Journal of Statistics Applications & Probability
*An International Journal*

http://dx.doi.org/10.12785/jsap/040315

# Some Properties of Exponentiated Weibull-Generalized Exponential Distribution

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Received: 7 Jun. 2014, Revised: 21 Sep. 2014, Accepted: 23 Sep. 2014.
Published online: 1 Nov. 2015.

**Abstract:** Several general methods have been developed for generating new flexible family of distributions. In this paper an attempt has been made to develop the Exponentiated Weibull-Generalized Exponential Distribution (EWGED). This distribution is an extension of Generalized Weibull-Exponential distribution. Various properties of this distribution has been discussed viz limiting behaviour, Shannon's entropy, moments, quantile function, hazard function, survival function. Skewness and kurtosis are discussed. Parameter estimation of exponentiated weibull-generalized exponential distribution by the maximum likelihood method is also provided.

**Keywords:** Exponentiated T-X distribution, Quantile function, Shannon's entropy, moments, Estimation, Hazard function, Survival function.

## 1 Introduction

Generalized distributions arise by adding parameters for more flexibility and can be used to describe real world phenomenon. Researchers have developed and studied many generalized classes of distributions to describe and to predict the real world scenarios. Various methods have been developed to generate new generalized distributions. Mudholkar and Srivastava [11] proposed the exponentiated weibull distribution to analyze bathtub failure data. Marshall and Olkin [12] proposed a new method by adding a parameter to a family of distributions with application to the exponential and weibull families. Gera AW [3] introduced the modified exponentiated-weibull distribution for life-time modelling. Gupta et al [4] introduced the exponentiated distributions with the additional parameter k by taking the cumulative distribution function of the random variable X as $G_k(x)=[F(x)]^k$. Nadarajah and Kotz [14] used the class of exponentiated distributions and created the new exponentiated Frechet distribution. Nadarajah and Kotz [15] studied some exponentiated type distributions. Alzaatreh et al [1] proposed new method called T-X family of distributions. In this method the two random variables T and X are used to generate many new distributions. Alzaatreh et al [2] generalizes the method of T-X family by inclusion of an additional parameter c which leads to a new family of exponentiated T-X distribution. Alzaatreh et al [2] studied the properties of exponentiated weibull-exponential distribution. Hamdy et al [8] extended the exponentiated weibull-exponential distribution to more general form. Gupta et al [9] studied some properties of exponentiated exponential distribution and obtained this distribution can be used as a possible alternative to a weibull or a gamma distribution. Smith [18] proposed an alternative approach for estimating the parameters of distribution. Different estimation procedures and properties of generalized exponential distribution given by Gupta and Kundu [7]. A detailed survey of exponentiated weibull distribution is presented by Saralees et al [17]. Most recently Suriya and Jan [20] obtained the structural properties of the T-X family of gamma-exponential distribution.

This paper is organised as follows. Section 2 defines exponentiated weibull-generalized exponential distribution. In section 3, moment generating function with mean and variance are obtained. Various properties of EWGED viz limiting behaviour, survival function, hazard function, quantile function, shannon's entropy, skewness and kurtosis are discussed in section 4. In section 5, we define the method of maximum likelihood for estimating the parameters of exponentiated weibull-generalized exponential distribution.

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## 2 Exponentiated Weibull-Generalized Exponential Distribution

Two random variables X, the transformer, and T, the transformed are used to develop the T-X family of distributions. Suppose a random variable X (the transformer) follows the two parameter generalized exponential distribution given by Gupta and Kandu [5] with probability density function (PDF) as

$$f(x)=\beta\tau e^{-\tau x}\left[1-e^{-\tau x}\right]^{\beta-1}; \quad x>0,\tau>0,\beta>0$$

and the cumulative distribution function (CDF) is given by

$$F(x)=\left(1-e^{-\tau x}\right)^{\beta}; \quad x>0,\beta>0,\tau>0 \tag{1}$$

Suppose T (the transformed) random variable follows weibull distribution with the following PDF & CDF

$$r(t)=\frac{\alpha}{\gamma^{\alpha}}t^{\alpha-1}e^{-\left(\frac{t}{\gamma}\right)^{\alpha}}; \quad t>0,\alpha>0,\gamma>0 \quad R(t)=1-e^{-\left(\frac{t}{\gamma}\right)^{\alpha}}; \quad t>0,\alpha>0,\gamma>0 \tag{2}$$

The PDF of Exponentiated Weibull-X distribution given by Alzaatreh [2] is

$$g(x)=\frac{c\alpha f(x)F^{c-1}(x)}{\gamma\left(1-F^{c}(x)\right)}\left[\frac{-\log\left(1-F^{c}(x)\right)}{\gamma}\right]^{\alpha-1}\exp\left[-\left\{\frac{-\log\left(1-F^{c}(x)\right)}{\gamma}\right\}\right]^{\alpha} \tag{3}$$

The PDF and CDF of EWGED is

$$g(x)=\frac{c\alpha\beta\tau e^{-\tau x}\left(1-e^{-\tau x}\right)^{\beta c-1}}{\gamma\left[1-\left(1-e^{-\tau x}\right)^{\beta c}\right]}\left[\frac{-\log\left(1-\left(1-e^{-\tau x}\right)^{\beta c}\right)}{\gamma}\right]^{\alpha-1}$$
$$\times\exp\left[-\left\{\frac{-\log\left(1-\left(1-e^{-\tau x}\right)^{\beta c}\right)}{\gamma}\right\}\right]^{\alpha} \tag{4}$$

$$G(x)=1-\exp\left[-\left\{\frac{-\log\left(1-\left(1-e^{-\tau x}\right)^{\beta c}\right)}{\gamma}\right\}\right]^{\alpha}; \quad x\geq 0,\alpha,\beta,\tau,c,\gamma>0 \tag{5}$$

Where α, β, c are the shape parameters and τ, $\gamma$ are the scale parameters.

