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An Application of an Information System via modern Topology

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An Application of an Information System via Modern Topology

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***Abstract*- This paper will discuss developing a new mathema-tical method through analysis of information system where we deal with some ambiguity problems. We will represent this knowledge into one or more types of classes then we can use these classes to make a topology called modern Topology. After that we will compare the base of modern Topology with other basis after removing some attributes then we will decide the core elements via the comparison. These elements will obviously be the true elements in the Fact.**

Keywords: Rough set, rough approximations, data reduc-tion, Core.

I. INTRODUCTION

In this section we will explain some expression that we use in this paper. Every object is associated with ascertain amount of knowledge and the object can be expressed by means of some obtained information this we call indiscernible object thus we can form blocks by these objects called element sets or the knowledge granularity. The key issue is to compute Lower and Upper approximations. We can also combine this approach with any other approach to uncertainty. The knowledge used in approximation would divide the universe into classes derived with respect to the decision attribute. We can put these data that concerned the conditions and decision in data set table or data base table. The advantage of this branch that it doesn't need any preliminary information about data (like probability theory).

II. FUNDAMENTAL OF ROUGH SET MODEL

The main purpose of studying rough set analysis is to identify partial or total dependencies in data and remove redundant data. We start to define Pawlak's rough sets.

Let U be a finite set (universe) and R called equivalence (relation) or U. then $U/R=\{Y_1,Y_2,Y_3,...,Y_m\}$ on U, where $Y_1,Y_2,...,Y_m$ are the equivalence classes generated by R. Then also called "elementary sets" of R and ϕ is empty set, for any $X\subseteq U$.

We can discrete X be the elementary sets of R and the two sets

$$\underline{\text{apr}}(X)=\bigcup\{Y_i\in U/R\,|Y_i\subseteq X\},$$

$$\overline{\text{apr}}(X)=\bigcup\{Y_i\in U/R\,|Y_i\cap X\neq\phi\}$$

Then,

$$B_R(X)=(\overline{\text{apr}}(X))-(\underline{\text{apr}}(X)).$$

The $\underline{\text{apr}}(X)$ and $\overline{\text{apr}}(X)$ are called lower and upper approximation of X, respectively, $B_R(X)$ and is called Boundary region.

Definition 2.1 [4]. Let (U,R) be an approximation space and $X\subseteq U$ then the lower approximation of X with respect to (R) is the set of observations that can be classified into this concept where (R) naked indiscernibility relation.

$$\underline{\text{apr}}(X)=\bigcup\{M_i\in U/R\,|\,M_i\subseteq X\}$$

It can also be named Positive region and denoted by $\text{Pos}_B(X)$.

Definition 2.2 [4, 5]. Let (U,R) be an approximation space and $X\subseteq U$ then the upper approximation of X with respect to (R) is all nonempty sets of equivalence classes that intersection with (X) or we may say that it may belong to (X)

$$\overline{\text{apr}}(X)=\bigcup\{M_i\in U/R\,|\,M_i\cap X\neq\phi\}.$$

Definition 2.3. (Boundary region). Rough set theory expresses impression of imperfect knowledge by boundary region

$$B_R(X)=\overline{\text{apr}}(X)-\underline{\text{apr}}(X),$$

it consists of elements that we can not decisively classify into (X) or out side (X). If $B_R(X)=\phi$ then it will be well defined rules.

Definition 2.4. (Negative region). $\text{NEG}_R(X)=U-\underline{\text{apr}}(X)$ is a set without ambiguity it also called "complement of (X)".

We can understand Definitions (2.1)-(3.4) via Fig. 1.

