Applied Mathematics & Information Sciences
An International Journal

Multichannel ARMA signal covariance intersection fusion Wiener filter for the two-sensor system with time-delayed measurements

Peng Zhang[1,2] *and Zili Deng*[1]

[1] Department of Automation, Heilongjiang University, Harbin 150080, People's Republic of China
[2] Department of Computer and Information Engineering, Harbin Deqiang College of Commerce, Harbin 150025, People's Republic of China

Received: Aug. 12, 2012; Revised Nov. 28, 2012; Accepted Dec. 8, 2012
Published online: 1 Mar. 2013

Abstract: For the two-sensor, multi-channel autoregressive moving average (ARMA) signals with measurement delays, the system with measurement delays is converted into the system without measurement delays by the measurement transformation method. When the filtering error cross-covariances are known, by the Kalman filtering method, based on the white noise Wiener filters and measurement Wiener predictors, the optimal fusion Wiener signal filters weighted by matrices, diagonal matrices and scalars are presented. When the filtering error cross-covariances are unknown, by the covariance intersection (CI) fusion method, the CI fusion Wiener signal filter is presented. It is rigorously proven that the actual accuracy of the CI Wiener signal fuser is higher than that of each local Wiener signal filter, and is lower than that of the optimal Wiener signal fuser with the matrix weights. The geometric interpretation of the above accuracy relations are presented based on the covariance ellipses. A Monte-Carlo simulation example shows that the actual accuracy of the CI fuser is close to that of the fuser with the matrix weights, so that it has higher accuracy and good performance.

Keywords: covariance intersection fusion, measurement delays, unknown cross-covariance, Wiener filter, covariance ellipse.

1. Introduction

In order to improve the state or signal estimation accuracy based on the single sensor, the multi-sensor information fusion has received great attention in recent years, which has been widely applied to many fields, such as guidance, defence, robotics, GPS positioning and signal processing [1]. For Kalman filter-based data fusion, there exist two methodologies [2,3]: the state fusion and measurement fusion method. The state fusion methods weight the local state estimators based on various individual sensors to obtain an improved fused state estimator. Whereas measurement fusion methods directly fuse the sensor measurements to obtain a weighted measurement, and then use a single Kalman filter to obtain the final fused state estimators.

The ARMA signal estimation problem is an important topic in the signal processing field. The method to solve ARMA signal estimation problem include the Kalman filtering method [4,5], Wiener filtering method [6–8], and modern time series analysis method [9,10]. For the ARMA signals with a single sensor and white measurement noises, the optimal estimation problem was solved by the Wiener filtering method [6–8] and modern time series method [9, 10], the distributed optimal information fusion Wiener filters for the single channel or multichannel ARMA signals were presented by the modern time series analysis method [11] and by the Kalman filtering method [12].

However, the standard Kalman filter is only suitable for the systems without measurement delays, and the above results did not consider the time delays of the measurements. But today, the systems with measurement delays appear in many application fields [13]. There are two basic methods to solve the estimation problem of the systems with measurement delays: the augmented state method [4, 5] and non-augmented state method [13–16].

* Corresponding author: e-mail: dzl@hlju.edu.cn

The limitation of the above results is that the cross-covariances among the local estimation errors is required. However in many theoretical and application problems, the cross-covariances are unknown, or the algorithms of the cross-covariances are very complex, or the computational burden of the cross-covariances is very large [16–19], or to find the formulae for computing the cross-covariances is very difficult. In order to overcome this limitation, the covariance intersection (CI) fusion method was presented and developed in [20–23], which can solve the fusion estimation problems with unknown cross-covariances, avoid the computation of the cross-covariances, and is widely applied into many fields [24–27]. The CI fusion method has the advantages: it gives an upper bound of the actual estimation error variances with arbitrary cross-covariances by the convex combination, so it has the robustness with respect to unknown cross-covariances.

In this paper, for the two-sensor system with measurement delays and unknown cross-covariances, a CI steady-state Wiener signal fuser is presented and its actual filtering error variance formula is presented. The theoretical accuracy relations among the CI fusers and three weighting signal fusers are presented and proved based on the unbiased linear minimum variance (ULMV) estimation criterion [28,29]. The geometric interpretation of these accuracy relations are given based on the covariance ellipse concept [20,22,30]. In order to verify the correctness of the theoretical accuracy relations, a Monte-Carlo simulation example for a two-sensor multi-channel ARMA signal system is given, which gives the accuracy comparison of the CI fuser , the local and three weighting fusers.

