Journal of Statistics Applications & Probability
An International Journal

http://dx.doi.org/10.18576/jsap/060206

On Rough Interval Three Level Large Scale Quadratic Integer Programming Problem

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Received: 18 Oct. 2016, Revised: 2 Mar. 2017, Accepted: 3 Mar. 2017
Published online: 1 Jul. 2017

Abstract: This paper focuses on the solution of a three level large scale quadratic integer programming problem (TLLSQIPP) where there are some or all of rough coefficients in the objective function and that has block angular structure of the constraints. An algorithm based on interval method, Taylor's series, decomposition algorithm and branch and bound method is suggested to find a compromised solution for the problem under consideration. Then, the proposed algorithm is compared to Frank and Wolfe algorithm to demonstrate its effectiveness. Finally, a numerical illustrative example is given to clarify the main results developed in this paper.

Keywords: Large Scale Problems; Quadratic Programming; Rough Interval; Three–level Programming; Decomposition Algorithm.

MSC 2010: 90C06; 90C20; 90C99.

1 Introduction

Rough set theory has been demonstrated to be a proficient mathematical tool to incomplete knowledge. Pawlak [1] has defined a new methodology in rough set. In this methodology, any ambiguous concept is developed by lower approximations, upper approximations and the boundary region of a set.

Interval programming based on the interval analysis has been created as a helpful and basic method to deal with classificatory analysis of ambiguous concepts; the rough interval is used to deal with partially vague or poorly characterized parameters [2].

Interval method has two features. First, the results are in type of intervals. Second, the interval method doesn't ignore any part of solution region. Thus, the interval method gives us solution with high precision [3].

Multi-level programming (MLP) problem is a sequence of multiple optimization problems in which the constraint area of one is decided by the solution of other decision makers. The sequence of the play is very important and the decision of the upper-level limitations affects the decision of the lower-levels [4,5,6,7].

In large scale programming (LSP) problem, distributing the choice space among several planning subunits. These sub-units connect through a set of common constraints involving the choice variables of all the divisions. The remaining constraints can be allocated to each subunit, with each constraint including only the choice variable of a single subunit [8].

Quadratic programming (QP) is one of the most well-known models used in decision-making and in optimization problems. QP problem goes for minimizing (maximizing) a quadratic objective function subject to a set of linear constraints. If the coefficients of the objective function are exactly known crisp value, then these models can be solved by traditional algorithms, else interval method can be used to convert rough nature to crisp [9].

Integer programming (IP) problems are optimization problems that minimize or maximize the objective function in the limits of equality or inequality constraints and integer variables. More widely application of integer programming can be used to appropriately describe the decision problems on the management and effective use of resources in engineering technology, business administration and numerous other areas [10].

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Rough set theory, introduced by Pawlak [11], presented ambiguity, not by means of membership, but utilizing a boundary region of a set. The theory of rough set defined the approximation of a discretionary subset of a universe by two determinable or detectable subsets called lower and upper approximations.

Kryskiewice [12] utilized a rough set theory to incomplete has discovered many interesting applications. Tsumoto [13] utilized the idea of lower and upper approximation in rough sets theory, knowledge covered up in information systems may be unwound and developed in the form of decision rules.

Pal [14] see that the rough set approach seems to be of major significance to psychological sciences, particularly in the fields of machine learning, decision analysis and expert systems. Xu et al. [15] transformed from random rough nature into equivalent crisp model and introduced interactive method to get decision maker satisfying solution, using a random rough simulation technique which can act with random rough objective functions and constraints, grouping with the genetic algorithm.

Lu et al. [16] introduced the concept of rough interval to express dual uncertain information of many parameters and the related solution method presented to solve rough interval fuzzy linear programming (LP) problems. Alolyan [17] tackled LP problems with fuzzy parameters in the objective function and the constraints based on preference relations between explored intervals.

Lin [18] tackled constrained optimization problems using genetic algorithm with the rough set theory, which is known as the rough penalty genetic algorithm (RPGA), with the intend to adequately accomplish powerful solutions and to resolve constrained optimization problems.

Jana et al. [19] handled fuzzy rough multi-item economic production quantity (EPQ) model and developed constant demand. Infinite production rate has adaptability and dependability consideration in production process, demand dependent unit production cost and shortages under the limitations on capacity region, by geometric programming (GP) technique tackled the problem.

Saad et al. [20] presented an algorithm for solving a three-level quadratic programming, where some or all of its coefficients in the objective function are rough intervals. Omran et al. [21] presented an algorithm for solving a three level fractional programming problem with rough coefficient in constraints.

Ma et al. [10] proposed a new branch and bound algorithm through a series of improvements on the traditional branch and bound algorithm, which can be used to solve integer quadratic programming problems effectively and efficiently. This algorithm employed a new linear relaxation and bound method and a rectangular deep bisection method. At the same time, a rectangular reduction strategy is used to improve the approximation degree and speed up the convergence of the algorithm.

This paper is organized as follows: Section 2 formulates the model of a three level large scale quadratic integer programming problem with rough interval coefficients in the objective function. The theories used to transform rough interval to crisp model are obtained in section 3. Section 4 discusses Taylor's series transformation. Section 5 presents a decomposition algorithm for a three level large scale linear programming problems and constraint method. Section 6 involves the concepts of Frank and Wolfe algorithm. An algorithm followed by a flowchart for solving the proposed problem is suggested in Section 7 and Section 8. In addition, a numerical example is provided in Section 9 to clarify the results. Finally, conclusion and future works are reported in Section 10.

