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Correlation Function and Business Cycle Turning Points: A Comparison with Markov Switching Approach

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Abstract: We present a new technical approach based on the autocorrelation function, widely used in physics, to determine and to analyze the business cycle turning points of an economic activity. This method is adapted to stochastic processes and does not require a smoothing technique. The application of this method to the industrial production seasonally adjusted of Tunisia, for the period 1994:4 –2006:8 gives similar results to these obtained by two-state Markov switching model.

Keywords: Business cycle, turning points, Markov switching models, correlation function.

1. Introduction

The notion of dating business cycles has a long history in economic researches. We notice that many empirical studies have proved that the business cycle is not periodic, and then there is no connection between the different phases of the cycle [4,3,5]. Furthermore, the determination of business cycle turning points is interesting for the policy interest centers on economic activity in any country in the world. So, dating the economic activity of a country is a very important issue. In fact, doing such work can help the experts and the economic agents to take the best decision. Dating cycles has two main objectives:

(i) Providing the basis for economic cyclical analysis.

(ii) Serving like a reference chronology to validate models aiming at "detecting" turning points.

For constructing a reference turning point chronology, we must choose a method which can detect the true points for the passage from a cyclical phase to another. For instance, starting from a single time series and applying two different methods can lead to different dating results. However, each method possesses its own advantages and drawbacks and it is not very clear which method should be used by practitioners, and that the dating methodology adds some noise to the signal, since dating depends on the chosen filter [5]. It may therefore be careful for the choice of the method of dating. Several works [6] are relevant to this specific topic, based on comparisons between the results computed and a reference dating chronology. Unfortunately, because of the lack of reference chronology, these works are specific only to the American economy. When a researcher develops a method to estimate the turning point chronology of a given country, the ultimate criteria to assess this method is to compare the resulting dating with a benchmark. However, in our paper, we tend to construct a reference dating chronology. Therefore, the assessment of diverse dating methods is not obvious. Some properties can help us to compare the proposed methods:

(i) Transparency: the dating method must be replicable to everyone.

(ii) Adaptability of the method to different series and countries.

(iii) Robustness to extreme values and to the sample.

(iv)The chronology must not be revised through time.

Medhioub [32,33] has proved that Markov switching models can capture the business cycle asymmetries of Tunisian economic activity. It was considered Markov switching regime models of two and three states to distinguish between different business cycle phases. It was concluded that, due to the property of business cycle asymmetries, the Tunisian

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business cycle can be estimated by the two-state switching models.

Contrary to the classical techniques, which use the techniques of smoothing for determining the peaks and the troughs; this proposed approach, much adapted to the stochastic processes, doesn't require a smoothing technique[1]. The idea behind this new method is fairly simple, and it is inspired from physics. To perform this approach, we use the notion of temporal delay and we consider all the pairs of points t and (t+τ) to determine the temporal correlation function [8–10, 31, 37, 28].

Section two of this paper is considered for presenting a brief review of the dating turning points while section three turns to the proposed approach which is based on the temporal correlation function. In section four we present and interpret the empirical results. Section five concludes the work.

2. Dating the turning points

A primordial question in which economic agents and policymakers are interested, concerns the detection and the forecast of the switch between the cyclical phases. To acquire such knowledge can be useful for the latter, in order to adjust their activities ahead of rises and of fallout of the economic cycles. In the United States of America, the National Bureau of Economic Research, NBER, is a reference organism which is interested in the determination of the different American economic activity phases.

The national offices are interested at the diffusion of the statistical information. This information has several well-known criteria, such as quality, exactitude, opportunity and uniformity. Among these practices it exists the identification issue of the business cycle turning points: dating, detecting and forecasting the turning points. These latter are a concept, which must be clearly defined. Arthur Okun, in his famous paper concerning the evaluation of the turning points [35], indicated that, “distinguishing wiggles from genuine turning points is a serious and difficult problem”. We need to find criteria to characterize the business cycles turning points, and to avoid focusing the fluctuations on the short term. Dating the cycles is adapted for two main objectives: first provide the essential for the cyclic economic analysis, and determine the chronology of reference to validate models “detecting” the turning points in a second way (such as the Markov-Switching model[20, 21].

In other words, a major problem of the analysis of the economic fluctuation is the detection of the business cycle turning points. We tend to determine the date where the economic activity changes from a phase to another (from a recession to an expansion or conversely). These two last decades, several econometric and statistic works were interested to develop new methods for the analysis of the business cycles which would make the problem of detecting the turning points more reliable. These methods concentrated on the concept of asymmetries of the cycles refer to the ideas of Burns and Mitchell [5] . In this paper, we try to apply a new tool which is very easy to adopt (we use the concept of correlation function [7, 11, 12]).

