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Mojgan Akbari
Department of Mathematics, Faculty of Mathematical Science, University of Guilan, Rasht, P.O.Box 1914, Iran, m_akbari@guilan.ac.ir

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Exact solutions of the coupled Higgs equation and the Maccari system using the modified simplest equation method

*Mojgan Akbari**

Department of Mathematics, Faculty of Mathematical Science, University of Guilan, Rasht, P.O.Box 1914, Iran

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Abstract: In this paper, the modified simplest equation method is successfully implemented to find travelling wave solutions of the coupled Higgs equation and the Maccari system. This method is direct, effective and easy to calculate, and it is a powerful mathematical tool for obtaining exact travelling wave solutions of the coupled Higgs equation and Maccari system and can be used to solve other nonlinear partial differential equations in mathematical physics.

Keywords: The modified simplest equation method, traveling wave solutions, homogeneous balance, solitary wave solutions, The coupled Higgs equation, The Maccari system.

1 Introduction

Consider the following coupled Higgs equation

$$u_{tt} - u_{xx} + |u|^2 u - 2uv = 0,$$
$$v_{tt} + v_{xx} - (|u|^2)_{xx} = 0. \quad (1)$$

Tajiri obtained N-soliton solutions to Eq. (1)in [1]. Zhao constructed more general traveling wave solutions of Eq.(1) in [2].
Recently, Attilio Maccari derived a new integrable (2+1)-dimensional nonlinear system [3]

$$iu_t + u_{xx} + uv = 0,$$
$$v_t + v_y + (|u|^2)_x = 0. \quad (2)$$

The integrability property was explicitly demonstrated and the Lax pairs were also obtained. Zhao also constructed more general traveling wave solutions of system Eq. (2) in [2].
In this work we apply the modified simplest equation method [4-10] to the coupled Higgs equation and Maccari system. The modified simplest equation method is one of the most powerful and direct methods for constructing solutions of nonlinear partial differential equations is the modified simplest equation method.

2 Modified simplest equation method

The modified simplest equation method is based on the assumptions that the exact solutions can be expressed by a polynomial in $\frac{F'}{F}$, such that $F = F(\xi)$ is an unknown linear ordinary equation to be determined later. This method consists of the following steps:

Step 1. Consider a general form of nonlinear partial differential equation (PDE)

$$P(u, u_x, u_t, u_{xx}, u_{xt}, \cdots) = 0. \quad (3)$$

Assume that the solution is given by $u(x,t) = U(\xi)$ where $\xi = x + ct$. Hence, we use the following changes:

$$\frac{\partial}{\partial t}(.) = c\frac{\partial}{\partial \xi}(.),$$
$$\frac{\partial}{\partial x}(.) = \frac{\partial}{\partial \xi}(.), \quad (4)$$
$$\frac{\partial^2}{\partial x^2}(.) = \frac{\partial^2}{\partial \xi^2}(.).$$

and so on for other derivatives. Using (4) changes the PDE (3) to an ODE

$$Q(U, U', U'', \cdots) = 0. \quad (5)$$

* Corresponding author e-mail: m_akbari@guilan.ac.ir

where $U = U(\xi)$ is an unknown function, Q is a polynomial in the variable U and its derivatives.

Step 2. We suppose that Eq. (5) has the following formal solution:

$$U(\xi) = \sum_{i=0}^{N} A_i(\frac{F'}{F})^i, \quad (6)$$

where a_i are arbitrary constants to be determined such that $A_N \neq 0$, while $F(\xi)$ is an unknown function to be determined later.

Step 3. We determine the positive integer N in (6) by balancing the highest order derivatives and the nonlinear terms in Eq.(5).

Step 4. We substitute (6) into (5), we calculate all the necessary derivatives $U', U'', \cdots$ and then we account the function $F(\xi)$. As a result of this substitution, we get a polynomial of $\frac{F'(\xi)}{F(\xi)}$ and its derivatives. In this polynomial, we equate with zero all the coefficients of it. This operation yields a system of equations which can be solved to find A_i and $F(\xi)$. Consequently, we can get the exact solution of Eq.(3).

3 Application the modified simplest equation method

In this section, we study the coupled Higgs equation and the Maccari system using the modified simplest equation method.

3.1 Coupled Higgs equation

Using the wave variables

$$u = e^{i\theta}U(\xi), \quad v = V(\xi), \quad \theta = px + rt, \quad \xi = x + ct. (7)$$

Substituting (7) into (1), we have

$$(c^2-1)U'' + (p^2 - r)U - 2UV + U^3 = 0,$$
$$(c^2+1)V'' - 2(U')^2 - 2UU'' = 0. \quad (8)$$