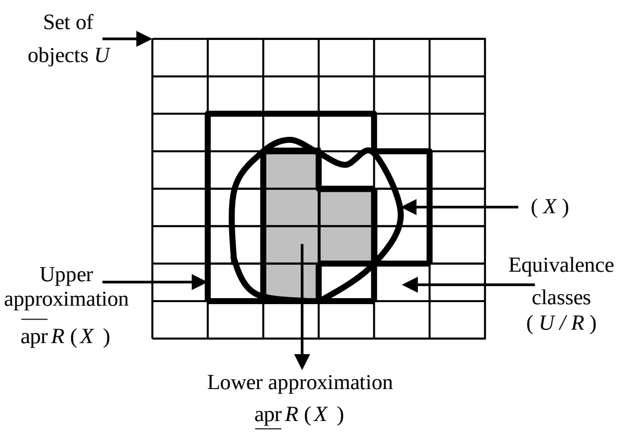

Fig. 1: A rough set in rough approximation space.

Proposition 2.1 [4, 5]. Let $(-X)$ be component of (X), then the following properties is true in rough set theory:

(1L) $\underline{apr}(U)=U$ → (Co-normality),

(1H) $\overline{apr}(U)=U$ → (Co-normality),

(2L) $\underline{apr}(\phi)=\phi$ → (Normality),

(2H) $\overline{apr}(\phi)=\phi$ → (Normality),

(3L) $\underline{apr}(X)\subseteq X$ → (Contraction),

(3H) $X\subseteq \overline{apr}(X)$ → (Extension),

(4L) $\underline{apr}(X\cap Y)=\underline{apr}(X)\cap \underline{apr}(Y)$ → (Multiplication),

(4H) $\overline{apr}(X\cup Y)=\overline{apr}(X)\cup \overline{apr}(Y)$ → (Addition),

(5L) $\underline{apr}(\underline{apr}(X))=\underline{apr}(X)$ → (Idempotency),

(5H) $\overline{apr}(\overline{apr}(X))=\overline{apr}(X)$ → (Idempotency),

(6L) $X\subseteq Y \Rightarrow \underline{apr}(X)\subseteq \underline{apr}(Y)$ → (Monotone),

(6H) $X\subseteq Y \Rightarrow \overline{apr}(X)\subseteq \overline{apr}(Y)$ → (Monotone),

(7L) $\underline{apr}(-\underline{apr}(X))=-\underline{apr}(X)$ → (Lower complem-ent relation),

(7H) $\overline{apr}(-\overline{apr}(X))=-\overline{apr}(X)$ → (Upper complem-ent relation),

(8LH) $\underline{apr}(-X)=-\overline{apr}(X)$ → (Duality),

(9LH) $\overline{apr}(-X)=-\underline{apr}(X)$ → (Duality),

(10L) $\forall\ K\in U/R$, $\underline{apr}(K)=K$ → (Granularity),

(10H) $\forall\ K\in U/R$, $\overline{apr}(K)=K$ → (Granularity).

The properties include all important properties of lower and upper approxim-ation and the other properties could be deduced from the above properties

Definition 2.5 [8]. (Modern topology). Let U be the universe (non-empty) set, R be an dependability relation on U and $\tau_R(X)=\{U,\phi,\underline{apr}(X),\overline{apr}(X),B_R(X)\}$; where $X\subseteq U$ satisfy these axioms:

(i) U and $\phi\in\tau_R(X)$.

(ii) The union of elements of any subcollection of $\tau_R(X)$ is in $\tau_R(X)$.

(iii) The intersection of any finite subcollection of $\tau_R(X)$ in $\tau_R(X)$, and the pair $(U,\tau_R(X))$ is called a modern topological space.

The elements of $\tau_R(X)$ is called (modern open set) in (L1) and the complement is called modern closed sets of (U) and dual modern topology of $\tau_R(X)$ is $[\tau_R(X)]^c$.

Remark 2.1. Thivager and Richard [7, 8] observed that the family $\beta=\{U;\ \overline{apr}(X);\ \overline{apr}(X);B_R(X)\}$ is the basis of $\tau_R(X)$ on U with respect to *X*.

Remark 2.2. Let $(U,\tau_R(X))$ be a modern topological space with respect to (X), $X\subseteq U$, and $R\rightarrow$ equivalence relation on U then U/R denotes the equivalence classes of (U) by (R).