2. Problem formulation

Consider the two-sensor multi-channel ARMA signals with measurement delays

$$A(q^{-1})s(t) = C(q^{-1})w(t) \tag{1}$$

$$z_i(t) = s(t-\tau_i) + \xi_i(t), i = 1,2 \tag{2}$$

where t is the discrete time, $z_i(t) \in R^m$ are the measurement, $\tau_i \geq 0$ are the time delays of the ith sensor, $s(t) \in R^m$ is the ARMA signals to be estimated. $w(t) \in R^r$ and $\xi_i(t) \in R^m$ are the white input and measurement noises, respectively. q^{-1} is the backward shift operator $q^{-1}s(t) = s(t-1)$, and $A(q^{-1})$ and $C(q^{-1})$ are polynomial matrices with the following form

$$\begin{aligned} &A(q^{-1}) = I_m + A_1 q^{-1} + \cdots + A_{n_a} q^{-n_a}, \\ &A_i = 0, i > n_a \end{aligned} \tag{3}$$

$$\begin{aligned} &C(q^{-1}) = C_1 q^{-1} + \cdots + C_{n_c} q^{-n_c}, \\ &C_0 = 0, C_i = 0, i > n_c \end{aligned} \tag{4}$$

where I_m is the $m \times m$ identity matrix, and we assume that $n_a \geq n_c$.

Assumption 1 $w(t)$ and $\xi_i(t)$ are uncorrelated white noises with zero means and variances Q_w and $Q_{\xi i}$, respectively, i.e.

$$\mathrm{E}\left\{ \begin{bmatrix} w(t) \\ \xi_i(t) \end{bmatrix} \begin{bmatrix} w^{\mathrm{T}}(k) \ \xi_j^{\mathrm{T}}(k) \end{bmatrix} \right\} = \begin{bmatrix} Q_w & 0 \\ 0 & Q_{\xi i}\delta_{ij} \end{bmatrix} \delta_{tk} \tag{5}$$

where $i = 1,2$, E and T denote the expectation and transpose, respectively. δ_{tk} is the Kronecker delta function, $\delta_{ii} = 1, \delta_{tk} = 0 (t \neq k)$.

Assumption 2 $(A(q^{-1}), C(q^{-1}))$ are left-coprime.

The aim is to find the local steady-state optimal Wiener signal filters $\hat{s}_i^z(t|t+N)$ of $s(t)$, $i = 1,2, N > 0, N = 0, N < 0$, which can be obtained by the local white noise estimators and the measurement predictors, and then to find the optimal weighting fusion Wiener signal filter $\hat{s}_m^z(t|t+N)$, $\hat{s}_s^z(t|t+N)$,$\hat{s}_d^z(t|t+N)$, which means the fusers weighted by matrices, scalars, diagonal matrices, respectively. Based on the CI method, the CI fusion Wiener signal filter $\hat{s}_{CI}^z(t|t+N)$ can be obtained.

3. Local steady-state optimal Wiener signal filters

The ARMA signal $s(t)$ can be converted into the state space model[4]

$$x(t+1) = \Phi x(t) + \Gamma w(t) \tag{6}$$

$$z_i(t) = Hx(t-\tau_i) + \xi_i(t), \ i = 1,2 \tag{7}$$

$$s(t) = Hx(t) \tag{8}$$

where

$$\Phi = \begin{bmatrix} -A_1 & & \\ \vdots & & I_{m(n_a-1)} \\ -A_{n_a} & 0 \cdots & 0 \end{bmatrix}, \Gamma = \begin{bmatrix} C_1 \\ \vdots \\ C_{n_a} \end{bmatrix}, \tag{9}$$

$$H = \begin{bmatrix} I_m & 0 \cdots 0 \end{bmatrix}$$

So the problem of signal estimation with measurement delays is converted into that of the state estimation with measurement delays.