Integrating the second equation in the system and neglecting the constant of integration we find

$$(c^2+1)V = U^2. \quad (9)$$

Substituting (9) into the first equation of the system and integrating we find

$$(c^4-1)U'' + (c^2+1)(p^2-r^2)U + (c^2-1)U^3 = 0, \quad (10)$$

where prime denotes differentiation with respect to ξ. By balancing the highest order derivative term U'' with the nonlinear term U^3 in (10), we obtain $N = 1$ in (6). So we assume that Eq.(10) has solution in the form

$$U(\xi) = A_0 + A_1(\frac{F'}{F}), \quad A_1 \neq 0. \quad (11)$$

Using (11), we obtain

$$U^3 = A_0^3 + 3A_0^2A_1(\frac{F'}{F}) + 3A_0A_1^2(\frac{F'}{F})^2 + A_1^3(\frac{F'}{F})^3, \quad (12)$$

$$U'' = A_1(\frac{F'''}{F} - \frac{F'F''}{F^2} + 2(\frac{F'}{F})^3). \quad (13)$$

Substituting (11) to (13) into Eq. (10) and setting the coefficients of $F^j (j = 0, -1, -2)$ to zero, we obtain

$$(c^2+1)(p^2-r^2)A_0 + (c^2-1)A_0^3 = 0 \quad (14)$$

$$(c^4-1)A_1F''' + (c^2+1)(p^2-r^2)A_1F'$$
$$+ 3(c^2-1)A_0^2A_1F' = 0 \quad (15)$$

$$-3A_1(c^4-1)F'F'' + 3(c^2-1)A_0A_1^2F'^2 = 0 \quad (16)$$

$$2(c^4-1)F'^3 + (c^2-1)A_1^3F'^3 = 0 \quad (17)$$

Eqs. (14) and (17) directly imply following solutions:

$$A_0 = \pm\sqrt{\frac{(c^2+1)(p^2-r^2)}{1-c^2}}, \quad A_1 = \pm i\sqrt{2(c^2+1)}$$

Thus, Eqs. (15) and (16) become

$$(c^4-1)F''' - 2(c^2+1)(p^2-r^2)F' = 0, \quad (18)$$

$$F'' - i\sqrt{\frac{2(p^2-r^2)}{1-c^2}}F' = 0. \quad (19)$$

By substituting Eq. (19) into Eq. (18) we get

$$F''' - i\sqrt{\frac{2(p^2-r^2)}{1-c^2}}F'' = 0. \quad (20)$$

The general solution of Eq. (20) is

$$F(\xi) = a_0 + a_1\xi + a_2 e^{i\sqrt{\frac{2(p^2-r^2)}{1-c^2}}\xi}$$

where $a_i (i = 0, 1, 2)$ are arbitrary constants.
Thus, we have

$$U(\xi) = \pm\sqrt{c^2+1}\Big(\sqrt{\frac{p^2-r^2}{1-c^2}} + i\sqrt{2}\frac{a_1 + ia_2\sqrt{\frac{2(p^2-r^2)}{1-c^2}}e^{i\sqrt{\frac{2(p^2-r^2)}{1-c^2}}\xi}}{a_0 + a_1\xi + a_2 e^{i\sqrt{\frac{2(p^2-r^2)}{1-c^2}}\xi}}\Big),$$

$$V(\xi) = \Big(\sqrt{\frac{p^2-r^2}{1-c^2}} + i\sqrt{2}\frac{a_1 + ia_2\sqrt{\frac{2(p^2-r^2)}{1-c^2}}e^{i\sqrt{\frac{2(p^2-r^2)}{1-c^2}}\xi}}{a_0 + a_1\xi + a_2 e^{i\sqrt{\frac{2(p^2-r^2)}{1-c^2}}\xi}}\Big)^2$$

Now, the exact solution of Eq. (1) has the form

$$u(x,t) = \pm e^{i(px+rt)}\sqrt{c^2+1}\Big(\sqrt{\frac{p^2-r^2}{1-c^2}} + i\sqrt{2}\frac{a_1+ia_2\sqrt{\frac{2(p^2-r^2)}{1-c^2}}e^{i\sqrt{\frac{2(p^2-r^2)}{1-c^2}}(x+ct)}}{a_0+a_1(x+ct)+a_2e^{i\sqrt{\frac{2(p^2-r^2)}{1-c^2}}(x+ct)}}\Big),$$

$$v(x,t) = \Big(\sqrt{\frac{p^2-r^2}{1-c^2}} + i\sqrt{2}\frac{a_1+ia_2\sqrt{\frac{2(p^2-r^2)}{1-c^2}}e^{i\sqrt{\frac{2(p^2-r^2)}{1-c^2}}(x+ct)}}{a_0+a_1(x+ct)+a_2e^{i\sqrt{\frac{2(p^2-r^2)}{1-c^2}}(x+ct)}}\Big)^2$$

If $a_1 = 0$ and $a_0 = a_2 = 1$, we have

$$u(x,t) = \pm e^{i(px+rt)}\sqrt{\frac{(c^2+1)(p^2-r^2)}{1-c^2}} \times \tan\Big(\sqrt{\frac{p^2-r^2}{2(1-c^2)}}(x+ct)\Big)$$

$$v(x,t) = \frac{p^2-r^2}{1-c^2}\tan^2\Big(\sqrt{\frac{p^2-r^2}{2(1-c^2)}}(x+ct)\Big)$$