2 Problem Formulation and Solution Concept

A three level large scale quadratic integer programming problem with rough interval coefficients in the objective function (TLLSQIPPRIC) may be formulated as follows:

[First Level]

$$\underset{x_1,x_2}{Max}\ F_1\ (x) = \sum_{j=1}^{m} (\left[\underline{a}_j^L, \underline{a}_j^U\right], \left[\overline{a}_j^L, \overline{a}_j^U\right])x_j + \frac{1}{2}\ x_j{}^T(\left[\underline{a}_j^L, \underline{a}_j^U\right], \left[\overline{a}_j^L, \overline{a}_j^U\right])x_j, \tag{1}$$

Where $x_3,\ldots,x_m$ solves

[Second Level]

$$\underset{x_3,x_4}{Max}\ F_2\ (x) = \sum_{j=1}^{m} (\left[\underline{b}_j^L, \underline{b}_j^U\right], \left[\overline{b}_j^L, \overline{b}_j^U\right])x_j + \frac{1}{2}\ x_j{}^T(\left[\underline{b}_j^L, \underline{b}_j^U\right], \left[\overline{b}_j^L, \overline{b}_j^U\right])x_j, \tag{2}$$

Where $x_5,\ldots,x_m$ solves

[Third Level]

$$\underset{x_5,x_6}{Max}\ F_3\ (x) = \sum_{j=1}^{m} (\left[\underline{c}_j^L, \underline{c}_j^U\right], \left[\overline{c}_j^L, \overline{c}_j^U\right])x_j + \frac{1}{2}\ x_j{}^T(\left[\underline{c}_j^L, \underline{c}_j^U\right], \left[\overline{c}_j^L, \overline{c}_j^U\right])\ x_{j,} \tag{3}$$

Where $x_7,\ldots,x_m$ solves

Subject to

$$x \in G = \left\{x \in R^{m\times n}\,|Ax \le b, x \ge 0\right\}. \quad (4)$$

Where

$$\begin{aligned} G=\{a_{01}x_1+a_{02}x_2 \quad & +a_{0m}x_m \le b_0,\\ d_1x_1 \quad & \le b_1,\\ d_2x_2 \quad & \le b_2,\\ & d_mx_m \le b_m,\\ & x_1,\ldots,x_m \ge 0 \text{and integer}\}. \end{aligned}$$

In the above Problem (1)-(4), $x_j \in R^n, (j=1,2,...,m)$ be a real vector variables, $\left[\underline{a}_j^L, \underline{a}_j^U\right]$, $\left[\overline{a}_j^L, \overline{a}_j^U\right]$, $\left[\underline{b}_j^L, \underline{b}_j^U\right]$, $\left[\overline{b}_j^L, \overline{b}_j^U\right]$, $\left[\underline{c}_j^L, \underline{c}_j^U\right]$ *and* $\left[\overline{c}_j^L, \overline{c}_j^U\right]$ are $m\times m$ matrix of rough interval coefficients of the objective function for the three levels, G is the large scale linear constraint set where, $b=(b_0,...,b_m)^T$ is $(m+1)$ vector, $A=m\times mn$ is the coefficients constraints matrix, and $a_{01},...,a_{0m},d_1,...,d_m$ are constants.

Therefore $F_i : R^m \to R, (i=1,2,3)$ be the first level, the second level, and the third level objective function, respectively. Moreover, FLDM has x_1,x_2 indicating the first decision level integer choice, SLDM and TLDM have x_3,x_4 and x_5,x_6 indicating the second and the third decision level integer choice, respectively.

To tackle Problem (1)-(4) and to deal with rough nature using the interval method to transform the rough coefficients in the objective functions into crisp number presented in Section 3.

3 The Equivalent Crisp Model for TLLSQIPPRIC

To solve the large scale quadratic integer programming problem, where there are some or all of rough coefficients in the objective function, directly using the problem base form without transformation is very complex. Valuable studies have been introduced in the area of the large scale quadratic programming [9], which relied on indirect methods by dealing with linear programs derived from the original programming problems, whose the solutions will be approximated to the solution of the original problems without accuracy. Currently, the challenging task for academic research is to solve the quadratic programming problems using direct method to demonstrate the effectiveness of the indirect methods.

Conversion of the proposed problem into upper and lower approximation is usually a hard work for many cases, but transformation process needs the following definitions to be known:
Definition 1. [3]
Rough Interval (RI) can be considered as a qualitative value from vague concept defined on a variable x in R.
Definition 2. [3]
The qualitative value A is called a rough interval when one can assign two closed intervals A_* and A^* on R to it where $A_* \subseteq A^*$.
Definition 3. [3]
A_* and A^* are called the lower approximation interval (LAI) and the upper approximation interval (UAI) of A, respectively. Further, A is denoted by $A = (A_*$ and $A^*)$.
Definition 4. [3]
Consider all of the corresponding linear programming with interval coefficients (LPIC) and LP of Problem (1)-(4):

1. The interval $\left[F_*^L, F_*^U\right]$ $\left(\left[F^{*L}, F^{*U}\right]\right)$ is called the surely (possibly) optimal range of Problem (1)-(4), if the optimal range of each LPIC is a superset (subset) of $\left[F_*^L, F_*^U\right]$ $\left(\left[F^{*L}, F^{*U}\right]\right)$.
2. Let $\left[F_*^L, F_*^U\right]$ $\left(\left[F^{*L}, F^{*U}\right]\right)$ be surely an optimal range of Problem (1)-(4), then the rough interval $\left(\left[F_*^L, F_*^U\right]\ \left[F^{*L}, F^{*U}\right]\right)$ is called the rough optimal range of Problem (1)-(4).
3. The optimal solution of each corresponding LPIC of Problem (1)-(4) which its optimal value belongs to $\left[F_*^L, F_*^U\right]$ $\left(\left[F^{*L}, F^{*U}\right]\right)$ is called a completely satisfactory (rather) solution of Problem (1)-(4).