In order to transform the systems with measurement delays into that without measurement delays, introducing the new measurements and noises

$$y_i(t) = z_i(t+\tau_i), v_i(t) = \xi_i(t+\tau_i) \tag{10}$$

So we have the measurement equation without measurement delays

$$y_i(t) = Hx(t) + v_i(t), \ i = 1,2 \tag{11}$$

Let linear space spanned by the stochastic variables $(z_i(t+N), z_i(t+N-1), \cdots)$ be denoted by $L(z_i(t+N), z_i(t+N-1), \cdots)$, and the linear space spanned by the stochastic variables $(y_i(t+N-\tau_i), y_i(t+N-\tau_i-1), \cdots)$ be denoted by $L(y_i(t+N-\tau_i), y_i(t+N-\tau_i-1), \cdots)$. Then we have the relation

$$\begin{aligned} &L\ (z_i(t+N), z_i(t+N-1), \cdots) \\ &= L(y_i(t+N-\tau_i), y_i(t+N-\tau_i-1), \cdots) \end{aligned} \tag{12}$$

Let linear minimum variance estimators based on the measurements $(y_i(t+N-\tau_i), y_i(t+N-\tau_i-1), \cdots)$ be denoted by $\hat{s}_i(t|t+N-\tau_i)$, and the linear minimum variance estimators based on the $(z_i(t+N), z_i(t+N-1), \cdots)$ be denoted by $\hat{s}_i^z(t|t+N)$. From (12) we have the relation

$$\hat{s}_i^z(t|t+N) = \hat{s}_i(t|t+N-\tau_i), N \geq 0, N < 0, i = 1,2 \quad (13)$$

Therefore the relation between the estimation errors $\tilde{s}_i^z(t|t+N) = s(t) - \hat{s}_i^z(t|t+N)$ and $\tilde{s}_i(t|t+N-\tau_i) = s(t) - \hat{s}_i(t|t+N-\tau_i)$ is given by

$$\tilde{s}_i^z(t|t+N) = \tilde{s}_i(t|t+N-\tau_i), N \geq 0, N < 0, i = 1,2 \quad (14)$$

Define the steady-state cross-covariance matrices and auto-covariance matrices among the estimation errors as

$$P_{ii}^z(N) = \mathrm{E}[\tilde{s}_i^z(t|t+N)\tilde{s}_i^{z\mathrm{T}}(t|t+N)] \quad (15)$$

$$P_{ij}(N-\tau_i, N-\tau_j) = \mathrm{E}[\tilde{s}_i(t|t+N-\tau_i)\tilde{s}_j^{\mathrm{T}}(t|t+N-\tau_j)] \quad (16)$$

$$P_{ii}(N-\tau_i) = P_{ii}(N-\tau_i, N-\tau_i) \quad (17)$$

$$P_{ij}^z(N) = P_{ij}(N-\tau_i, N-\tau_j) \quad (18)$$

Hence the problem is converted into that of finding the multi-step local optimal Wiener estimators$\hat{s}_i(t|t+N-\tau_i)$,$N \geq 0, N < 0, \tau_i > 0, i = 1,2$ of the signal $s(t)$ using the measurements $(y_i(t+N-\tau_i), y_i(t+N-\tau_i-1), \cdots)$ for the system (6), (8) and (11) without measurement delays. From (13), we can obtain the local filters $\hat{s}_i^z(t|t+N)$ using the measurements $(z_i(t+N), z_i(t+N-1), \cdots), i = 1,2$.

From (8) and (11), we have

$$y_i(t) = s(t) + v_i(t), \ i = 1,2 \quad (19)$$

According to the projection theory[5] yields the relation

$$\hat{s}_i(t|t+N-\tau_i) = \hat{y}_i(t|t+N-\tau_i) - \hat{v}_i(t|t+N-\tau_i), i = 1,2 \quad (20)$$

Thus the problem is converted into that of finding the white noise estimators $\hat{v}_i(t|t+N-\tau_i)$ and the measurement estimators $\hat{y}_i(t|t+N-\tau_i)$. They can be obtained by the following Kalman filtering method.

