1$ and β_2 are constants.
In the case, when both population means $\bar{X}$ and $\bar{Z}$ are unknown, complete information on the auxiliary variables is collected for all the units in the first-phase sample. The collected information becomes available for sub sampling phases. We proposed modified generalized chain ratio in regression type estimator for population mean using two phase sampling in the presence of non-response, which is given as follow:

$$T_5 = \bar{y}^* + b_{yx} \left\{ \bar{x}' \left(\frac{\bar{z}'}{\bar{z}^*}\right)^{p_1} \left(\frac{\bar{z}}{\bar{z}'}\right)^{p_2} - \bar{x}^* \right\} \tag{11}$$

where p_1 and p_2 are constants and b_{yx} is regression coefficient.

3 Mean Square Error of the Proposed Estimator

In order to derive the expressions for mean square error of the estimators-
Let $\bar{y}^* = \bar{Y}(1+\varepsilon_0)$,$\bar{x}^* = \bar{X}(1+\varepsilon_1)$,$\bar{x}' = \bar{X}(1+\varepsilon_2)$,$\bar{z}^* = \bar{Z}(1+\varepsilon_3)$,$\bar{z}' = \bar{Z}(1+\varepsilon_4)$ and $\bar{z} = \bar{Z}(1+\varepsilon_5)$, such that$E(\varepsilon_l) = 0$ and $\varepsilon_l < 1$, l= 0, 1, 2, 3, 4,5. Now, using SRSWOR scheme of sampling,we have

$$E(\varepsilon_0^2) = \frac{f}{n}C_y^2 + \frac{W_2(k-1)}{n}C_{y(2)}^2, \quad E(\varepsilon_1^2) = \frac{f}{n}C_x^2 + \frac{W_2(k-1)}{n}C_{x(2)}^2, \quad E(\varepsilon_2^2) = \frac{f'}{n'}C_x^2,$$
$$E(\varepsilon_3^2) = \frac{f}{n}C_z^2 + \frac{W_2(k-1)}{n}C_{z(2)}^2, \quad E(\varepsilon_4^2) = \frac{f'}{n'}C_z^2, \quad E(\varepsilon_5^2) = \frac{f}{n}C_z^2,$$
$$E(\varepsilon_0\varepsilon_1) = \frac{f}{n}C_{yx} + \frac{W_2(k-1)}{n}C_{yx(2)}, \quad E(\varepsilon_0\varepsilon_2) = \frac{f'}{n'}C_{yx}, \quad E(\varepsilon_0\varepsilon_3) = \frac{f}{n}C_{yz} + \frac{W_2(k-1)}{n}C_{yz(2)},$$
$$E(\varepsilon_0\varepsilon_4) = \frac{f'}{n'}C_{yz}, \quad E(\varepsilon_0\varepsilon_5) = \frac{f}{n}C_{yz}, \quad E(\varepsilon_1\varepsilon_2) = \frac{f'}{n'}C_x^2, \quad E(\varepsilon_1\varepsilon_3) = \frac{f}{n}C_{xz} + \frac{W_2(k-1)}{n}C_{xz(2)},$$
$$E(\varepsilon_1\varepsilon_4) = \frac{f'}{n'}C_{xz}, \quad E(\varepsilon_1\varepsilon_5) = \frac{f}{n}C_{xz}, \quad E(\varepsilon_2\varepsilon_3) = \frac{f'}{n'}C_{xz}, \quad E(\varepsilon_2\varepsilon_4) = \frac{f'}{n'}C_{xz},$$
$$E(\varepsilon_2\varepsilon_5) = \frac{f}{n}C_{xz}, \quad E(\varepsilon_3\varepsilon_4) = \frac{f'}{n'}C_z^2, \quad E(\varepsilon_3\varepsilon_5) = \frac{f}{n}C_z^2, \quad E(\varepsilon_4\varepsilon_5) = \frac{f'}{n'}C_z^2$$