Now, the equivalent problem of the first level by using interval method [3] can obtained by getting the surely optimal range of Problem (1) and (4) by solving two large scale quadratic integer programming(LSQIP) as Follows [3]:

$$\underline{F}^L = max \sum_{j=1}^{m} \underline{a}_j^L x_j + \frac{1}{2} \; x_j{}^T \underline{a}_j^L x_j, \tag{5}$$

subject to
$x \in G.$

$$\underline{F}^U = max \sum_{j=1}^{m} \underline{a}_j^U x_j + \frac{1}{2} \; x_j{}^T \underline{a}_j^U x_j, \tag{6}$$

subject to
$x \in G.$

While the possibly optimal range of Problem (1) and (4) can obtain by solving two LSQIPs as follows [3]:

$$\overline{\mathrm{F}}^{\mathrm{L}} = max \sum_{j=1}^{m} \overline{\mathrm{a}}_{\mathrm{j}}^{\mathrm{L}} x_j + \frac{1}{2} \, x_j{}^T \overline{a}_j^L \, x_j, \tag{7}$$

subject to
$x \in G.$

$$\overline{\mathrm{F}}^{\mathrm{U}} = max \sum_{j=1}^{m} \overline{\mathrm{a}}_{\mathrm{j}}^{\mathrm{U}} x_j + \frac{1}{2} \, x_j{}^T \overline{a}_j^U x_j, \tag{8}$$

subject to
$x \in G.$

After using interval method [3] to convert Problem (1) and (4) for the first level from rough nature to crisp that resulted in four LSQIP problems. These steps will be repeated for second and third level, so the problem of TLLSQIPPRIC (1)-(4) converted into twelve LSQIP with four problems at each level. Then each level has his\her own optimal solution using Taylor's series and decomposition algorithm together with constraint method.

4 Taylor's Series Approach [9]

To solve LSQIP problem using decomposition algorithm very complex problem, Taylor series can overcome this problem by obtaining polynomial objective functions which are equivalent to quadratic objective functions.

$$H_i(x) \cong F_i(x_j^*) + \sum_{j=1}^{m} (x_j - x_j^*) \frac{\partial f_i(x_j^*)}{\partial x_j}, (i = 1, 2, 3). \tag{9}$$

So the equivalent large scale linear integer programming (LSLIP) problem can be written as:

$$Max \; H_i \left(x \right), \left(i = 1, 2, 3 \right), \tag{10}$$

Subject to

$x \in G.$

5 A Decomposition Algorithm for Three Level Large Scale Linear Integer Programming (TLLSLIP) Problem

To solve the TLLSLIP problem based on the decomposition algorithm [9] and constraint method. The FLDM gets the optimal solution using decomposition algorithm by breaking the large scale problem into n-sub problems that can be solved directly. Then by inserting the FLDM decision variable to the SLDM for him/her to seek the optimal solution using decomposition method. Finally, the TLDM does the same action till he/she obtains the optimal solution of his problem.

6 Frank and Wolfe algorithm [22]

This method deals with the following problem in which all constraints are linear:

$$Max \; Z = f\left(X \right), \tag{11}$$

Subject to

$$AX \leq b, X \geq 0.$$

Let X^kbe the feasible trial point at iteration k.the objective function $f(X)$can be expanded in the neighborhood of X^kusing Taylor series. This gives

$$f(X) \cong f\left(X^k\right) + \partial f\left(X^k\right)\left(X - X^k\right) = \left(f\left(X^k\right) - \partial f\left(X^k\right)X^k\right) + \partial f\left(X^k\right)X. \quad (12)$$

The procedure calls for determining a feasible point $X = X^*$ such that $f(X)$ is maximized subject to the linear constraints of the problem. Because $f(X^k) - \partial f(X^k)X^k$ is a constant, the problem for determining X^* reduces for solving the linear program:

$$Max\ w_k(X) = \partial f(X^k)X, \quad (13)$$

Subject to

$$AX \leq b, X \geq 0.$$

Given w_k is constructed from the gradient of $f(X)$ at X^k, an improved solution point can be secured if and only if $w_k(X^*) \succ w_k\left(X^k\right)$. From Taylor expansion, the condition does not that $f(X^*) \succ f\left(X^k\right)$unless X^* is in the neighborhood of X^k. However, given $w_k(X^*) \succ w_k\left(X^k\right)$, there must exist a point X^{k+1} on the line segment $\left(X^k, X^*\right)$ such that $f\left(X^{k+1}\right) \succ f\left(X^k\right)$. The objective is to determine X^{k+1}. Define

$$X^{k+1} = (1-r)X^k + rX^* = X^k + r\left(X^* - X^k\right), 0 \prec r \leq 1. \quad (14)$$

This means that X^{k+1} is a linear combination of X^k and X^*. Because X^k and X^* are two feasible point in a convex solution space, X^{k+1} is also feasible. The parameter r represents the step size.
The point X^{k+1} is determined such that $f(X)$ is maximized. Because X^{k+1} is a function of r only, X^{k+1} is determined by maximizing

$$h(r) = f\left(X^k + r\left(X^* - X^k\right)\right). \quad (15)$$

The procedure is repeated until, at the kth iteration, $w_k(X^*) \leq w_k\left(X^k\right)$ at this point , no further improvements are possible, and process terminates with X^k as the best solution point.