For the two-sensor system (6) and (11) with assumptions 1 and 2, the local steady-state Kalman predictor $\hat{x}_i(t+1|t)$ is given by[5]

$$\hat{x}_i(t+1|t) = \Psi_{pi}\hat{x}_i(t|t-1) + K_{pi}y_i(t), i = 1,2 \quad (21)$$

$$\Psi_{pi} = \Phi - K_{pi}H, \quad K_{pi} = \Phi\Sigma_{ii}H^{\mathrm{T}}Q_{\varepsilon i}^{-1} \quad (22)$$

$$\varepsilon_i(t) = y_i(t) - H\hat{x}_i(t|t-1) \quad (23)$$

$$Q_{\varepsilon i} = H\Sigma_{ii}H^{\mathrm{T}} + Q_{vi}, \quad Q_{vi} = Q_{\xi i} \quad (24)$$

where the prediction error variance matrices Σ_{ii} satisfy the Riccati equations

$$\Sigma_{ii} = \Phi[\Sigma_{ii} - \Sigma_{ii}H^{\mathrm{T}}(H\Sigma_{ii}H^{\mathrm{T}} + Q_{vi})^{-1}H\Sigma_{ii}]\Phi^{\mathrm{T}} + \Gamma Q_w \Gamma^{\mathrm{T}} \quad (25)$$

and the cross-covariances between local prediction errors $\Sigma_{ij} = \mathrm{E}[\tilde{x}_i(t+1|t)\tilde{x}_j^{\mathrm{T}}(t+1|t)]$, satisfy the Lyapunov equations

$$\Sigma_{ij} = \Psi_{pi}\Sigma_{ij}\Psi_{pj}^{\mathrm{T}} + \Gamma Q_w \Gamma^{\mathrm{T}}, i,j = 1,2, i \neq j \quad (26)$$

where $\tilde{x}_i(t+1|t) = x(t+1) - \hat{x}_i(t+1|t)$.

Theorem 1 The two-sensor system (6) and (11) with assumptions 1 and 2 has the local steady-state optimal white noise estimators

$$\hat{v}_i(t|t+N-\tau_i) = L_{i,N-\tau_i}^v(q^{-1})\varepsilon_i(t+N-\tau_i) \quad (27)$$

where

$$\begin{cases} L_{i,N-\tau_i}^v(q^{-1}) = \sum_{j=0}^{N-\tau_i} M_j^{(i)} q^{j-(N-\tau_i)}, N \geq \tau_i \\ L_{i,N-\tau_i}^v(q^{-1}) = 0, N < \tau_i \\ M_j^{(i)} = -Q_{vi}K_{pi}^{\mathrm{T}}\Psi_{pi}^{(j-1)\mathrm{T}}H^{\mathrm{T}}Q_{\varepsilon i}^{-1}, j \geq 1 \\ M_0^{(i)} = Q_{vi}Q_{\varepsilon i}^{-1} \end{cases} \quad (28)$$

which can be rewritten in the Wiener filter form as

$$\hat{v}_i(t|t+N-\tau_i) = \frac{L_{i,N-\tau_i}^v(q^{-1})\Lambda_i(q^{-1})}{\psi_i(q^{-1})} y_i(t+N-\tau_i) \quad (29)$$

with the definitions

$$\psi_i(q^{-1}) = \det(I_n - q^{-1}\Psi_{pi}) \quad (30)$$

$$\Lambda_i(q^{-1}) = \psi_i(q^{-1})I_m - H adj(I_n - q^{-1}\Psi_{pi})K_{pi}q^{-1} \quad (31)$$

where the notation “adj” denotes the adjoint matrix.

The steady-state white noise error variance matrices $P_{vi}(N-\tau_i) = \mathrm{E}[\tilde{v}_i(t|t+N-\tau_i)\tilde{v}_i^{\mathrm{T}}(t|t+N-\tau_i)]$ with $\tilde{v}_i(t|t+N-\tau_i) = v_i(t) - \hat{v}_i(t|t+N-\tau_i)$ are given as

$$\begin{cases} P_{vi}(N-\tau_i) \\ = Q_{vi} - \sum_{j=0}^{N-\tau_i} M_j^{(i)} Q_{\varepsilon i} M_j^{(i)\mathrm{T}}, N \geq \tau_i \\ P_{vi}(N-\tau_i) = Q_{vi}, N < \tau_i \end{cases} \quad (32)$$

Proof. Applying the standard Kalman filer[5] yields

$$\tilde{x}_i(t+1|t) = \Psi_{pi}\tilde{x}_i(t|t-1) + \Gamma w(t) - K_{pi}v_i(t) \quad (33)$$