The contribution of the terms involving the powers in ε_0 ,ε_1 ,ε_2 ,ε_3 ,ε_4 and ε_5 of order higher than two in mean square error are assumed to be negligible. Hence, the expression for mean square error of the estimator T_5 upto the terms of order $(1/n)$ is given by

$$\begin{aligned} MSE(T_5) = \; & V(\bar{y}^*) + \lambda b\bar{X}\left[b\bar{X}C_z^2 + p_1^2 b\bar{X}C_z^2 - 2\bar{Y}C_{yx} - 2p_1\bar{Y}C_{yz} + 2p_1 b\bar{X}C_{xz} + 2p_1p_2\bar{Y}C_{yz} - 2p_2 b\bar{X}C_{xz}\right. \\ & \left.+ p_2^2 b\bar{X}C_z^2 - 2p_1p_2 b\bar{X}C_z^2\right] + \theta b\bar{X}\left[b\bar{X}C_{z(2)}^2 + p_1^2 b\bar{X}C_{z(2)}^2 - 2\bar{Y}C_{yx(2)} - 2p_1\bar{Y}C_{yz(2)} + 2p_1 b\bar{X}C_{xz(2)}\right. \\ & \left.+ 2p_1p_2\bar{Y}C_{yz(2)} - 2p_2 b\bar{X}C_{xz(2)} + p_2^2 b\bar{X}C_{z(2)}^2 - 2p_1p_2 b\bar{X}C_{z(2)}^2\right] \end{aligned} \tag{12}$$

where p_1 and p_2 are optimum values and b is regression coefficient.

$$p_{1(opt)} = \frac{\lambda\left(\bar{Y}C_{yz} - b\bar{X}C_{xz} - p_2 b\bar{X}C_z^2 + p_2\bar{Y}C_{yz}\right) + \theta\left(\bar{Y}C_{yz(2)} - b\bar{X}C_{xz(2)}\right)}{b\bar{X}\left(\lambda C_z^2 + \theta C_{z(2)}^2\right)} \tag{13}$$

$$p_{2(opt)} = \frac{b\bar{X}C_{xz} - \bar{Y}C_{yz} + Mb\bar{X}C_z^2}{bXC_z^2 - QbXC_z^2} \tag{14}$$

$$where, \quad \lambda = \left(\frac{1}{n} - \frac{1}{n'}\right), \qquad \theta = \frac{W_2(k-1)}{n}, \qquad b = \frac{\bar{Y}\rho_{yx(2)C_{y(2)}}}{XC_{x(2)}} \tag{15}$$

$$M = \frac{\lambda(\bar{Y}C_{yz} - b\bar{X}C_{xz}) + \theta(\bar{Y}C_{yz(2)} - b\bar{X}C_{xz(2)})}{b\bar{X}(\lambda C_z^2 + \theta C_{z(2)}^2)}, \qquad Q = \frac{\lambda C_z^2}{(\lambda C_z^2 + \theta C_{z(2)}^2)} \tag{16}$$

Mean square error of the estimators T_1,T_2, T_3 and T_4 upto the terms of order $(1/\text{n})$ is given by

$$MSE(T_1) = V(\bar{y}^*) - \bar{Y}^2\left[\lambda\rho_{yx}^2C_y^2 + \theta\rho_{yx(2)}^2C_{y(2)}^2\right] \tag{17}$$

$$MSE(T_2) = V(\bar{y}^*) - \bar{Y}^2\left[\lambda\rho_{yx}^2C_y^2\right] \tag{18}$$

$$MSE(T_3) = V(\bar{y}^*) - \bar{Y}^2\left[\lambda\rho_{yx}^2C_y^2 + \theta\rho_{yx(2)}^2C_{y(2)}^2 + \eta\rho_{yz}^2C_y^2\right] \tag{19}$$