7 An Algorithm for Solving TLLSQIPPRIC

A solution algorithm to solve TLLSQIPPRIC is described in a series of steps. This algorithm uses interval method [3] to convert the interval rough parameters into real numbers to overcome the complexity nature of the proposed problem and uses the constraint method of the three level optimization to facility the large scale nature. Inserting the variables value of every higher level decision maker to his lower level decision maker break the difficulty faces the problem.

The suggested algorithm can be summarized in the following manner.

Step 1. The FLDM converts Problem (1) and (4) into Problems (5)-(8) by using interval method [3], which resulted in four LSQIP problems.

Step 2. Apply Taylor's series approach to obtain polynomial objective function in Formula (9), which results in four LSLIP problems.

Step 3. Use the decomposition algorithm [9] to solve the four LSLIP problems by breaking the large scale problem into n-sub problems that can be solved directly, then the optimal solution is reached.

Step 4. If the solution of the problem is integer optimal solution, go to Step 6, otherwise, go to Step 5.

Step 5. Using branch and bound method [10] to find integer optimal solution.

Step 6. If the SLDM obtains his optimal solution, then go to Step 8, otherwise $\left[\underline{x}_1^L, \underline{x}_1^U\right]$, $\left[\overline{x}_1^L, \overline{x}_1^U\right]$, $\left[\underline{x}_2^L, \underline{x}_2^U\right]$, $\left[\overline{x}_2^L, \overline{x}_2^U\right]$ must be assigned to the SLDM constraints.

Step 7. The SLDM converts Problem (2) and (4) into Problems (5)-(8) by using interval method [3], go to Step 2.

Step 8. If the TLDM obtains his optimal solution, then go to Step 10, otherwise $\left[\underline{x}_1^L, \underline{x}_1^U\right]$, $\left[\overline{x}_1^L, \overline{x}_1^U\right]$, $\left[\underline{x}_2^L, \underline{x}_2^U\right]$, $\left[\overline{x}_2^L, \overline{x}_2^U\right]$, $\left[\underline{x}_3^L, \underline{x}_3^U\right]$, $\left[\overline{x}_3^L, \overline{x}_3^U\right]$, $\left[\underline{x}_4^L, \underline{x}_4^U\right]$, $\left[\overline{x}_4^L, \overline{x}_4^U\right]$ must be assigned to the TLDM constraints.

Step 9. The TLDM converts Problem (3) and (4) into Problem (5)-(8) by using interval method [3], go to Step 2.

Step10. Set the optimal solution of the TLDM $\left[\underline{x}_1^L, \underline{x}_1^U\right]$, $\left[\overline{x}_1^L, \overline{x}_1^U\right]$, $\left[\underline{x}_2^L, \underline{x}_2^U\right]$, $\left[\overline{x}_2^L, \overline{x}_2^U\right]$, $\left[\underline{x}_3^L, \underline{x}_3^U\right]$, $\left[\overline{x}_3^L, \overline{x}_3^U\right]$, $\left[\underline{x}_4^L, \underline{x}_4^U\right]$, $\left[\overline{x}_4^L, \overline{x}_4^U\right]$, $\left[\underline{x}_5^L, \underline{x}_5^U\right]$, $\left[\overline{x}_5^L, \overline{x}_5^U\right]$, $\left[\underline{x}_6^L, \underline{x}_6^U\right]$, $\left[\overline{x}_6^L, \overline{x}_6^U\right]$ as the compromised solution of the TLLSQIPPRIC, then stop.

8 A Flowchart for Solving TLLSQIPPRIC

A flowchart to explain the suggested algorithm is described as follows:

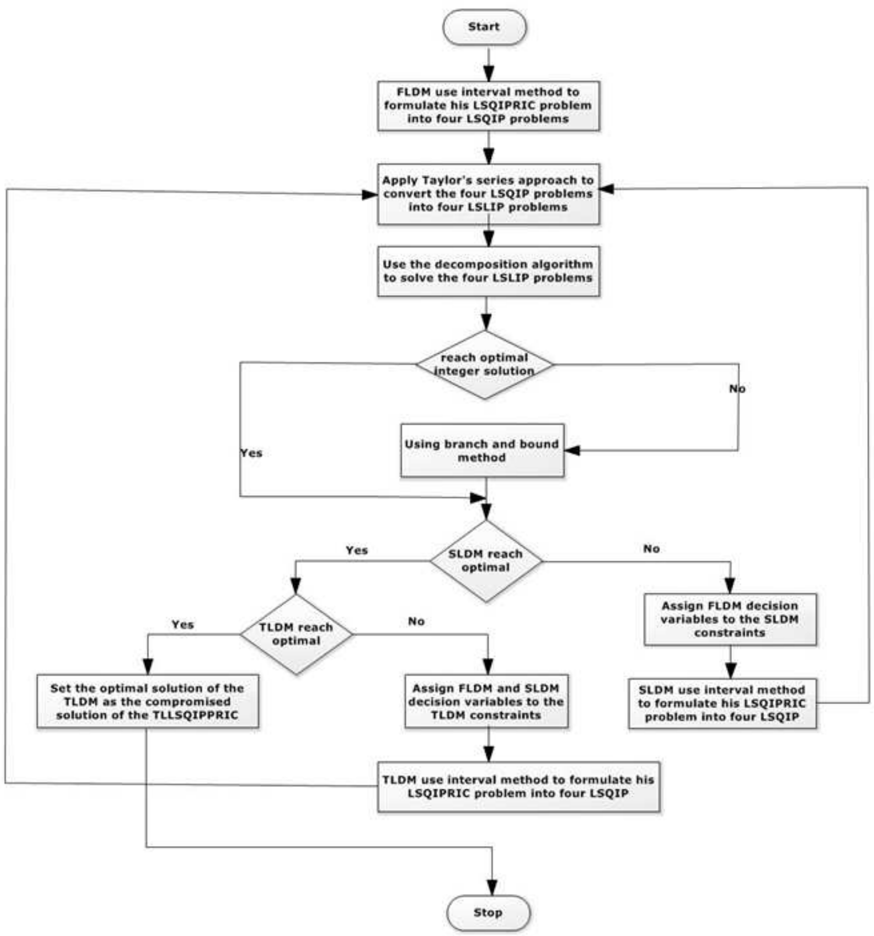

Fig. 1: flowchart to Explain The Suggested Algorithm.

Remark1. For TLLSQIPPRIC, the Lingo package is suggested as a basic solution tool.

9 Numerical Example

To demonstrate the solution method for TLLSQIPPRIC in the objective, let us consider the following numerical example:

[FLDM]

$$\underset{x_1,x_2}{Max}\ F_1(x) = \underset{x_1,x_2}{Max}\, 4([4,6],[3,7])x_1 - x_1^2 + 4([2,4],[1,4])x_2 - x_2^2 + 2x_4 + x_5 + x_6,$$

Where x_3, x_4, x_5, x_6 solves

[SLDM]

$$\underset{x_3,x_4}{Max}\ F_2(x) = \underset{x_3,x_4}{Max}\, x_1^2 + 3x_2 + 6([2,4],[1,6])x_3 - x_3^2 + ([0,3],[0,4])x_4 + x_6,$$

Where x_5, x_6 solves

[TLDM]

$$\underset{x_5,x_6}{Max}\ F_3(x) = \underset{x_5,x_6}{Max}\, 6x_1 + ([1,2],[1,3])x_2 + 4([3,4],[2,5])x_5 - x_5^2 + 2([5,7],[4,8])x_6 - x_6^2,$$

Subject to

$$\begin{aligned}
&x_1 + x_2 + x_3 + x_4 + x_5 + x_6 \le 50,\\
&2x_1 + x_2 \le 40,\\
&5x_3 + x_4 \le 12,\\
&x_5 + x_6 \le 20,\\
&x_5 + 5x_6 \le 80,\\
&x_1, x_2, x_3, x_4, x_5, x_6 \ge 0, x_1, x_2, x_3, x_4, x_5, x_6 \ge 0, \text{and integers.}
\end{aligned}$$

FLDM problem using Taylor's series and decomposition algorithms

The equivalent problem of the first level programming problem with rough coefficients in objective function by using interval method can be written as:-

Table 1: The Equivalent Problem of The FLDM using Interval Method.

Upper	**Lower**
P1: $Max 12x_1 - x_1^2 + 4x_2 - x_2^2 + 2x_4 + x_5 + x_6$ *Subject to* $x_1 + x_2 + x_3 + x_4 + x_5 + x_6 \le 50,$ $2x_1 + x_2 \le 40,$ $5x_3 + x_4 \le 12,$ $x_5 + x_6 \le 20,$ $x_5 + 5x_6 \le 80,$ $x_1, x_2, x_3, x_4, x_5, x_6 \ge 0.$	P3: $Max 16x_1 - x_1^2 + 8x_2 - x_2^2 + 2x_4 + x_5 + x_6$ *Subject to* $x_1 + x_2 + x_3 + x_4 + x_5 + x_6 \le 50,$ $2x_1 + x_2 \le 40,$ $5x_3 + x_4 \le 12,$ $x_5 + x_6 \le 20,$ $x_5 + 5x_6 \le 80,$ $x_1, x_2, x_3, x_4, x_5, x_6 \ge 0.$
P2: $Max 28x_1 - x_1^2 + 16x_2 - x_2^2 + 2x_4 + x_5 + x_6$ *Subject to* $x_1 + x_2 + x_3 + x_4 + x_5 + x_6 \le 50,$ $2x_1 + x_2 \le 40,$ $5x_3 + x_4 \le 12,$ $x_5 + x_6 \le 20,$ $x_5 + 5x_6 \le 80,$ $x_1, x_2, x_3, x_4, x_5, x_6 \ge 0.$	P4: $Max 24x_1 - x_1^2 + 16x_2 - x_2^2 + 2x_4 + x_5 + x_6$ *Subject to* $x_1 + x_2 + x_3 + x_4 + x_5 + x_6 \le 50,$ $2x_1 + x_2 \le 40,$ $5x_3 + x_4 \le 12,$ $x_5 + x_6 \le 20,$ $x_5 + 5x_6 \le 80,$ $x_1, x_2, x_3, x_4, x_5, x_6 \ge 0.$

Then, the objective functions of the FLDM in Table (1) are transformed by using 1^{st} order Taylor polynomial series to linear functions as follows:

Table 2: Transformation of The FLDM Objective Functions to Linear Functions.

Upper	**Lower**
IF $p_1(0,20,0,10,10,0) = -290$, then P1: $Max 12x_1 - 36x_2 + 2x_4 + x_5 + x_6 + 400$ *Subject to* $x \in G$.	IF $p_3(0,21,0,5,10,0) = -253$, then P3: $Max 16x_1 - 34x_2 + 2x_4 + x_5 + x_6 + 441$ *Subject to* $x \in G$.
IF $p_2(0,25,0,5,15,0) = -200$, then P2: $Max 28x_1 - 34x_2 + 2x_4 + x_5 + x_6 + 625$ *Subject to* $x \in G$.	IF $p_4(0,22,0,8,15,0) = -101$, then P4: $Max 24x_1 - 28x_2 + 2x_4 + x_5 + x_6 + 484$ *Subject to* $x \in G$.