### *2.1 Some special cases of EWGED*

**Case (2.1.1):** If $\beta=1, c=c, \alpha=\alpha, \tau=\tau$ and $\gamma=\gamma$ in (4), we get the PDF of generalized weibull-exponential distribution given by Hamdy et al [8].

$$g(x)=\left(\frac{c\alpha\tau e^{-\tau x}\left(1-e^{-\tau x}\right)^{c-1}}{\gamma\left(1-\left(1-e^{-\tau x}\right)^{c}\right)}\right)\left[\frac{-\log\left(1-\left(1-e^{-\tau x}\right)^{c}\right)}{\gamma}\right]^{\alpha-1}$$

$$\times\exp\left[-\left\{\frac{-\log\left(1-\left(1-e^{-\tau x}\right)^{c}\right)}{\gamma}\right\}\right]^{\alpha};x>0,c,\alpha,\tau,\gamma>0$$

**Case (2.1.2):** When $c=\beta=\alpha=\tau=1$ and $\gamma=\gamma$ in (4), we get the PDF of the exponential distribution with parameters $\left(\frac{1}{\gamma}\right)$

$$g(x)=\frac{1}{\gamma}e^{-\frac{x}{\gamma}};x>0,\gamma,\tau>0$$

**Case (2.1.3):** Putting $\beta=c=\tau=1,\alpha=\alpha$ and $\gamma=\gamma$ in (4), then it reduces to the weibull distribution with parameters $(\gamma,\alpha)$

$$g(x)=\frac{\alpha}{\gamma^{\alpha}}x^{\alpha-1}e^{-\left(\frac{x}{\gamma}\right)^{\alpha}};x>0,\alpha,\gamma>0$$

**Case (2.1.4):** Suppose $\alpha=\beta=\gamma=1,c=c$ and $\tau=\tau$ in (4), then the density function of the two parameter generalized exponential distribution with parameters $c$ and $\tau$ can be obtained

$$g(x)=c\tau e^{-\tau x}\left(1-e^{-\tau x}\right)^{c-1};x>0,c,\tau>0$$

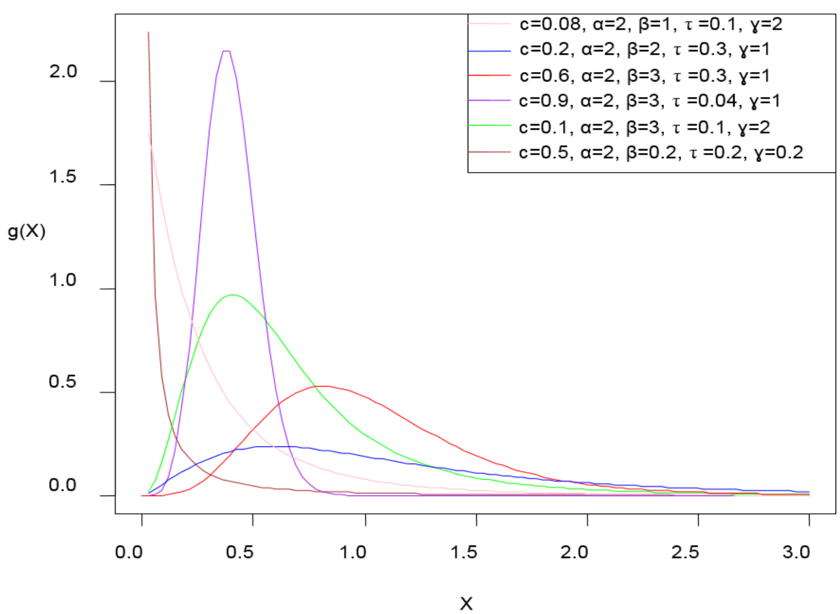


**Figure 1**: Different shapes for pdf of EWGED with some values of $c,\beta,\tau$ and $\gamma$ when $\alpha=2$.

## 3 Moment Generating Function

Let X follows the EWGED then moment generating function can be obtained as:

$$M_x(t) = E\left(e^{tx}\right) = \int_0^\infty e^{tx} g(x)dx$$

$$= \int_0^\infty e^{tx} \frac{c\alpha\beta\tau\left(1-e^{-\tau x}\right)^{\beta c-1}}{\gamma\left[1-\left(1-e^{-\tau x}\right)^{\beta c}\right]}\left[\frac{-\log\left(1-\left(1-e^{-\tau x}\right)^{\beta c}\right)}{\gamma}\right]^{\alpha-1} \exp\left[-\left\{\frac{-\log\left(1-\left(1-e^{-\tau x}\right)^{\beta c}\right)}{\gamma}\right\}\right]^{\alpha} dx \qquad (6)$$