For the simplest case, generally considered, a business cycle is composed of two phases: a phase of expansion for which the economic activity tends to be reinforced, and a phase of recession for which the economic activity tends to weaken. The passage from one cyclical phase to another is marked by a peak or a trough. These extreme points represent in fact the turning points of the economic activity.

In this proposal, we tent to date cycle turning points by using a new methodology based on the temporal correlation function. This method is fairly simple to manipulate and it is based on two fundamental concepts: the correlation function and the notion of temporal delay between observations of time series. In the next section we expose this approach.

3. Markov switching model

A great panoply of techniques of non linear time series have been deployed to model the different economic cycle characteristics since the linear models cannot capture the cyclical asymmetries. A great importance is recently attached to the non linear specifications in which we have introduced a significant distinction between the expansion and the recession phases. These models are sufficiently flexible and they allow us to take into consideration the different specifications and the relations corresponding to each phase. Among these non-linear models, we can mention the autoregressive threshold models (Tar)[38], the SETAR models [38] and the regime switching models (Markov switching models) [18]. In this paper, we will focus on Markov switching models[2].

According to Hamilton, it is supposed that the economic growth is divided into two phases: a negative growth rate phase and a positive phase. He also supposes in his model that the economy shifts from a process to another is governed by a ”latent state” variable.

To capture these asymmetry concepts, we consider the two state Markov switching model, called MS(2)-AR(p), as developed by Hamilton [20]. By supposing a mean regime switching, the two state model for the output growth Δy_t can be expressed as:

$$\Delta y_t = \mu_{s_t} + \sum_{i=1}^{2} \varphi_i \left(\Delta y_{t-i} - \mu_{s_t-i}\right) + \varepsilon_t \tag{1}$$

$$\mu_{s_t} = \begin{cases} \mu_1 \text{ if } S_t = 0 \text{ (recession phase)} \\ \mu_2 \text{ if } S_t = 1 \text{ (expansion phase)} \end{cases} \text{[3]} \tag{2}$$

Where μ_{s_t} represent the mean growth rate corresponding

1 We use the correlation function which is not directly sensible to perturbations.

2 The Markov switching models are known for their empirical success to analyze business cycle asymmetries.

to the state S_t.This latter is an unobservable variable that is governed by a first order Markov process with fixed transition probabilities:

$$P\left[S_t = j \,/ S_{t-1} = i\right] = p_{ij} \qquad \forall i, j = 1,2 \qquad (3)$$

which satisfy: $\sum_{j=1}^{2} p_{ij} = 1$.

To estimate this model, Hamilton [21] has proposed to use the EM algorithm and to assign to each observation the state m which corresponds to the highest "smoothed" probability:

$$J^* = \arg\max_j P\left(S_t = j \,/\, \Psi_T\right) \qquad (4)$$

Hamilton has developed an algorithm decomposed in two stages. In the first stage, he estimated the parameters that corresponds to the joint probability density of the state variable. Secondly, filtered and smoothed probabilities are estimated. To date turning points, smoothed probabilities are used

$$P\left[S_t = j \,/\, y_T, y_{T-1}, \ldots, y_1\right]. \qquad (5)$$

The classification rule is very simple and consists in assigning each observation in the regime with the highest probability. In a more general way, the turning points dating are achieved as follows:

i- Peak at the date t if:

$$P\left[S_t = 2 \,/\, y_T, y_{T-1}, \ldots, y_1\right] > 0.5 \qquad (6)$$

and

$$P\left[S_{t+1} = 2 \,/\, y_T, y_{T-1}, \ldots, y_1\right] < 0.5 \qquad (7)$$

ii - Trough at the date t if:

$$P\left[S_t = 1 / \, y_T, y_{T-1}, \ldots, y_1\right] > 0.5 \qquad (8)$$

and

$$P\left[S_{t+1} = 2 \,/\, y_T, y_{T-1}, \ldots, y_1\right] < 0.5 \qquad (9)$$

4. The proposed method

Thanks to their advantages for exploring the internal dynamic of the studied system, the techniques of correlation function are used in several scientific fields such as electronics [30], telecommunication[24], cryptography[39], astrophysics[29], plasma physics[13], statistical mechanics [34], atomic physics [26–28] and condensed matter physics [14,15,19,16,17].
For stochastic processes, a correlation function is the correlation between random variables at different points [31]. Correlation function contains information about the distribution of points or events.
In astrophysics, Hunbury Brown and Twiss proposed for the first time the use of autocorrelation function in order to determine the angular distance for stern measurements [25].
This method proved a better accuracy of measurement compared to classical methods [25]. In This paper we propose to adapt the autocorrelation technique for detecting the business cycle turning points of economic activity.