After that, apply the decomposition algorithm on the FLDM to solve linear large scale integer programming problem in Table (2) and get the following results:

Table 3: Results of Applying The decomposition Algorithm on Linear Functions of The FLDM.

Upper	**Lower**
$\overline{F}_1^L = 682$, where $\left(x_1^F, x_2^F, x_3^F, x_4^F, x_5^F, x_6^F\right) = (20,0,0,12,18,0)$.	$\underline{F}_1^L$=803, where $\left(x_1^F, x_2^F, x_3^F, x_4^F, x_5^F, x_6^F\right) = (20,0,0,12,18,0)$.
$\overline{F}_1^U$=1227, where $\left(x_1^F, x_2^F, x_3^F, x_4^F, x_5^F, x_6^F\right) = (20,0,0,12,18,0)$.	$\underline{F}_1^U$=1006, where $\left(x_1^F, x_2^F, x_3^F, x_4^F, x_5^F, x_6^F\right) = (20,0,0,12,18,0)$.

SLDM problem using Taylor's series and decomposition algorithms

Now set $x_1^F = ([20,20],[20,20])$=20 and $x_2^F = ([0,0],[0,0]) = 0$ to the SLDM constraints. Then, the equivalent problem of the second level programming problem with rough interval coefficients in objective function by using interval method can be written as:

Table 4: The Equivalent Problem of The SLDM using Interval Method.

Upper	**Lower**
P1: $Max 6x_3 - x_3^2 + x_6 + 400$ *Subject to* $x_3 + x_4 + x_5 + x_6 \leq 30$, $5x_3 + x_4 \leq 12$, $x_5 + x_6 \leq 20$, $x_5 + 5x_6 \leq 80$, $x_3, x_4, x_5, x_6 \geq 0$.	P3: $Max 12x_3 - x_3^2 + x_6 + 400$ *Subject to* $x_3 + x_4 + x_5 + x_6 \leq 30$, $5x_3 + x_4 \leq 12$, $x_5 + x_6 \leq 20$, $x_5 + 5x_6 \leq 80$, $x_3, x_4, x_5, x_6 \geq 0$.
P2: $Max 36x_3 - x_3^2 + 4x_4 + x_6 + 400$ *Subject to* $x_3 + x_4 + x_5 + x_6 \leq 30$, $5x_3 + x_4 \leq 12$, $x_5 + x_6 \leq 20$, $x_5 + 5x_6 \leq 80$, $x_3, x_4, x_5, x_6 \geq 0$.	P4: $Max 24x_3 - x_3^2 + 3x_4 + x_6 + 400$ *Subject to* $x_3 + x_4 + x_5 + x_6 \leq 30$, $5x_3 + x_4 \leq 12$, $x_5 + x_6 \leq 20$, $x_5 + 5x_6 \leq 80$, $x_3, x_4, x_5, x_6 \geq 0$.

Then, do the same action on the SLDM and get the following results:

Table 5: Results of Applying The decomposition Algorithm on Linear Functions of The SLDM.

Upper	Lower
$\overline{F}_2^L = 426.6$, where $\left(x_3^S, x_4^S, x_5^S, x_6^S\right) = (2.4, 0, 0, 16)$. Apply branch and bound algorithm to get integer optimal solution. So, $\overline{F}_2^L$=425, where $\left(x_3^S, x_4^S, x_5^S, x_6^S\right) = (2, 0, 0, 16)$.	$\underline{F}_2^L$=441, where $\left(x_3^S, x_4^S, x_5^S, x_6^S\right) = (2.4, 0, 0, 16)$. Apply branch and bound algorithm to get integer optimal solution. So, $\underline{F}_2^L$=437, where $\left(x_3^S, x_4^S, x_5^S, x_6^S\right) = (2, 0, 0, 16)$.
$\overline{F}_2^U$=503.4, where $\left(x_3^S, x_4^S, x_5^S, x_6^S\right) = (2.4, 2, 0, 16)$. Apply branch and bound algorithm to get integer optimal solution. So, $\overline{F}_2^U$=497, where $\left(x_3^S, x_4^S, x_5^S, x_6^S\right) = (2, 2, 0, 16)$.	$\underline{F}_2^U$=469.8, where $\left(x_3^S, x_4^S, x_5^S, x_6^S\right) = (2.4, 0, 0, 16)$. Apply branch and bound algorithm to get integer optimal solution. So, $\underline{F}_2^U$=465, where$\left(x_3^S, x_4^S, x_5^S, x_6^S\right) = (2, 2, 0, 16)$.

TLDM problem using Taylor's series and decomposition algorithms

Now set $x_1^F = ([20,20],[20,20]) = 20, x_2^F = ([0,0],[0,0]) = 0, x_3^S = ([2,2],[2,2]) = 2$ and $x_4^F = ([0,2],[0,2])$ to the TLDM constraints. Then, the equivalent problem of the third level programming problem with rough interval coefficients in objective function by using interval method can be written as:

Table 6: The Equivalent Problem of The TLDM using Interval Method.