Definition 2.6. If $(U,\tau_R(X))$ is modern topological space where $X\subseteq U$ and if $A\cap U$, then:

(i) The modern interior of (A) is the union of all modern open set contained in (A); $m\,\mathrm{Int}(A)$ that is the largest modern open subsets of (A).

(ii) The modern closure of the set A is defined as the intersection of all modern closed sets containing (A), $m\,cl(A)$, $m\,cl(A)$ is the smallest modern closed set containing (A).

1- One can define reduct as a smallest independent attribute subset that has the same equivalence relations as the overall attribute set. Thus it essential system of the information system to distinguish all objects in information system.

2- We also can define the comm on part of all reducts as (*A* core).

III. Fundamental of Rough Set Model

While studying information information systems a question faced is whether some of the condition attributes may be removed without altering the basic properties of the system, that is whether there is some superfluous data. Rough set model deduces rules by reducing the noaber of attributes. The process is referred as attribute reduction or is context of machine learning as feature selection [1, 2, 3, 6].

The main idea of redacts is to get the minimum possible subsets of attributes the preserves the information of interest. The procedure adopted is shown as in Fig. 2.

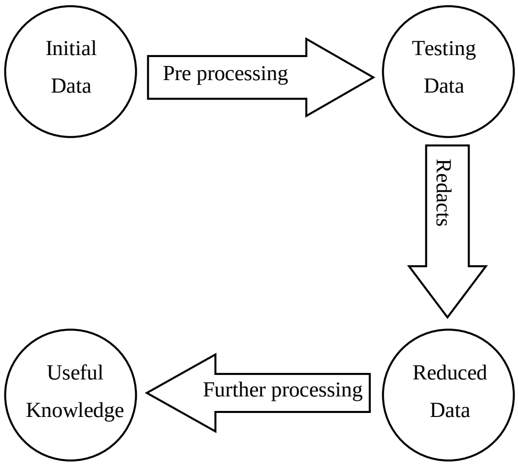

Fig. 2: Attribute reduction framework.

Analysis Example 3.1. knowledge system of patient with symptom of congenital Anomaly

Table 1. The effect of some symptoms of congenital anomaly disease on patients

U	a_1	a_2	a_3	a_4	a_5	a_6	Decision
P_1	No	Normal	Megalo	Yes	Yes	Long	Aarskog
P_2	Yes	Hyper	Megalo	Yes	Yes	Long	Aarskog
P_3	Yes	Hyper	Normal	No	No	Normal	Down
P_4	Yes	Hyper	Normal	No	No	Normal	Down
P_5	Yes	Hyper	Large	Yes	Yes	Long	Aarskog
P_6	No	Hyper	Megalo	Yes	No	Long	Cat-cry

where, a_1 = Round, a_2 = Telorism, a_3 = Cornea, a_4 = Slanting, a_5 = Iris defects, a_6 = Eyelashes.

$U/a_1 = \{\{P_1, P_6\}, \{P_2, P_3, P_4, P_5\}\}$,
$U/a_2 = \{\{P_1\}, \{P_2, P_3, P_4, P_5, P_6\}\}$,
$U/a_3 = \{\{P_1, P_2, P_6\}, \{P_3, P_4\}, \{P_5\}, \{P_6\}\}$,
$U/a_4 = \{\{P_1, P_2, P_5, P_6\}, \{P_3, P_4\}\}$,
$U/a_5 = \{\{P_1, P_2, P_5\}, \{P_3, P_4, P_6\}\}$,
$U/a_6 = \{\{P_1, P_2, P_5, P_6\}, \{P_3, P_4\}\}$.

(1) Now we begin with patients with Aarskog decision. $X_A = \{\{P_1, P_2, P_5\}\}$ where X_A (denoted patient with Aarsjog)

$U/\text{ind}(A) = \{\{P_1\}, \{P_2\}, \{P_3, P_4\}, \{P_5\}, \{P_6\}\}$
$= \{X_1, X_2, X_3, X_4, X_5\}$,
$\underline{\text{apr}}(X_A) = \{\{P_1, P_2, P_5\}\}$,
$\overline{\text{apr}}(X_A) = \{P_1, P_2, P_5\}$,
$Bnd(X_A) = \phi$.