Let $\left[\frac{-\log\left(1-\left(1-e^{-\tau x}\right)^{\beta c}\right)}{\gamma}\right]^{\alpha} = z$, then

$$\frac{c\alpha\beta\tau e^{-\tau x}\left(1-e^{-\tau x}\right)^{\beta c-1}}{\gamma\left[1-\left(1-e^{-\tau x}\right)^{\beta c}\right]}\left[\frac{-\log\left(1-\left(1-e^{-\tau x}\right)^{\beta c}\right)}{\gamma}\right]^{\alpha-1} dx = dz$$

Equation (6) reduces to

$$M_x(t) = \int_0^\infty e^{-z} \exp\left[-\frac{t}{\tau}\log\left(1-\left(1-e^{-z^{\frac{1}{\alpha}}\gamma}\right)^{\frac{1}{\beta c}}\right)\right]^{-\frac{t}{\tau}} dz$$

$$= \int_0^\infty e^{-z}\left[1-\left(1-e^{-\frac{\gamma}{\tau}z^{\frac{1}{\alpha}}}\right)^{\frac{1}{\beta c}}\right]^{-t} dz$$

$$M_x(t) = 1+\sum_{l=1}^{\infty}\frac{(t)_l}{l!}\int_0^\infty e^{-z}\left(1-e^{-\frac{\gamma}{\tau}z^{\frac{1}{\alpha}}}\right)^{\frac{l}{\beta c}} dz \qquad (7)$$

Where $(t)_l = t(t+1)\ldots(t+l-1)$, and $\left[1-\left(1-e^{-\frac{\gamma}{\tau}z^{\frac{1}{\alpha}}}\right)^{\frac{1}{\beta c}}\right]^{-t} = \sum_{l=0}^{\infty}\binom{t+l-1}{l}\left(1-e^{-\frac{\gamma}{\tau}z^{\frac{1}{\alpha}}}\right)^{\frac{l}{\beta c}}$

$$\left(1-e^{-\frac{\gamma}{\tau}z^{\frac{1}{\alpha}}}\right)^{\frac{l}{\beta c}} = \sum_{m=0}^{\infty}\frac{(-1)^m}{m!}\left(\frac{l}{\beta c}\right)_m e^{-m\frac{\gamma}{\tau}z^{\frac{1}{\alpha}}} \qquad (8)$$

Using (8) in (7), it becomes

$$M_x(t) = 1+\sum_{l=1}^{\infty}\frac{(t)_l}{l!}\left(\sum_{m=0}^{\infty}\frac{(-1)^m}{m!}\left(\frac{l}{\beta c}\right)_m\right)\int_0^\infty e^{-z}e^{-m\frac{\gamma}{\tau}z^{\frac{1}{\alpha}}} dz \qquad (9)$$

Using series expansion of

$$e^{-m\frac{\gamma}{\tau}z^{\frac{1}{\alpha}}}=\frac{\sum_{n=0}^{\infty}\left(m\frac{\gamma}{\tau}z^{\frac{1}{\alpha}}\right)^n}{n!}=\sum_{n=0}^{\infty}\frac{(-1)^n m^n}{n!}\left(\frac{\gamma}{\tau}\right)^n z^{\frac{n}{\alpha}}$$

The integral in (9) reduces to

$$M(t)=1+\sum_{l=1}^{\infty}\frac{(t)_l}{l!}\left(\sum_{m=0}^{\infty}\frac{(-1)^m}{m!}\left(\frac{l}{\beta c}\right)_m\right)\left(\sum_{n=0}^{\infty}\frac{(-1)^n m^n}{n!}\left(\frac{\gamma}{\tau}\right)^n\right)\int_0^{\infty}e^{-z}z^{\frac{n}{\alpha}}dz$$

$$M(t)=\sum_{l=1}^{\infty}\frac{(t)_l}{l!}\left(\sum_{m=0}^{\infty}\frac{(-1)^m}{m!}\left(\frac{l}{\beta c}\right)_m\right)\left(\sum_{n=0}^{\infty}\frac{(-1)^n m^n}{n!}\left(\frac{\gamma}{\tau}\right)^n\right)\Gamma\left(1+\frac{n}{\alpha}\right) \qquad (10)$$

Putting t=0, the sth derivative of (10) can be obtained as

$$E\left(X^s\right)=\sum_{l=1}^{\infty}\frac{d^s}{dt^s}\frac{(t)_l}{l!}\left(\sum_{m=0}^{\infty}\frac{(-1)^m}{m!}\left(\frac{l}{\beta c}\right)_m\right)\left(\sum_{n=0}^{\infty}\frac{(-1)^n m^n}{n!}\left(\frac{\gamma}{\tau}\right)^n\right)\Gamma\left(1+\frac{n}{\alpha}\right)$$