3.2 Maccari system

We next consider the Maccari system (2). Let us assume the travelling wave solution of (2) has the form

$$u = e^{i\theta}U(\xi),\ v = V(\xi),\ \theta = px+qy+rt,\quad \xi = x+y+ct. \tag{21}$$

Substituting (21) into (2), we have

$$U'' - (r+p^2)U + UV = 0,\quad (c+1)V' + 2UU'' = 0. \tag{22}$$

Integrating the second equation in the system and neglecting the constant of integration we find

$$-(c+1)V = U^2. \tag{23}$$

Substituting (23) into the first equation of the system and integrating we find

$$(c+1)U'' - (c+1)(r-p^2)U - U^3 = 0, \tag{24}$$

where prime denotes differentiation with respect to ξ. By using (6) and balancing U'' terms with U^3 in (24) gives

$$m+2 = 3m,$$

so that

$$m = 1.$$

So we assume that Eq.(24) has solution in the form

$$U(\xi) = A_0 + A_1\Big(\frac{F'}{F}\Big),\quad A_1 \neq 0. \tag{25}$$

Using (25), we obtain

$$U^3 = A_0^3 + 3A_0^2A_1\Big(\frac{F'}{F}\Big) + 3A_0A_1^2\Big(\frac{F'}{F}\Big)^2 + A_1^3\Big(\frac{F'}{F}\Big)^3, \tag{26}$$

$$U'' = A_1\Big(\frac{F'''}{F} - \frac{F'F''}{F^2} + 2\Big(\frac{F'}{F}\Big)^3\Big). \tag{27}$$

Substituting (25) to (27) into Eq. (24) and setting the coefficients of $F^j (j = 0,-1,-2)$ to zero, we obtain

$$-(c+1)(r-p^2)A_0 - A_0^3 = 0 \tag{28}$$

$$(c+1)A_1F''' - (c+1)(r-p^2)A_1F' - 3A_0^2A_1F' = 0 \tag{29}$$

$$-3(c+1)A_1F'F'' - 3A_0A_1^2F'^2 = 0 \tag{30}$$

$$2(c+1)A_1F'^3 - A_1^3F'^3 = 0 \tag{31}$$

Eqs. (28) and (31) directly imply following solutions:

$$A_0 = \pm\sqrt{(c+1)(p^2-r)},\quad A_1 = \pm\sqrt{2(c+1)}.$$

Thus, Eqs. (29) and (30) become

$$F''' - 2(p^2-r)F' = 0, \tag{32}$$

$$F'' + \sqrt{2(p^2-r)}F' = 0. \tag{33}$$

By substituting Eq. (33) into Eq. (32) we get

$$F''' + \sqrt{2(p^2-r)}F'' = 0. \tag{34}$$

The general solution of Eq. (34) is

$$F(\xi) = a_0 + a_1\xi + a_2e^{-\sqrt{2(p^2-r)}\xi}$$

where $a_i (i = 0,1,2)$ are arbitrary constants.
Thus, we have

$$U(\xi) = \pm\sqrt{c+1}\Big(\sqrt{p^2-r} + \sqrt{2}\frac{a_1 - a_2\sqrt{2(p^2-r)}e^{-\sqrt{2(p^2-r)}\xi}}{a_0+a_1\xi+a_2e^{-\sqrt{2(p^2-r)}\xi}}\Big),$$

$$V(\xi) = -\Big(\sqrt{p^2-r} + \sqrt{2}\frac{a_1 - a_2\sqrt{2(p^2-r)}e^{-\sqrt{2(p^2-r)}\xi}}{a_0+a_1\xi+a_2e^{-\sqrt{2(p^2-r)}\xi}}\Big)^2.$$

Now, the exact solution of Eq. (2) has the form

$$u(x,y,t) = \pm e^{i\theta}\sqrt{c+1}\Big(\sqrt{p^2-r} + \sqrt{2}\frac{a_1 - a_2\sqrt{2(p^2-r)}e^{-\sqrt{2(p^2-r)}(x+y+ct)}}{a_0+a_1(x+y+ct)+a_2e^{-\sqrt{2(p^2-r)}(x+y+ct)}}\Big),$$

$$v(x,y,t) = -\Big(\sqrt{p^2-r} + \sqrt{2}\frac{a_1 - a_2\sqrt{2(p^2-r)}e^{-\sqrt{2(p^2-r)}(x+y+ct)}}{a_0+a_1(x+y+ct)+a_2e^{-\sqrt{2(p^2-r)}(x+y+ct)}}\Big)^2.$$

If $a_1 = 0$ and $a_0 = a_2 = 1$, we have

$$u(x,t) = \pm e^{i(px+qy+rt)}\sqrt{(c+1)(p^2-r)} \times \tanh\left(\sqrt{\frac{p^2-r}{2}}(x+y+ct)\right),$$

$$v(x,t) = (r-p^2)\tanh^2\left(\sqrt{\frac{p^2-r}{2}}(x+y+ct)\right).$$

Conclusion

In this paper, the modified simplest equation method is applied successfully for solving the coupled Higgs equation and the Maccari system. The results show that this method is efficient in finding the exact solutions of nonlinear differential equations.

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