This set (X_A) is crisp.

(2) When we take patients with (Down) decision.

$X_D = \{P_3, P_4\}$,
$\underline{\text{apr}}(X_D) = \{\{P_3, P_4\}\}$,
$\overline{\text{apr}}(X_D) = \{P_3, P_4\}$,
$Bnd(X_D) = \phi$.

This set also crisp (nont rough set).

(3) We will take patents with (Cat-cry) decision.

$X_C = \{P_6\}$.

This set is also cirsp when we try to make reduction of attributes (symptoms).

We first remove the first attribute (a_1)

$U/\text{ind}(A - a_1) = \{\{P_1\}, \{P_2\}, \{P_3, P_4\}, \{P_5\}, \{P_6\}\}$,
$U/\text{ind}(A) = U/\text{ind}(A - a_1)$.

So, (a_1) is not necessary for the (illness) $\rightarrow$ Aarskog, $a_1 \notin \text{Core}(A)$.

So, the attribute (a_1) (round) is redundant.

(4) Second, we will remove (a_2) from the Table (1),

$U/\text{ind}(A - a_2) = \{\{P_1\}, \{P_2\}, \{P_3, P_4\}, \{P_5\}, \{P_6\}\}$,

then

$U/\text{ind}(A - a_2) = U/\text{ind}(A)$.

So we can remove this attribute from the system.

The attribute (Telorism) is redundant $\rightarrow a_2 \notin \text{Core}(A)$.

(5) Thirdly, we will remove (a_3) from the attribute and calculate the indiscernible classes.

$U/\text{ind}(A - a_3) = \{\{P_1\}, \{P_2, P_5\}, \{P_3, P_4\}, \{P_6\}\}$,
$U/\text{ind}(A) \neq U/\text{ind}(A - a_3)$.

So, the attribute (Cornea) is essential attribute to know the patient with symptom of congenital Anomaly of Aarskog.

So $a_3 \in \text{Core}(A)$.

(6) We remove (a_4) from the attributes

$U/\text{ind}(A - a_4) = \{\{P_1\}, \{P_2\}, \{P_3, P_4\}, \{P_5\}, \{P_6\}\}$,

consequently

$U/\text{ind}(A) = U/\text{ind}(A - a_4)$.

So, $a_4 \notin \text{Core}(A)$.

(7) We remove (a_5) from the table [Iris defects], then

$U/\text{ind}(A - a_5) = \{\{P_1\}, \{P_2\}, \{P_3, P_4\}, \{P_5\}, \{P_6\}\}$,.

Thus,

$U/\text{ind}(A) = U/\text{ind}(A - a_5)$.

So (a_5) is redundant $[a_5 \notin \text{Core}(A)]$.

(8) We remove (a_6) from the Table (1) (Eyelashes) consequently

$U/\text{ind}(A - a_6) = \{\{P_1\}, \{P_2\}, \{P_3, P_4\}, \{P_5\}, \{P_6\}\}$,
$U/\text{ind}(A) = U/\text{ind}(A - a_6)$.

So $a_6 \notin \text{Core}(A)$.

From the above analysis we denote that the attribute (a_3) only is essential for the (illness).

(Cornea) is not indispensable or it is essential. Thus we can diagnose the (Aarskog) only from the attribute (Cornea),

Core (A) = [Cornea].

Also, this Core (A) is similar to that when we take $(X_D) \rightarrow$ patients with (Down) and $X_C \rightarrow$ patients with Cat-cry.

As knowledge granularity $(U/\text{ind}(A))$ doesn't depend on decision attributes.

Analysis Example 3.2. Consider the following information given in table 2 about five students (U) in a school having an exam in three different languages [English(E), French (F), Germany (G)], respectively.