Hence the mean and the variance of EWGED becomes

$$E(X)=\sum_{l=1}^{\infty}\frac{1}{l}\left(\sum_{m=0}^{\infty}\frac{(-1)^m}{m!}\left(\frac{l}{\beta c}\right)_m\right)\left(\sum_{n=0}^{\infty}\frac{(-1)^n m^n}{n!}\left(\frac{\gamma}{\tau}\right)^n\right)\Gamma\left(1+\frac{n}{\alpha}\right)$$

$$V(X)=\sum_{l=1}^{\infty}\frac{2(-\psi(1)+\psi(l))}{l}\left(\sum_{m=0}^{\infty}\frac{(-1)^m}{m!}\left(\frac{l}{\beta c}\right)_m\right)\left(\sum_{n=0}^{\infty}\frac{(-1)^n m^n}{n!}\left(\frac{\gamma}{\tau}\right)^n\right)\Gamma\left(1+\frac{n}{\alpha}\right)$$

## 4 Properties of EWGED

### *4.1 Limiting behaviour.*

**Theorem (4.1.1):** The limit of exponentiatedweibull-generalized exponential distribution as x goes to infinity is zero, and the limit when $x\to 0$, is

$$\lim_{x\to 0}g(x)=\begin{cases}0, & \alpha\beta c>1\\ \frac{\tau}{\gamma^{\alpha}}, & \alpha\beta c=1\\ \infty & \alpha\beta c<1\end{cases} \qquad (11)$$

Proof: Rearranging the terms of (4), the limit of pdf of EWGED when $x\to 0$ can be simplified as

$$\lim_{x\to 0}g(x)=\frac{c\alpha\beta\tau}{\gamma}\lim_{x\to 0}\frac{e^{-\tau x}\left(1-e^{-\tau x}\right)^{\beta c-1}}{1-\left(1-e^{-\tau x}\right)^{\beta c}}\left[\frac{-\log\left(1-\left(1-e^{-\tau x}\right)^{\beta c}\right)}{\gamma}\right]^{\alpha-1}$$

$$\times\exp\left[-\left\{\frac{-\log\left(1-\left(1-e^{-\tau x}\right)^{\beta c}\right)}{\gamma}\right\}\right]^{\alpha}$$

$$=\frac{c\alpha\beta\tau}{\gamma^{\alpha}}\lim_{x\to 0}\left[-\log\left(1-\left(1-e^{-\tau x}\right)^{\beta c}\right)\right]^{\alpha-1}\left(1-e^{-\tau x}\right)^{\beta c-1}$$

$$=\frac{c\alpha\beta\tau}{\gamma^{\alpha}}\lim_{x\to 0}\left[\sum_{k=0}^{\infty}\left\{\frac{\left(1-e^{-\tau x}\right)^{k\beta c}}{k}\right\}^{\alpha-1}\left(1-e^{-\tau x}\right)^{\beta c-1}\right]$$

$$=\frac{c\alpha\beta\tau}{\tau}\lim_{x\to 0}\left\{\left[1+\frac{\left(1-e^{-\tau x}\right)^{\beta c}}{2}+\frac{\left(1-e^{-\tau x}\right)^{2\beta c}}{3}+....\right]^{\alpha-1}\left(1-e^{-\tau x}\right)^{c\alpha\beta-1}\right\} \tag{12}$$

$$\lim_{x\to 0}\left[1+\frac{\left(1-e^{-\tau x}\right)^{\beta c}}{2}+\frac{\left(1-e^{-\tau x}\right)^{2\beta c}}{3}+...\right]^{\alpha-1}=1$$

$$\lim_{x\to 0}g\left(x\right)=\frac{c\alpha\beta\tau}{\gamma^{\alpha}}\lim_{x\to 0}\left[\left(1-e^{-\tau x}\right)^{\alpha\beta c-1}\right] \tag{13}$$

In (12), and it reduces to

When αβc>1, (13) approaches to zero, when αβc<1, it approaches to infinity and when αβc=1, it reduces to $\frac{\tau}{\gamma^{\alpha}}$ .

## *4.2 Survival and Hazard function for EWGED*

The survival function of the EWGED can be obtained as

$$R\left(x\right)=1-G\left(x\right)=\exp\left[-\left\{\frac{-\log\left(1-\left(1-e^{-\tau x}\right)^{\beta c}\right)}{\gamma}\right\}\right]^{\alpha} \tag{14}$$

And the hazard function is

$$h(x)=\frac{g(x)}{R(x)}=\frac{c\alpha\beta\tau e^{-\tau x}\left(1-e^{-\tau x}\right)^{\beta c-1}}{\gamma\left[1-\left(1-e^{-\tau x}\right)^{\beta c}\right]}\left[\frac{-\log\left(1-\left(1-e^{-\tau x}\right)^{\beta c}\right)}{\gamma}\right]^{\alpha-1} \tag{15}$$

**Theorem (4.2.1):** When $x\to 0$ and $x\to\infty$, the limit of the hazard function for EWGED is

$$\lim_{x\to 0}h\left(x\right)=\begin{cases}0, & \alpha\beta c>1\\ \frac{\tau}{\gamma^{\alpha}}, & \alpha\beta c=1\\ \infty, & \alpha\beta c<1\end{cases} \tag{16}$$

$$\lim_{x\to\infty}h\left(x\right)=\begin{cases}0, & \alpha\beta c>1\\ \frac{\tau}{\gamma}, & \alpha\beta c=1\\ \infty, & \alpha\beta c<1\end{cases} \tag{17}$$

Proof. The proof of this theorem immediately follows from the theorem 4.1.1 with different shapes of hazard functions as in Figure 2.