$$MSE(T_4) = V(\bar{y}^*) - \bar{Y}^2\left[\lambda\rho_{yx}^2C_y^2 + \eta\rho_{yz}^2C_y^2\right] \tag{20}$$

where,

$$C_{yx} = \rho_{yx}C_yC_x, \qquad C_{yx(2)} = \rho_{yx(2)}C_{y(2)}C_{x(2)}, \qquad C_{yz} = \rho_{yz}C_yC_z,$$

$$C_y = S_y/\bar{Y}, \qquad C_x = S_x/\bar{X}, \qquad C_z = S_z/\bar{Z}, \qquad C_{y(2)} = S_{y(2)}/\bar{Y}, \qquad C_{x(2)} = S_{x(2)}/\bar{X},$$

$$C_{z(2)} = S_{z(2)}/\bar{Z}, \qquad S_y^2 = \frac{1}{(N-1)}\sum_{j=1}^{N}(Y_j - \bar{Y})^2, \qquad S_x^2 = \frac{1}{(N-1)}\sum_{j=1}^{N}(X_j - \bar{X})^2,$$

$$S_z^2 = \frac{1}{(N-1)}\sum_{j=1}^{N}(Z_j - \bar{Z})^2, \qquad S_{y(2)}^2 = \frac{1}{(N_2-1)}\sum_{j=1}^{N_2}(Y_j - \bar{Y_2})^2,$$

$$S_{x(2)}^2 = \frac{1}{(N_2-1)}\sum_{j=1}^{N}(X_j - \bar{X_2})^2, \qquad S_{z(2)}^2 = \frac{1}{(N_2-1)}\sum_{j=1}^{N_2}(Z_j - \bar{Z_2})^2, \qquad \eta = \frac{f'}{n'}$$

$(\rho_{yx}, \rho_{yx(2)})$ are the correlation coefficients between (y,x) for whole population and non-responding part of the population, $(\rho_{yz}, \rho_{yz(2)})$ is the correlation coefficient between (y,z) for whole population and $(\bar{Y_2}, \bar{X_2})$ are the means of the study character y and auxiliary character x and auxiliary character z for the non-responding part of the population.

4 Comparison of the Proposed Estimator T_5 with Respect to the Relevant Estimators

$$
\begin{aligned}
& MSE(T_5) < MSE(\bar{y}^*) \\
if \quad & \frac{-A_2-\sqrt{A_2^2-4A_1A_3}}{2A_1} < p_1 < \frac{-A_2+\sqrt{A_2^2-4A_1A_3}}{2A_1} \\
and \quad & \frac{-B_2-\sqrt{B_2^2-4B_1B_3}}{2B_1} < p_2 < \frac{-B_2+\sqrt{B_2^2-4B_1B_3}}{2B_1}
\end{aligned} \tag{21}
$$

$$
\begin{aligned}
& MSE(T_5) < MSE(T_1) \\
if \quad & \frac{-A_2-\sqrt{A_2^2-4A_1A_4}}{2A_1} < p_1 < \frac{-A_2+\sqrt{A_2^2-4A_1A_4}}{2A_1} \\
and \quad & \frac{-B_2-\sqrt{B_2^2-4B_1B_4}}{2B_1} < p_2 < \frac{-B_2+\sqrt{B_2^2-4B_1B_4}}{2B_1}
\end{aligned} \tag{22}
$$

$$
\begin{aligned}
& MSE(T_5) < MSE(T_2) \\
if \quad & \frac{-A_2-\sqrt{A_2^2-4A_1A_5}}{2A_1} < p_1 < \frac{-A_2+\sqrt{A_2^2-4A_1A_5}}{2A_1} \\
and \quad & \frac{-B_2-\sqrt{B_2^2-4B_1B_5}}{2B_1} < p_2 < \frac{-B_2+\sqrt{B_2^2-4B_1B_5}}{2B_1}
\end{aligned} \tag{23}
$$