From the previous table we have $U = \{S_1, S_2, S_3, S_4, S_5\}$ and the knowledge base is:

$U/R = \{\{S_1, S_4\}, \{S_2\}, \{S_3\}, \{S_5\}\}$.

Case 1: Let $X = \{S_3, S_4\}$ then one can deduce that

$\underline{\text{apr}}_R(X) = \{S_3\}$,
$\overline{\text{apr}}_R(X) = \{S_1, S_3, S_4\}$,
$Bnd_R(X) = \{S_1, S_4\}$,
$\beta(\tau_R(X)) = \{U, \{S_3\}, \{S_1, S_4\}\}$,

we begin to make reduction:

Step 1: If we remove the attribute "English" from the set of conditions, then

$U/R - E = \{\{S_1, S_4\}, \{S_2, S_3\}, \{S_5\}\}$,
$\underline{\text{apr}}_{R(A-E)}(X) = \{\phi\}$,
$\overline{\text{apr}}_{R(A-E)}(X) = \{S_1, S_2, S_3, S_4\}$,
$\beta(\tau_R(X)) = \{U, \phi, \{S_1, S_2, S_3, S_4\}\} \neq \beta_R[R(X)]$
$\Rightarrow E \in$ Core (conditions attribute).

Step 2: If we remove the attribute French from the set of conditions attribute, then

$U/R - F = \{\{S_1, S_4\}, \{S_2, S_5\}, \{S_3\}\}$,
$\underline{\text{apr}}_{R(A-F)}(X) = \{S_3\}$,
$\overline{\text{apr}}_{R(A-F)}(X) = \{S_1, S_3, S_4\}$,
$\beta(\tau_R(X)) = \{U, \phi, \{S_3\}, \{S_1, S_4\}\} = \beta_R(R(X))$,

$\Rightarrow F \notin$ Core (cond. attribute).

Step 3: If we remove the attribute Germany (G) from the set of conditions then:

$U/R-G=\{\{S_1,S_4\},\{S_2\},\{S_3\},\{S_5\}\}$,

$\underline{apr}_{R(A-G)}(X)=\{S_3\}$,

$\overline{apr}_{R(A-G)}(X)=\{S_1,S_3,S_4\}$,

$\beta(\tau_R(X))=\{U,\phi,\{S_3\},\{S_1,S_4\}\}=\beta_R(R(X))$,

$\Rightarrow G \notin$ Core (conditions attribute).

Observation: From the above example we conclude that English is the key attribute that it is necessary to decide weather "Student can pass the exam or not" and the other two languages are redundant.

Table 2. Exam results of three languages for five students

U Students	English (E)	French (F)	Germany (G)	Decision attribute results
Student 1	true	false	false	×
Student 2	false	true	true	×
Student 3	true	true	true	✓
Student 4	true	false	false	✓
Student 5	false	false	true	×

Table 3. The effect of some conditions on plans

U plants	Conditions attributes (A)				D
	Temperature (T)	Soil (S)	Water (W)	Sunlight (Su)	
P_1	Normal	Red	Medium	Normal	High
P_2	High	Red	Medium	High	Low
P_3	Normal	Sand	Medium	Normal	High
P_4	High	Loose soil	Medium	High	High
P_5	Normal	Red	Medium	Low	Low
P_6	Normal	Hard	Large	Low	High
P_7	High	Loose soil	Large	High	Low
P_8	High	Loose soil	Medium	High	Low

where D = Decision attribute (Production Yield).

Analysis Example 3.3. Let $U=\{P_1,P_2,P_3,P_4,P_5,P_6,P_7,P_8\}$ be the set of eighth plants, $A=\{$Temperature (T), Soil (S), Water (W), Sunlight $(Su)\}$ be the set of attributes (symptoms).