Upper	Lower
P1: $Max 8x_5 - x_5^2 + 8x_6 - x_6^2 + 120$ *Subject to* $x_5 + x_6 \leq 28$, $x_5 + x_6 \leq 20$, $x_5 + 5x_6 \leq 80$, $x_5, x_6 \geq 0$.	P3: $Max 12x_5 - x_5^2 + 10x_6 - x_6^2 + 120$ *Subject to* $x_5 + x_6 \leq 28$, $x_5 + x_6 \leq 20$, $x_5 + 5x_6 \leq 80$, $x_5, x_6 \geq 0$.
P2: $Max 20x_5 - x_5^2 + 16x_6 - x_6^2 + 120$ *Subject to* $x_5 + x_6 \leq 26$, $x_5 + x_6 \leq 20$, $x_5 + 5x_6 \leq 80$, $x_5, x_6 \geq 0$.	P4: $Max 16x_5 - x_5^2 + 14x_6 - x_6^2 + 120$ *Subject to* $x_5 + x_6 \leq 26$, $x_5 + x_6 \leq 20$, $x_5 + 5x_6 \leq 80$, $x_5, x_6 \geq 0$.

Then, do the same action on the TLDM and get the following results:

Table 7: Results of Applying The decomposition Algorithm on Linear Functions of The TLDM.

Upper	Lower
$\overline{F}_3^L = 250$, where $\left(x_5^T, x_6^T\right) = (20, 0)$.	$\underline{F}_3^L$=337, where $\left(x_5^T, x_6^T\right) = (20, 0)$.
$\overline{F}_3^U$=506,where $\left(x_5^T, x_6^T\right) = (20, 0)$.	$\underline{F}_3^U$=426, where $\left(x_5^T, x_6^T\right) = (20, 0)$.

FLDM problem using frank and Wolfe algorithm combined with decomposition algorithm

apply Frank and Wolfe algorithm combined with decomposition algorithm on the FLDM to solve quadratic large scale integer programming problem in Table (1) with direct steps and get the following results:

Table 8: Results of Applying Frank and Wolfe Algorithm Combined with Decomposition Algorithm on The FLDM.

Upper	Lower
$\overline{F}_1^L = 84$, where $\left(x_1^F, x_2^F, x_3^F, x_4^F, x_5^F, x_6^F\right) = (6,2,0,12,10,10)$.	$\underline{F}_1^L$=124, where $\left(x_1^F, x_2^F, x_3^F, x_4^F, x_5^F, x_6^F\right) = (8,4,0,12,10,10)$.
$\overline{F}_1^U$=300.5, where $\left(x_1^F, x_2^F, x_3^F, x_4^F, x_5^F, x_6^F\right) = (13.5,7.5,0,12,8.5,8.5)$. Apply branch and bound algorithm to get integer optimal solution. So, $\overline{F}_1^U$=300, where $\left(x_1^F, x_2^F, x_3^F, x_4^F, x_5^F, x_6^F\right) = (13,7,0,12,8,10)$.	$\underline{F}_1^U$=250.5, where $\left(x_1^F, x_2^F, x_3^F, x_4^F, x_5^F, x_6^F\right) = (11.5,7.5,0,12,9.5,9.5)$. Apply branch and bound algorithm to get integer optimal solution. So, $\underline{F}_1^U$=250, where $\left(x_1^F, x_2^F, x_3^F, x_4^F, x_5^F, x_6^F\right) = (11,7,0,12,9,11)$.

SLDM problem using frank and Wolfe algorithm combined with decomposition algorithm Now set $x_1^F = ([8,11]),[6,13])$, and $x_2^F = ([4,7]),[2,7])$ to the SLDM constraints. Then, the equivalent problem of the second level programming problem with rough interval coefficients in objective function by using interval method can be written as:

Table 9: The Equivalent Problem of The SLDM using Interval Method.

Upper	Lower
P1: $Max 6x_3 - x_3^2 + x_6 + 42$ *Subject to* $x_3 + x_4 + x_5 + x_6 \leq 42$, $5x_3 + x_4 \leq 12$, $x_5 + x_6 \leq 20$, $x_5 + 5x_6 \leq 80$, $x_3, x_4, x_5, x_6 \geq 0$.	P3: $Max 12x_3 - x_3^2 + x_6 + 76$ *Subject to* $x_3 + x_4 + x_5 + x_6 \leq 38$, $5x_3 + x_4 \leq 12$, $x_5 + x_6 \leq 20$, $x_5 + 5x_6 \leq 80$, $x_3, x_4, x_5, x_6 \geq 0$.
P2: $Max 36x_3 - x_3^2 + 4x_4 + x_6 + 190$ *Subject to* $x_3 + x_4 + x_5 + x_6 \leq 30$, $5x_3 + x_4 \leq 12$, $x_5 + x_6 \leq 20$, $x_5 + 5x_6 \leq 80$, $x_3, x_4, x_5, x_6 \geq 0$.	P4: $Max 24x_3 - x_3^2 + 3x_4 + x_6 + 142$ *Subject to* $x_3 + x_4 + x_5 + x_6 \leq 32$, $5x_3 + x_4 \leq 12$, $x_5 + x_6 \leq 20$, $x_5 + 5x_6 \leq 80$, $x_3, x_4, x_5, x_6 \geq 0$.

After that, apply Frank and Wolfe algorithm combined with decomposition algorithm on the SLDM to solve quadratic large scale integer programming problem in Table (9) with direct steps and get the following results:

Table 10: Results of Applying Frank and Wolfe Algorithm Combined with Decomposition Algorithm on The SLDM.