$$
\begin{aligned}
& MSE(T_5) < MSE(T_3) \\
if \quad & \frac{-A_2-\sqrt{A_2^2-4A_1A_6}}{2A_1} < p_1 < \frac{-A_2+\sqrt{A_2^2-4A_1A_6}}{2A_1} \\
and \quad & \frac{-B_2-\sqrt{B_2^2-4B_1B_6}}{2B_1} < p_2 < \frac{-B_2+\sqrt{B_2^2-4B_1B_6}}{2B_1}
\end{aligned} \tag{24}
$$

$$
\begin{aligned}
& MSE(T_5) < MSE(T_4) \\
if \quad & \frac{-A_2-\sqrt{A_2^2-4A_1A_7}}{2A_1} < p_1 < \frac{-A_2+\sqrt{A_2^2-4A_1A_7}}{2A_1} \\
and \quad & \frac{-B_2-\sqrt{B_2^2-4B_1B_7}}{2B_1} < p_2 < \frac{-B_2+\sqrt{B_2^2-4B_1B_7}}{2B_1}
\end{aligned} \tag{25}
$$

where,

$$
\begin{aligned}
A_1 &= b\bar{X}(\lambda C_z^2+\theta C_{z(2)}^2), \qquad A_2=\lambda(-2\bar{Y}C_{yz}+2b\bar{X}C_{xz}-2Pb\bar{X}C_z^2)+\theta(-2\bar{Y}C_{yz(2)}+2b\bar{X}C_{xz(2)}), \\
A_3 &= \lambda(b\bar{X}C_x^2-2\bar{Y}C_{yx}+2Pb\bar{Y}C_{yz}+2Pb\bar{X}C_{xz}-b\bar{X}PC_z^2)+\theta(b\bar{X}C_{x(2)}^2-2\bar{Y}C_{yx(2)}), \\
B_1 &= \lambda b\bar{X}C_z^2(1-Q)^2+\theta Q^2b\bar{X}C_{z(2)}^2, \\
B_2 &= \lambda(2bQM\bar{X}C_z^2-2Q\bar{Y}C_{yz}+2QPb\bar{X}C_{xz}-2b\bar{X}C_{xz}-2b\bar{X}MC_z^2)+Q\theta(Mb\bar{X}C_{z(2)}^2-2\bar{Y}C_{yz(2)}+2b\bar{X}C_{xz(2)}), \\
B_3 &= \lambda(b\bar{X}C_x^2-2\bar{Y}C_{yx}-2M\bar{Y}C_{yz}+2Mb\bar{X}C_{xz}+bM^2\bar{X}C_x^2)+\theta(b\bar{X}C_{x(2)}^2-2\bar{Y}C_{yx(2)}-2\bar{Y}MC_{yz(2)}+2Mb\bar{X}C_{xz(2)}+bM^2\bar{X}C_{x(2)}^2), \\
A_4 &= A_3+\theta b\bar{X}C_{x(2)}^2, \qquad B_4=B_3+\theta b\bar{X}C_{x(2)}^2, \\
A_5 &= A_3+\lambda b\bar{X}C_{x(2)}^2, \qquad B_5=B_3+\lambda b\bar{X}C_{x(2)}^2, \\
A_6 &= A_3+\ b\bar{X}C_{x(2)}^2(\lambda+\theta)+\eta\rho_{yz}^2C_y^2, \qquad B_6=B_3+\ b\bar{X}C_{x(2)}^2(\lambda+\theta)+\eta\rho_{yz}^2C_y^2, \\
A_7 &= A_3+\lambda b\bar{X}C_{x(2)}^2+\eta\rho_{yz}^2C_y^2, \qquad B_7=B_3+\lambda b\bar{X}C_{x(2)}^2+\eta\rho_{yz}^2C_y^2 \\
M &= \frac{\lambda(\bar{Y}C_{yz}-b\bar{X}C_{xz})+\theta(\bar{Y}C_{yz(2)}-b\bar{X}C_{xz(2)})}{b\bar{X}(\lambda C_z^2+\theta C_{z(2)}^2)}, \qquad Q=\frac{\lambda C_z^2}{(\lambda C_z^2+\theta C_{z(2)}^2)}, \\
P &= \frac{b\bar{X}C_{yz}-\bar{Y}C_{yz}+Mb\bar{X}C_z^2}{b\bar{X}C_z^2-Qb\bar{X}C_z^2}
\end{aligned}
$$