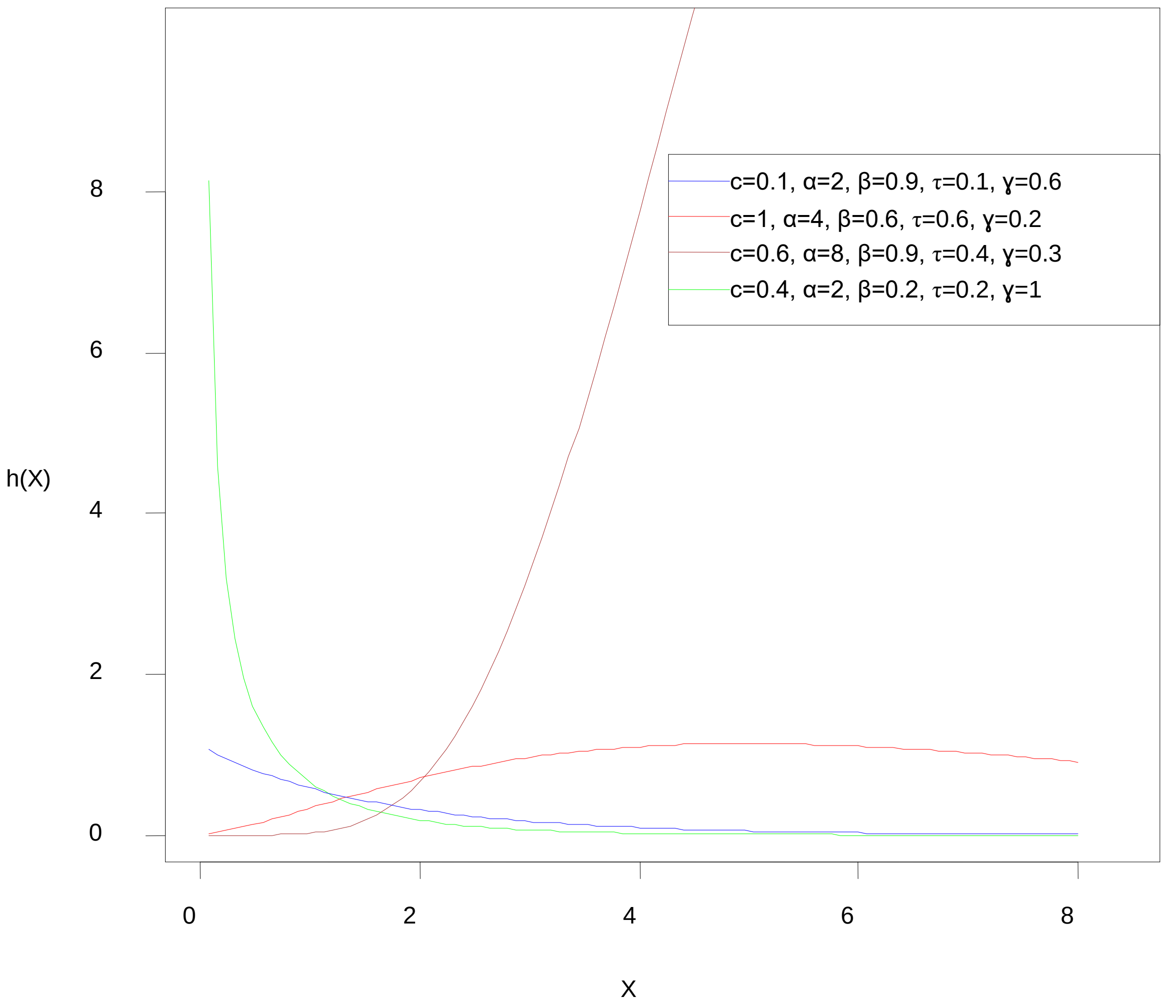


**Figure 2:** Different shapes for hazard function of the EWGED with some values of $c, \alpha, \beta, \tau$ and $\gamma$ .

### *4.3 Relationship between EWGED with some distributions*

**Lemma (I):** Suppose Y be a standard exponential random variable, then by using the transformation

$X = -\frac{1}{\lambda}\log\left[1-\left(1-e^{-\gamma Y^{\frac{1}{\alpha}}}\right)^{\frac{1}{\beta c}}\right]$ Results the PDF of (4).

Proof: On solving the above transformation, the following expression is obtained

$$Y = \left[-\frac{1}{\gamma}\log\left\{1-\left(1-e^{-\lambda x}\right)^{\beta c}\right\}\right]^{\alpha} \tag{18}$$

After taking the derivative of (18), it becomes

$$\frac{dY}{dX}=\frac{c\alpha\beta\lambda e^{-\lambda x}\left(1-e^{-\lambda x}\right)^{\beta c-1}}{\gamma\left[1-\left(1-e^{-\lambda x}\right)^{\beta c}\right]}\left[\frac{-\log\left(1-\left(1-e^{-\lambda x}\right)^{\beta c}\right)}{\gamma}\right]^{\alpha-1} \tag{19}$$

$s(x)=\frac{dY}{dX}f(y)$ follows the pdf of (4).