Upper	**Lower**
$\overline{F}_2^L = 66.64$, where $\left(x_3^S, x_4^S, x_5^S, x_6^S\right) = (2.4, 0, 0, 16)$. Apply branch and bound algorithm to get integer optimal solution. So, $\overline{F}_2^L$=66, where $\left(x_3^S, x_4^S, x_5^S, x_6^S\right) = (2, 0, 0, 16)$.	$\underline{F}_2^L$=115.04, where $\left(x_3^S, x_4^S, x_5^S, x_6^S\right) = (2.4, 0, 0, 16)$. Apply branch and bound algorithm to get integer optimal solution. So, $\underline{F}_2^L$=112, where $\left(x_3^S, x_4^S, x_5^S, x_6^S\right) = (2, 0, 0, 16)$.
$\overline{F}_2^U$=286.64, where $\left(x_3^S, x_4^S, x_5^S, x_6^S\right) = (2.4, 0, 0, 16)$. Apply branch and bound algorithm to get integer optimal solution. So, $\overline{\mathrm{F}}_2^{\mathrm{U}}$=282, where $\left(x_3^S, x_4^S, x_5^S, x_6^S\right) = (2, 2, 0, 16)$.	$\underline{F}_2^U$=209.84,where $\left(x_3^S, x_4^S, x_5^S, x_6^S\right) = (2.4, 0, 0, 16)$. Apply branch and bound algorithm to get integer optimal solution. So, $\underline{\mathrm{F}}_2^{\mathrm{U}}$=208, where$\left(x_3^S, x_4^S, x_5^S, x_6^S\right) = (2, 2, 0, 16)$.

TLDM problem using frank and Wolfe algorithm combined with decomposition algorithm

Now set $x_1^F = ([8,11]), [6,13]), x_2^F = ([4,7],[2,7]), x_3^S = ([2,2],[2,2]) = 2$and $x_4^F = ([0,2],[0,2])$ to the TLDM constraints. Then, the equivalent problem of the third level programming problem with rough interval coefficients in objective function by using interval method can be written as:

Table 11: The Equivalent Problem of The TLDM using Interval Method.

Upper	**Lower**
P1: $Max 8x_5 - x_5^2 + 8x_6 - x_6^2 + 38$ *Subject to* $x_5 + x_6 \leq 40$, $x_5 + x_6 \leq 20$, $x_5 + 5x_6 \leq 80$, $x_5, x_6 \geq 0$.	P3: $Max 12x_5 - x_5^2 + 10x_6 - x_6^2 + 52$ *Subject to* $x_5 + x_6 \leq 36$, $x_5 + x_6 \leq 20$, $x_5 + 5x_6 \leq 80$, $x_5, x_6 \geq 0$.
P2: $Max 20x_5 - x_5^2 + 16x_6 - x_6^2 + 99$ *Subject to* $x_5 + x_6 \leq 26$, $x_5 + x_6 \leq 20$, $x_5 + 5x_6 \leq 80$, $x_5, x_6 \geq 0$.	P4: $Max 16x_5 - x_5^2 + 14x_6 - x_6^2 + 80$ *Subject to* $x_5 + x_6 \leq 28$, $x_5 + x_6 \leq 20$, $x_5 + 5x_6 \leq 80$, $x_5, x_6 \geq 0$.

After that, apply Frank and Wolfe algorithm combined with decomposition algorithm on the TLDM to solve quadratic large scale integer programming problem in Table (11) with direct steps and get the following results:

Table 12: Results of Applying Frank and Wolfe Algorithm Combined with Decomposition Algorithm on The TLDM.

Upper	**Lower**
$\overline{F}_3^L = 70$, where $\left(x_5^T, x_6^T\right) = (4,4)$.	$\underline{F}_3^L$=113, where $\left(x_5^T, x_6^T\right) = (6,5)$.
$\overline{F}_3^U$=263, where $\left(x_5^T, x_6^T\right) = (10,8)$.	$\underline{F}_3^U$=193, where $\left(x_5^T, x_6^T\right) = (8,7)$.

Finally, getting the following results

Level	*The possibly using Taylor*	*The surely using Taylor*	*The possibly using Frank and Wolfe*	*The surely using Frank and Wolfe*
FLDM	[664,1209]	[785, 988]	[48, 280]	[95,225]
SLDM	[409,477]	[421, 451]	[54, 274]	[101,199]
TLDM	[250, 506]	[337,426]	[70, 263]	[113,193]

The proposed algorithm produces an approximated, in accurate, but fast solutions. These solutions can be used in fields such as agricultural decisions.

The Frank and Wolfe algorithm introduces accurate but slow solutions. These solutions can serve in fields such as medical and financial decisions.

10 Conclusion and Future Points

This paper suggested an algorithm to solve TLLSQIPPRIC. The suggested algorithm has used interval method at each level to define a crisp model, then all decision makers attempt to optimize their problems separately as a large scale quadratic programming using Dantzig and Wolfe decomposition method and Taylor's series together with constraint method. Then, compared the proposed algorithm to Frank and Wolfe algorithm to demonstrate its effectiveness

The solution algorithm has a few features:

1. It combines interval method, Taylor's series, decomposition algorithm, branch and bound and constraint method to obtain a compromised solution for the TLLSQIPPRIC.
2. The results are in the form of intervals and the interval method doesn't ignore any part of solution area.
3. It can be efficiently coded.

Finally, a numerical example was given to clarify the main results developed in this paper.

However, there are many other aspects, which should by explored and studied in the area of a large scale multi-level optimization such as:

1. Large scale multi-level fractional programming problem with rough interval parameters in the objective functions and in the constraints and with integrality conditions.
2. Large scale multi-level fractional programming problem with rough fuzzy number in the objective functions and in the constraints and with integrality conditions.
3. Large scale multi-level quadratic programming problem with rough fuzzy number in the objective functions and in the constraints and with integrality conditions.

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