5 An empirical study

To study the performance of the proposed estimator, we use an empirical studie given by Khare and Sinha [22].The description of the population is given below: The data on physical growth of upper socio-economic group of 95 school children of Varanasi under an ICMR study, Department of Pediatrics, B.H.U., during 1983-84 has been taken under study. The first 25% (i. e. 24 children) units have been considered as non-responding units. Here we have taken the study character (y) , auxiliary character (x) and the additional auxiliary character (z) are taken as follows:
y: weight (in kg.) of the children.
x: skull circumference (in cm) of the children.
z: chest circumference (in cm) of the children.
The values of the parameters of the characters for the given data are given as follows:

$$\bar{Y} = 19.4968, \quad \bar{X} = 51.1726, \quad \bar{Z} = 55.8611, \quad C_y = 0.15613,$$
$$C_x = 0.03006, \quad C_z = 0.05711, \quad C_{y(2)} = 0.12075, \quad C_{x(2)} = 0.032479,$$
$$C_{z(2)} = 0.05403, \quad \rho_{yx} = 0.328, \quad \rho_{yz} = 0.868, \quad \rho_{xz} = 0.305,$$
$$\rho_{yx(2)} = 0.477, \quad \rho_{yz(2)} = 0.729, \quad \rho_{xz(2)} = 0.507, \quad W_2 = 0.25,$$
$$N = 95, \quad n = 35.$$

Table 1, Relative efficiency (in %) of the estimators with respect to $\bar{y}^*$ for the fixed values of n', n and different values of k(N=95,n'=70 and n=35).

Estimators	1/k		
	1/4	1/3	1/2
$\bar{y}^*$	100(0.28597)*	100(0.24638)	100(0.20679)
T_1	116(0.24569)	115(0.21490)	112(0.18403)
T_2	105(0.27172)	106(0.23213)	107(0.19255)
T_3	130(0.21847)	131(0.18789)	132(0.15730)
T_4	117(0.24546)	120(0.20588)	124(0.16629)
T_5	185(0.15489)	186(0.13268)	188(0.10980)

*Figures in parenthesis given the MSE(.).

6 Conclusion

From Table 1, we observed that the proposed estimator T_5 is more efficient than $\bar{y}^*, T_1, T_2, T_3$ and T_4 for the different values of k. The MSE of all estimators decrease the value of k decrease. The relative efficency of T_2, T_3, T_4 and T_5 with respect to $\bar{y}^*$ increase as k decrease but relative efficency of T_1 with respect to $\bar{y}^*$ decrease as k decrease. This is due to the fact that with the decrease in the value of k, the varience of $\bar{y}^*$ decrease with the faster rate in comparison to T_1. Finally we observe that the proposed estimator is more effective than the relevent estimators for the different values of k. Hence we prefer the use of T_5 in practice.