Case I: [Plants with High Production], $X=\{P_1,P_3,P_4,P_6\}$. Let (R) be the equivalence relation on U with respect to the condition attribute $A=\{T,S,Su,W\}$, then the classes relation determined by (R) corresponding to (A) is given by:

$U/R(A)=\{\{P_1\},\{P_2\},\{P_3\},\{P_4,P_8\},\{P_5\},\{P_6\},\{P_7\}\}$,

then one can deduce that: For the set of high production $X=\{P_1,P_3,P_4,P_6\}$

$\underline{apr}_{R(A)}(X)=\{P_1,P_3,P_6\}$,

$\overline{apr}_{R(A)}(X)=\{P_1,P_3,P_4,P_6,P_8\}$,

and

$B_R(X)=\overline{apr}(X)-\underline{apr}(X)=\{P_4,P_8\}$.

Hence

$\beta(\tau_R(X))=\{U,\{P_4,P_8\},\{P_1,P_3,P_6\}\}$

and

$\tau_R(X)=\{U,\phi,\{P_1,P_3,P_6\},\{P_1,P_3,P_4,P_6,P_8\},\{P_4,P_8\}\}$.

To (Make reduction).

Step 1: We can remove the attribute (T) from the set of attributes $A_1=\{S,Su,W\}$ then the equivalences classes corresponding to A_1 is

$U/(R(A_1))=\{\{P_1\},\{P_2\},\{P_3\},\{P_4,P_8\},\{P_5\},\{P_6\},\{P_7\}\}$.

Then, one can deduce that

$\underline{apr}_{R(A_1)}(X)=\{P_1,P_3,P_6\}$,

$\overline{apr}_{R(A_1)}(X)=\{P_1,P_3,P_4,P_6,P_8\}$,

$B_{R(A_1)}(X)=\{P_4,P_8\}$,

then

$\beta(\tau_{(R-T)}(X))=\{U,\{P_4,P_8\},\{P_1,P_3,P_6\}\}$.

So $\beta(\tau(X))=\beta(\tau_{R_T}(X))$. $\Rightarrow$ the attribute (T) (Temperature) is redundant and we can remove it from set of attributes. So,

$T \notin$ Core $\{A\}$. (1)

Consequently plant can live without temperature.

Step 2: We can remove soil (S) from conditions, then $A_2=\{T,Su,W\}$, and

$U/(R(A_2))=\{\{P_1,P_3\},\{P_2,P_4,P_8\},\{P_5\},\{P_6\},\{P_7\}\}$.

one can deduce that

$\underline{apr}_{R(A_2)}(X)=\{P_1,P_3,P_6\}$,

$\overline{apr}_{R(A_2)}(X)=\{P_1,P_2,P_3,P_4,P_6,P_8\}$,

$B_{R(A_2)}(X)=\{P_2,P_4,P_8\}$,

then

$\beta(\tau_{(R(A_2))}(X))=\{U,\{P_1,P_3,P_6\},\{P_2,P_4,P_8\}\}$.

So $\beta(\tau_{R(A_2)}(X))\neq\beta(\tau_{R(A)}(X))$. So the attribute soil (S) is not reduce or (S) is dispensible $S\in$Core, we deduce that plant can't live without soil.

Step 3: We will remove the attribute (water) (W) from the set of conditions $A_3=\{T,S,Su\}$, then

$U/R-W=U/R(A_3)=\{\{P_1\},\{P_3\},\{P_3\},\{P_4,P_7,P_8\},\{P_5\},\{P_6\}\}$.

Then one can deduce that

$\underline{apr}_{R(A_3)}(X)=\{P_1,P_3,P_6\}$,

$\overline{apr}_{R(A_3)}(X)=\{P_1,P_3,P_4,P_6,P_7,P_8\}$,

$B_{R(A_3)}(X)=\{P_4, P_7, P_8\}$,

So $\beta(\tau_{R(A_3)}(X))=\{U, \{P_1, P_3, P_6\}, \{P_4, P_7, P_8\}\} \neq \beta(\tau_{R(A)}(X))$, so, $W \in$ Core (A) and water is not redundant attribute.

We deduce that plant can't live without water.