**Lemma (II):** Suppose Y be a weibull random variable with parameter α and $\gamma$ , then by using the transformation $X=-\frac{1}{\lambda}\log\left[1-\left(1-e^{-Y}\right)^{\frac{1}{\beta c}}\right]$ results the PDF of (4).

**Lemma (III):** Suppose Y be a uniform random variable, then by using the $X=-\frac{1}{\lambda}\log\left[1-\left(1-e^{-\gamma(-\log(Y))^{\frac{1}{\alpha}}}\right)^{\frac{1}{\beta c}}\right]$ results the PDF of (4).

**Proof:** The proofs of lemma I&II are obtained as in lemma I.

## *4.4 Quantile function*

**B**ased on the CDF of two random variables X and T the quantile function for the exponentiated T-X distribution given by Alzaatreh et al [2] as

$$Q(\lambda)=F^{-1}\left(1-e^{-R^{-1}(\lambda)}\right)^{\frac{1}{c}} \tag{20}$$

Using (1) & (2), the following expressions can be obtained as

$$R^{-1}(\lambda)=\gamma\left[-\log(1-\lambda)\right]^{\frac{1}{\alpha}} \tag{21}$$

$$F^{-1}(x)=-\log\left(1-y^{\frac{1}{\beta}}\right)^{\frac{1}{\tau}} \tag{22}$$

Using (20), (21) & (22) the quantile function for EWGED can be written as

$$Q(\lambda)=-\log\left[1-\left(1-e^{-\gamma(-\log(1-\lambda))^{\frac{1}{\alpha}}}\right)^{\frac{1}{\beta c}}\right]^{\frac{1}{\tau}} \tag{23}$$

Using the quantile function in (23), the skewness and kurtosis given by Galton [6] & Moors [10], for the EWGED can be obtained as

$$S=\frac{Q(3/4)-2Q(1/2)+Q(1/4)}{Q(3/4)-Q(1/4)}$$

$$K=\frac{Q(7/8)-Q(5/8)+Q(3/8)-Q(1/8)}{Q(6/8)-Q(2/8)}$$

## *4.5 Shannon entropy*

The entropy of a random variable X is a measure of variation of uncertainty given by Renyi [16]. Shannon entropy [19] for a random variable X with PDF $g(x)$ is defined as $E[-\log(g(x))]$. Shannon showed important applications of this entropy in communication theory and many applications have been used in different areas such as engineering, physics, biology and economics.

Alzaatreh et al [2] presented the Shannon entropy for the exponentiated T-X distribution as

$$\eta_X = -\log c - E\left[\log f\left(F^{-1}\left(1-e^{-T}\right)^{\frac{1}{c}}\right)\right] + \frac{1-c}{c}E\left(\log\left(1-e^{-T}\right)\right) - \mu_T + \eta_T \tag{24}$$

Shannon entropy for EWGED using (24) is given by

$$\eta_X = -\left(\log\left(\frac{\beta}{c}\right)\right) + \tau E\left[-\log\left(1-x^{\frac{1}{\beta}}\right)^{\frac{1}{\tau}}\right]^{\frac{1}{c}} - (\beta-1)E\left[\log\left(1-e^{-\tau\left(-\log\left(1-x^{\frac{1}{\beta}}\right)^{\frac{1}{\tau}}\right)}\right)^{\frac{1}{c}}\right] +$$

$$\frac{(1-c)}{c}\sum_{l=o}^{\infty}\frac{(-1)^{l+1}}{\gamma^{\alpha(l+1)}l!}\left[\sum_{m=1}^{\infty}\frac{\Gamma(\alpha(l+1))}{n^{\alpha(l+1)+1}}\right] + \gamma\left\{\Gamma\left(1+\frac{1}{\alpha}\right)+1-\frac{1}{\alpha}\right\} + \log\left(\frac{\gamma}{c\alpha}\right) + 1$$

## 5 Parameter Estimation

This section contains expression for the parameter estimation of exponentiatedweibull-generalized exponential distribution using the method of maximum likelihood. The likelihood function for the EWGED in (4) is given by

$$L(c,\alpha,\beta,\tau,\gamma) = \prod_{i=1}^{n}\frac{c\alpha\beta\tau e^{-\tau x}\left(1-e^{-\tau x}\right)^{\beta c-1}}{\gamma\left(1-\left(1-e^{-\tau x}\right)^{\beta c}\right)}\left[\frac{-\log\left(1-\left(1-e^{-\tau x}\right)^{\beta c}\right)}{\gamma}\right]^{\alpha-1}$$

$$\times\exp\left[-\left\{\frac{-\log\left(1-\left(1-e^{-\tau x}\right)^{\beta c}\right)}{\gamma}\right\}\right]^{\alpha} \tag{25}$$