References

[1] Hansen, M. H. and Hurwitz, W. N.(1946). The problem of non-response in sample surveys. J. Amer. Statist. Assoc., 41, 517-529.
[2] Cohran, W. G. (1977). Sampling Techniques, 3rd edn. Wiley, New York.
[3] Rao, P. S. R. S. (1986). Ratio estimation with subsampling the nonrespondents. Survey Methodology 12(2): 217-230.
[4] Rao, P. S. R. S. (1987). Ratio and regression Estimates with subsampling the non-respondents, Paper presented at a special contributed session of the International Statistical Association Meeting, September, 2-16, Tokyo, Japan.
[5] Rao, P. S. R. S. (1990). Regression estimators with sub sampling of nonrespondents. In Data quality control theory and Pragmatics (eds.) Gumer E. Liepins and V. R. R. Uppuluri, Marcel Dekker, New York, 191-208.
[6] Khare, B. B. and Srivastava, S. (1996). Transformed product type estimators for population mean in presence of softcore observations. Proc. Math. Soc. B. H.U., 12, 29-34.

[7] Khare, B. B. and Srivastava, S. (1997). Transformed ratio type estimators for population mean in presence. Commun. Statist. Theory Meth., 26(7), 1779-1791.
[8] Khare, B. B. and Srivastava, S.(1993). Estimation of population mean using auxiliary character in presence of non-response. Nat. Acad. Sci. Letters, India, 16(3), 111-114.
[9] Khare, B. B. and Srivastava, S.(1995). Study of conventional and alternative two phase sampling ratio, product and regression estimators in presence of non-response. Proc. Nat. Acad. Sci., India, Sect. A 65(a) II, 195-203.
[10] Khare, B.B., Kumar, A., Sinha, R.R. and Pandey, S.K. (2008). Two phase sampling estimators for population mean using auxiliary character in presence of non-response in sample surveys. Jour. of Sci. Res., 52,271-281.
[11] Khare, B.B. and Kumar, S.(2009). Transformed two phase sampling ratio and product type estimators for population mean in the presence of non response. Aligarh J. Stats., 29, 91-106.
[12] Khare, B. B. and Srivastava, S.(2010). Generalized two phase sampling estimators for the population mean in the presence of nonresponse. Aligarh J. Stats., 30, 39-54.
[13] Chand, L. (1975). Some ratio-type estimators based on two or more auxiliary variables. Ph.D. Thesis submitted to Iowa State University, Ames, IOWA.
[14] Kiregyera, B. (1980). A chain ratio type estimator in finite population double sampling using two auxiliary variables. Metrika, 27, 217-223.
[15] Kiregyera, B. (1984). Regression - type estimators using two auxiliary variables and model of double sampling from finite populations. Metrika, 31, 215-226.
[16] Srivastava, S. Rani, Khare, B.B. and Srivastava, S.R. (1990). A generalized chain ratio estimator for mean of finite population. Jour. Ind. Soc. Agri. Statist., 42(1), 108-117.
[17] Sahoo, J., Sahoo, L.N. and Mohanty, S. (1993). A regression approach to estimation in two-phase sampling using two auxiliary variables. Curr. Sci.,65(1), 73-75.
[18] Singh, A.K., Singh, H.P. and Upadhyaya, L.N. (2001). A generalized chain estimator for finite population mean in two phase sampling. Jour. Ind. Soc. Ag. Statist., 54(3), 370-375.
[19] Das, P.R. and Mishra, G. (2011): An improved class of estimators in two phase sampling using two auxiliary variables. Commun. Stats.-Theo. Meth., 40(24), 4347-4352.
[20] Khare, B. B. and Kumar, S. (2010). Chain regression type estimators using additional auxiliary variable in two phase sampling in the presence of non-response. Nat. Acad. Sci. Lett. India 33(11&12): 369-375.
[21] Khare, B.B., Srivastava, U. and Kumar, Kamlesh (2011). Generalized chain estimators for the population mean in the presence of non response, Proc. Nat. Acad. Sci., India, 81A(III), 231-238.
[22] Khare, B.B. and Sinha, R.R. (2007). Estimation of the ratio of the two population means using multi-auxiliary characters in the presence of non-response. In Statistical Technique in Life Testing, Reliability, Sampling Theory and Quality Control. Edited by B.N. Pandey, Narosa publishing house, New Delhi, 163-171.