Step 4: We can remove that last attribute sun light (Su) from set of attribute $A_4=\{W, S, T\}$, then

$$U/R(A_4)=\{\{P_1, P_5\}, \{P_2\}, \{P_3\}, \{P_4, P_8\}, \{P_6\}, \{P_7\}\},$$

and one can deduce that

$$\underline{\text{apr}}_{R(A_4)}(X)=\{P_3, P_6\},$$

$$\overline{\text{apr}}_{R(A_4)}(X)=\{P_1, P_3, P_4, P_5, P_6, P_8\},$$

$$B_{R(A_4)}(X)=\{P_1, P_4, P_5, P_8\},$$

$$\beta(\tau_{R(A_4)}(X))=\{U, \{P_3, P_6\}, \{P_1, P_4, P_5, P_8\}\} \neq \beta(\tau_{R(A)}(X)).$$

So, $Su \in$ Core (A) that is sunlight is not redundant attribute. We deduce that plant can not live without sunlight.

From the above analysis we have

$$\text{Core}(A)=\{S, W, Su\}.$$

Case II: Plants with low production. Let $X=\{P_2, P_5, P_7, P_8\}$ represents the set of plants with low production yield.

Let R be the set of equivalence relation on U with respect to the condition attribute $A=\{T, S, Su, W\}$ is given by

$$U/R(A)=\{\{P_1\}, \{P_2\}, \{P_3\}, \{P_4, P_8\}, \{P_5\}, \{P_6\}, \{P_7\}\}.$$

Then, one can deduce that

$$\underline{\text{apr}}_{R(A)}(X)=\{P_2, P_5, P_7\},$$

$$\overline{\text{apr}}_{R(A)}(X)=\{P_2, P_4, P_7, P_8\},$$

$$B_{R(A)}(X)=\{P_4, P_8\},$$

$$\beta(\tau_{R(A)}(X))=\{U, \{P_4, P_8\}, \{P_2, P_5, P_7\}\}.$$

When we start to make reduction to attribute set.

1st: We begin to remove the first attribute (temperature) $\Rightarrow$ $A_1=\{Su, W, S\}$,

$$U/R(A_1)=\{\{P_1\}, \{P_2\}, \{P_3\}, \{P_4, P_8\}, \{P_5\}, \{P_6\}, \{P_7\}\}=U/R(A).$$

Then one can deduce that:

$$\underline{\text{apr}}_{R(A_1)}(X)=\{P_2, P_5, P_7\},$$

$$\overline{\text{apr}}_{R(A_1)}(X)=\{P_2, P_4, P_5, P_7, P_8\},$$

$$B_{R(A_1)}(X)=\{P_4, P_8\},$$

$$\beta(\tau_{R(A_1)}(X))=\{U, \{P_4, P_8\}, \{P_2, P_5, P_7\}\} = \beta(\tau_{R(A)}(X)).$$

So, $T \notin$ Core (A).

Obviously, we can remove that attribute (T) from the set of attributes as it is not necessary for plants to give high production yield.

2nd: We begin to remove the second attribute set (soil) $A_2=\{T, Su, W\}$.

So ,

$$U/R(A_2)=\{\{P_1, P_3\}, \{P_2, P_4, P_8\}, \{P_5\}, \{P_6\}, \{P_7\}\}.$$

$$\underline{\text{apr}}_{R(A_2)}(X)=\{P_5, P_7\},$$

$$\overline{\text{apr}}_{R(A_2)}(X)=\{P_2, P_4, P_5, P_7, P_8\},$$

$$B_{R(A_2)}(X)=\{P_2, P_4, P_8\},$$

$$\beta(\tau_{R(A_1)}(X))=\{U, \{P_4, P_8\}, \{P_2, P_5, P_7\}\}$$

$$\beta(\tau_{R(A_2)}(X))=\{U, \{P_5, P_7\}, \{P_2, P_4, P_8\}\} \neq \beta(\tau_{R(A)}(X)).$$

So, soil is necessary for plants, we cannot remove it from attributes, so, $S \in$ Core (A).