The log-likekihood function of (25), is

$$\log L(c,\alpha,\beta,\tau,\gamma)=\ell(c,\alpha,\beta,\tau,\gamma) \quad =\sum_{i=1}^{n}\log\left[\frac{c\alpha\beta\tau e^{-\tau x}\left(1-e^{-\tau x}\right)^{\beta c-1}}{\gamma\left[1-\left(1-e^{-\tau x}\right)^{\beta c}\right]}\right]+$$

$$\sum_{i=1}^{n}\log\left[\left[\frac{-\log\left(1-\left(1-e^{-\tau x}\right)^{\beta c}\right)}{\gamma}\right]^{\alpha-1}\right]+$$

$$\sum_{i=1}^{n}\log\left[\exp\left[-\left\{\frac{-\log\left(1-\left(1-e^{-\tau x}\right)^{\beta c}\right)}{\gamma}\right\}\right]^{\alpha}\right]$$

$$\ell(c,\alpha,\beta,\tau,\gamma)=n\log c+n\log\alpha+n\log\beta+n\log\tau-\tau\sum_{i=1}^{n}x_i+(\beta c-1)\sum_{i=1}^{n}\log\left(1-e^{-\tau x_i}\right)-$$

$$n\log\gamma-\sum_{i=1}^{n}\log\left(1-\left(1-e^{-\tau x_i}\right)^{\beta c}\right)+(\alpha-1)\sum_{i=1}^{n}\log\left(-\log\left(1-\left(1-e^{-\tau x_i}\right)^{\beta c}\right)\right)+$$

$$n(\alpha-1)\log\gamma \quad +\sum_{i=1}^{n}\left[-\left\{\frac{-\log\left(1-\left(1-e^{-\tau x_i}\right)^{\beta c}\right)}{\gamma}\right\}\right]^{\alpha} \tag{26}$$

The derivatives of (26) with respect to c ,α ,β, τ and $\gamma$ are given by

$$\frac{\partial\ell}{\partial c}=\frac{n}{c}+\beta\sum_{i=1}^{n}\log\left(1-e^{-\tau x_i}\right)+\beta\sum_{i=1}^{n}\frac{\left(1-e^{-\tau x_i}\right)^{\beta c}}{1-\left(1-e^{-\tau x_i}\right)^{\beta c}}+$$

$$\beta(\alpha-1)\sum_{i=1}^{n}\frac{\left(1-e^{-\tau x_i}\right)}{\left(1-\left(1-e^{-\tau x_i}\right)\right)\left(-\log\left(1-\left(1-e^{-\tau x_i}\right)^{\beta c}\right)\right)}+$$

$$\sum_{i=1}^{n}\frac{\alpha\beta\left(1-e^{-\tau x_i}\right)}{\gamma\left(1-\left(1-e^{-\tau x_i}\right)\right)}\left[-\left\{\frac{-\log\left(1-\left(1-e^{-\tau x_i}\right)^{\beta c}\right)}{\gamma}\right\}\right]^{\alpha-1} \tag{27}$$

$$\frac{\partial\log L}{\partial\alpha}=\frac{n}{\alpha}+\sum_{i=1}^{n}\log\left(-\log\left(1-\left(1-e^{-\tau x_i}\right)^{\beta c}\right)\right)-n\log\gamma+$$

$$\alpha\sum_{i=1}^{n}\left[-\left\{\frac{-\log\left(1-\left(1-e^{-\tau x_i}\right)^{\beta c}\right)}{\gamma}\right\}\right]^{\alpha-1} \tag{28}$$

$$\frac{\partial \log L}{\partial \beta}=\frac{n}{\beta}+c\sum_{i=1}^{n}\log\left(1-e^{-\tau x_i}\right)+c\sum_{i=1}^{n}\frac{\left(1-e^{-\tau x_i}\right)^{\beta c}}{\left(1-\left(1-e^{-\tau x_i}\right)^{\beta c}\right)}-$$

$$c(\alpha-1)\sum_{i=1}^{n}\frac{\left(1-e^{-\tau x_i}\right)}{\left(1-\left(1-e^{-\tau x_i}\right)^{\beta c}\right)\log\left(1-\left(1-e^{-\tau x_i}\right)^{\beta c}\right)}-$$

$$\frac{\alpha c}{\gamma}\sum_{i=1}^{n}\frac{\left(1-e^{-\tau x_i}\right)^{\beta c}}{\gamma\left(1-\left(1-e^{-\tau x_i}\right)\right)}\left[-\left\{\frac{-\log\left(1-\left(1-e^{-\tau x_i}\right)^{\beta c}\right)}{\gamma}\right\}\right]^{\alpha-1} \tag{29}$$

$$\frac{\partial \ell}{\partial \tau}=\frac{n}{\tau}-\sum_{i=1}^{n}x_i+\tau(\beta c-1)\sum_{i=1}^{n}\frac{e^{-\tau x_i}}{\left(1-e^{-\tau x_i}\right)}+\beta c\tau\sum_{i=1}^{n}\frac{e^{-\tau x_i}\left(1-e^{-\tau x_i}\right)^{\beta c-1}}{\left(1-\left(1-e^{-\tau x_i}\right)^{\beta c}\right)}-$$

$$\beta c\tau(\alpha-1)\sum_{i=1}^{n}\frac{e^{-\tau x_i}\left(1-e^{-\tau x_i}\right)^{\beta c-1}}{\left(1-\left(1-e^{-\tau x_i}\right)^{\beta c}\right)\left(-\log\left(1-\left(1-e^{-\tau x_i}\right)^{\beta c}\right)\right)}-$$

$$\frac{\alpha\beta c\tau}{\gamma}\sum_{i=1}^{n}\frac{e^{-\tau x_i}\left(1-e^{-\tau x_i}\right)^{\beta c-1}}{\gamma\left(1-\left(1-e^{-\tau x_i}\right)^{\beta c}\right)}\left[-\left\{\frac{-\log\left(1-\left(1-e^{-\tau x_i}\right)^{\beta c}\right)}{\gamma}\right\}\right]^{\alpha-1} \tag{30}$$

$$\frac{\partial \ell}{\partial \gamma}=-\frac{n}{\gamma}-\frac{n(\alpha-1)}{\gamma}-\frac{\alpha}{\gamma^2}\sum_{i=1}^{n}\log\left(1-\left(1-e^{-\tau x_i}\right)^{\beta c}\right)\left[-\left\{\frac{-\log\left(1-\left(1-e^{-\tau x_i}\right)^{\beta c}\right)}{\gamma}\right\}\right]^{\alpha-1} \tag{31}$$