The autocorrelation function can be defined for a random variable X with discrete time evolution as:

$$G(\tau) = E_t(x(t)x(t+\tau)) \qquad (10)$$

Where E represents the temporal mean.
Since we are interested in the determination of turning points, especially we need to detect the peaks and the troughs of the temporal series cycles. We define two new functions, namely:

$$Gmax(\tau) = [max_t(x(t)x(t+\tau))]/G(\tau) \qquad (11)$$
$$Gmin(\tau) = [min_t(x(t)x(t+\tau))]/G(\tau) \qquad (12)$$

From these two latter functions, we can determine the timing of passage from a cyclical phase to another.
According to the rules censoring of non parametric methods for dating, the following rules are considered[4]:

i- Peaks and troughs must alternate.
ii- A phase must last at least six months.
iii- A cycle must have a minimum duration of fifteen months[5] [4].

In the economic literature, we can determine the dating of turning points by using non parametric methods [22, 23] or parametric methods [1,2,6]. For example, Anas and Ferrara [1] had used Markov switching model (parametric method) and Bry and Boschan algorithm [4], (non parametric method) to detect the turning points of the Eurozone economy by using the seasonally adjusted industrial production index. They find that the two methods give very similar results. Medhioub [32] found that parametric and non parametric methods give very similar results for dating turning points of the Tunisian industrial production index. In the next section, a comparative assessment on our proposed method and parametric method based on the two-state Markov switching model for dating the Tunisian cycle is considered.

5. Empirical results

We try here to determine the expansion and recession periods of the Tunisian economic activity. In fact, in this country, which is developing, there doesn't exist any committee which gets interested in this kind of work, and even the

[4] Also, we can find in the literature some other censoring rules that may also be considered, like, in the presence of a double turning point, we choose the last turning point.

[5] These two latter rules are considered as the main censoring rules, which concern the duration of phases and cycles, developed by Bry and Boschan

economic agents and the political decision makers anticipate the crisis periods by intuition. So, there doesn't exist any advanced economic indicators neither coincident nor leading, so as to judge the nature of the activities.

In this paper, we use the time series corresponding to the index of the industrial production to determine the cyclical turning. The choice of this variable is due to two reasons: the first one is that, in most of the works, we have used this series as series reference of the economic activity. The second is the validity of these series observations into a monthly frequency. The data base used in this work corresponds to the monthly data of the industrial production seasonally adjusted covering the period 1994:04 - 2006:08[6].

Firstly, we apply our new method and we determine the temporal maximum and minimum correlation functions from which we find the turning points of the economic activity in Tunisia for the period 1994-2006. Secondly, for analyzing the performance of this method, we apply the methodology of Markov switching described above and we compare the results of these two methods.

In table 1, the dating of the industrial cycle turning points are presented.

Table1: Turning points dating

Correlation function		Switching model	
Peak	***Trough***	***Peak***	***Trough***
1997:01	1995:04	1997:03	1995:08
1995:05	1998:09	1999:02	1998:04
2001:08	2000:05	2001:07	2000:09
—	2003:12	2004:09	2003:12

Looking at the dates of peaks and troughs of these two methods, we can conclude a very strong concordance of the dates detected by the two methods. Only, the peak of September 2004 doesn't be detected by the temporal correlation function. In term of average duration of the two phases of business cycle, we can notice that we obtain very similar durations by the classical and this new method. Then we can conclude that this method can be considered as a good tool to detect the business cycle turning points of economic activity. So, this method can be also applied to the time series of developed countries to verify the robustness of this easy approach.

Furthermore, by using the temporal correlation function applied widely in physics we can easily analyze the fluctuation of the economic activity. The advantage which we can note in favor of this method in comparison to the non-parametric or the parametric methods that this new method is more adapted to the stochastic processes and does not require a smoothing technique[7].

[6] In most of the works concerning the cyclical asymmetries, we use this industrial production series as a reference series. Also, in this work, we have used these series because we don't have, up to this moment, monthly data or even quarterly data of the GDP.

6. Conclusion

Parametric and non parametric methods of dating were developed in order to determine the turning points of the business cycles such as the parametric algorithm based on the two state Markov switching approach. The most of parametric and non parametric methods are based on the concepts of smoothing and filtering data in order to extract signals for turning points. In this paper, we have proposed and applied a new method to detect the turning points of business cycles. The attractiveness of this methodology lies in the fact that it yields an automatic method for identifying the turning points much easy than the informal procedures used by the NBER.

The dating process that we propose here is, a result of a non parametric algorithm based on the temporal correlation function inspired from physics, which is not based on smoothing concept. By applying this method to the Tunisian industrial production index, seasonally adjusted series, we find results very closed to those obtained from two-state Markov switching model.

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