3rd: We begin to remove the attribute (ware) (W), so

$$U/R(A_3)=\{\{P_1\}, \{P_2\}, \{P_3\}, \{P_4, P_7, P_8\}, \{P_5\}, \{P_6\}\},$$

Consequently,

$$\underline{\text{apr}}_{R(A_3)}(X)=\{P_2, P_5\},$$

$$\overline{\text{apr}}_{R(A_3)}(X)=\{P_2, P_4, P_5, P_7, P_8\},$$

and,

$$B_{R(A_3)}(X)=\{P_4, P_7, P_8\},$$

hence

$$\beta(\tau_{R(A_3)}(X))=\{U, \{P_1, P_5\}, \{P_4, P_7, P_8\}\} \neq \beta(\tau_{R(A)}(X)).$$

Then, $W \in$ Core (A).

So, we can't remove water as it is necessary for growing plants.

4th: We begin to remove the attribute sunlight from the set of attributes, so $A_4=\{T, S, W\}$. The equivalence classes of (U) with respect to A_4 is given by:

$$U/R(A_4)=\{\{P_1, P_5\}, \{P_2\}, \{P_3\}, \{P_4, P_8\}, \{P_6\}, \{P_7\}\}.$$

Subsequently we have

$$\underline{\text{apr}}_{R(A_4)}(X)=\{P_2, P_7\},$$

$$\overline{\text{apr}}_{R(A_4)}(X)=\{P_1, P_2, P_4, P_7, P_8\},$$

$$B_{R(A_4)}(X)=\{P_1, P_4, P_5, P_8\},$$

and

$$\beta(\tau_{R(A_4)}(X))=\{U, \{P_1, P_7\}, \{P_1, P_4, P_5, P_8\}\} \neq \beta(\tau_{R(A)}(X)).$$

So, $Su \in$ Core (A). Thus Core $(A)=\{S, W, Su\}$.

Observation: From both cases of the above example we observe that the Core(A) is found to be [soil, water, sunlight] which have close connection with plants in nature.

Plants must be inthese conditions to grow and give high production yield thus, [rough set model] can describe and interpret natural, physical and electrical phenomena.

IV. Conclusion and Future Work

We hope that this paper is just a beginning of a new structure. It will inspire many to contribute to the cultivation of modern topology in the field of mathematical structure of modern approximations. Practically, rough set theory can be viewed as a new method of intelligent data analysis. Rough set model has found many applications in medical data analysis, voice recognition, image processing and others. The proposed modern topology can be applied to more general and complex information systems for future research.

References

[1] F. Jhanf and Goliu, Rough relational operators and rough entropy relational database, In Second Int, Woehshop on Knowledge Discovery and Data Mining, 761-764, (2009).
[2] A. A. Nasef, A. I. Aggour and S. M. Darwish, On some classes of nearly open sets in nano topological spaces, Journal of the Egyptian Mathematical Society, 1-5, (2016).
[3] A. A. Nasef and M. K. El-Sayed, Rough set theory based on two relations and its application, Asian Journal of Mathematics and Applications, 1-7, (2017).
[4] Z. Pawlak, Rough sets, International Journal of Information and Computer Information Sciences, 11(5), 341-356, (1982).
[5] Z. Pawlak, Rough Sets, Theoretical Aspects of Reasoning About Data, Kluwer Academic Publishers, Boston, 1991.
[6] Q. Shen and R. Jensen, Rough sets, their extensions and applications, Int. Jour. of Automation and Comp. 4(3), 217-228, (2007).
[7] M. L. Thivager and C. Richard, On modern form of weakly open sets, International Journal of Mathematics and Statics Invention, 1(1), 31-37, (2013).
[8] M. L. Thivager, C. Richard and N. R. Poul, Mathematical Innovations of a Modern Topology in Hedical Events, International Journal of Information Sciences, 2(4), 33-36, (2012).