The above equations have complicated expressions which cannot be solved easily, so we use Newton Raphson numerical iterative method. Newton Raphton method [13] is a method to solve nonlinear equations. To estimate the parameters of EWGED setting (27-31) to zero, the MLE of $\hat{c},\hat{\alpha},\hat{\beta},\hat{\tau}$ and $\hat{\gamma}$ can be obtained by solving the resulting equations using Newton Raphson numerical procedure.

## 6 Conclusion

There has always been an interest in developing more flexible distributions in statistics. This paper defined a new flexible five-parameter Exponentiated Weibull-Generalized Exponential Distribution. Several properties of EWGED including hazard function, limiting behaviour, survival function, moments, quantile function, shannon·s entropy, skewness and kurtosis have been obtained in detail. Maximum likelihood method is proposed for estimating the parameters of exponentiatedweibull-generalized exponential distribution.

## Acknowledgement

The authors are highly thankful to the comments and suggestions made by associate editor which improved the quality of this research paper.

## References

[1] A. Alzaatreh, C. Lee & F .Femoye, A new method for generating families of continuous distributions, METRON **71**. 63-79 (2013a).

[2] C. Lee, F. Femoye, & A. Alzaatreh ,Exponentiated T-X Family of Distributions with some Applications, International Journal

of Statistics and Probability, **2**(3). 31-49 (2013b).

[3] AW .Gera, the modified exponentiated-weibull distribution for life-time modelling, Proceedings of the Annual Reliability and Maintainability Symposium, Philadelphia, 149-152 (1997).

[4] R.C. Gupta, P. Gupta & R. D Gupta, Modeling failure time data by Lehmaan alternatives, Communications in Statistics-Theory and Methods, **27**, 887-904 (1998), http://dx.doi.org/10.1080/03610929808832134.

[5] R.D.Gupta and D Kundu, Generalized exponential distributions, Australian New Zealand Journal of Statistics **41**, 173-188 (1999).

[6] F. Galton, Enquiries into Human Faculty and its Development, London: Macmillan & Company (1883).

[7] R.D. Gupta RD and D. Kundu, Generalized exponential distribution: existing results and some recent developments, J Stat Plann Inference, **137**:3537-3547 (2007).

[8] M. Hamdy Salem and Mahmoud Ali Selim, the Generalized Weibull-Exponential Distribution: Properties and Applications, International Journal of Statistics and Applications, **4**(2): 102-112 (2014).

[9] D. Kundu& R. D. Gupta, Exponentiated exponential family: an alternative to gamma and weibull, Biom J, **43**: 117-130 (2001).

[10] J.J Moors, A quantile alternative for kurtosis, Journal of the Royal Statistical Society D, **37**, 25-32 (1988), http://dx.doi.org/10.2307/2348376 .

[11] G.S. Mudholkar&D.K.Srivastava, Exponentiated Weibull family for analyzing bathtub failure data, IEEE Transactions on Reliability, **42**, 299-302 (1988), http://dx.doi.org/10.1109/24.229504.

[12] A.W.Marshall and I.Olkin, A new method for adding a parameter to a family ofdistributions with application to the exponential and weibull families, Biometrika, **84**:641-652 (1997).

[13] M.D Bressoud, a Radical Approach to Real Analysis, 2ndEdn, Cambridge Univ Press, Cambridge (2005).

[14] S. Nadarajah& S. Kotz, The exponentiatedFrechet distribution, InterStat (2003), retrieved from http://interstat.statjournals.net/YEAR/2003/abstracts/0312001.php .

[15] S. Nadarajah and S. kotz, the exponentiated type distributions, ActaAppl Math, **92**:97-111 (2006).

[16] A. Renyi, on measures of entropy and information. In: Proceedings of the Fourth Berkeley Symposium on Mathematical Statistics and Probability, I, university of California Press, Berkeley, 547-561 (1961).

[17] N. Saralees, M.C. Gauss &EMM Ortega, The exponentiated Weibull distribution: a survey, Stat Pap (2012), doi: 10.1007/s00362-012-0466-x.

[18] L. R. Smith, Maximum likelihood estimation in a class of nonregular cases, Biometrika, **72**(I), 67-90 (1985).

[19] C. E. Shannon, A mathematical theory of communication, Bell System Technical Journal, **27**, 379-432 (1948).

[20] J.Suriya and T. R Jan, T-X family of gamma-exponential distribution and its structural properties, Proceedings of International Conference on Advances in Computers, Communication and Electronic Engineering, university of Kashmir, 207-213 (2015).