Applying the projection formula[5] , we have

$$\begin{cases} \hat{v}_i(t|t+N-\tau_i) = \sum_{j=0}^{N-\tau_i} M_j^{(i)}\varepsilon_i(t+j), N \geq \tau_i \\ \hat{v}_i(t|t+N-\tau_i) = 0, N < \tau_i \end{cases} \quad (34)$$

$$M_j^{(i)} = \mathrm{E}[v_i(t)\varepsilon_i^{\mathrm{T}}(t+j)]Q_{\varepsilon i}^{-1} \quad (35)$$

Iterating (33) yields the relation

$$\tilde{x}_i(t+j|t+j-1) = \Psi_{pi}^j\tilde{x}_i(t|t-1) + \sum_{r=1}^{j}\Psi_{pi}^{j-r}[\Gamma w(t+r-1) - K_{pi}v_i(t+r-1)] \quad (36)$$

From (23), we have

$$\varepsilon_i(t+j) = H\tilde{x}_i(t+j|t+j-1) + v_i(t+j) \quad (37)$$

Substituting (36) into (37), substituting (37) into (34), and applying the assumption 1 yield (27) and (28), and using (27) and (28) we can obtain (32).

From (21), we can have the transfer function representation

$$\hat{x}_i(t|t-1)=(I_n-q^{-1}\Psi_{pi})^{-1}K_{pi}q^{-1}y_i(t) \tag{38}$$

Applying the formula for the inverse on matrix

$$\begin{aligned}&(\ I_n-q^{-1}\Psi_{pi})^{-1}\\&=adj(I_n-q^{-1}\Psi_{pi})/\det(I_n-q^{-1}\Psi_{pi})\end{aligned} \tag{39}$$

and substituting (39) and (38) into (23) yields the local ARMA innovation models

$$\Lambda_i(q^{-1})y_i(t)=\psi_i(q^{-1})\varepsilon_i(t) \tag{40}$$

where $\psi_i(q^{-1})$ and $\Lambda_i(q^{-1})$ are defined in (30) and (31).

From (40) we have

$$\varepsilon_i(t)=\frac{\Lambda_i(q^{-1})}{\psi_i(q^{-1})}y_i(t) \tag{41}$$

Substituting (41) into (27) yields (29). The proof is completed.

Theorem 2 The two-sensor system (6) and (11) with assumptions 1 and 2 has the local measurement predictors

$$\begin{aligned}&\hat{y}_i\ (t|t+N-\tau_i)\\&=\psi_i^{-1}(q^{-1})G^{(i)}_{-(N-\tau_i)}(q^{-1})y(t+N-\tau_i),\\&\quad N\geq 0, N<0, \tau_i>0\end{aligned} \tag{42}$$

where the polynomial matrices $F^{(i)}_{\tau_i}(q^{-1})$ and $G^{(i)}_{\tau_i}(q^{-1})$ are determined by the Diophantine equation

$$\begin{cases}\psi_i(q^{-1})I_m=F^{(i)}_{-(N-\tau_i)}(q^{-1})\Lambda_i(q^{-1})+\\ q^{N-\tau_i}G^{(i)}_{-(N-\tau_i)}(q^{-1}), N<\tau_i\\ F^{(i)}_{-(N-\tau_i)}(q^{-1})=F^{(i)}_{-(N-\tau_i),0}+F^{(i)}_{-(N-\tau_i),1}q^{-1}+\\ \cdots+F^{(i)}_{-(N-\tau_i),-(N-\tau_i)-1}q^{N-\tau_i+1}\\ F^{(i)}_{-(N-\tau_i),0}=I_m\end{cases} \tag{43}$$

$$\begin{cases}G^{(i)}_{-(N-\tau_i)}(q^{-1})=G^{(i)}_{-(N-\tau_i),0}+G^{(i)}_{-(N-\tau_i),1}q^{-1}\\ +\cdots+G^{(i)}_{-(N-\tau_i),n_{gi}}q^{-n_{gi}}, N<\tau_i\\ G^{(i)}_{-(N-\tau_i)}(q^{-1})=\psi_i(q^{-1})q^{-(N-\tau_i)}, N\geq\tau_i\end{cases} \tag{44}$$

where the order $n_{gi}=\max(n_{\lambda i}-1, n_{\psi i}-\tau_i)$, and $n_{\lambda i}$ is the order of $\Lambda_i(q^{-1})$.

The prediction error $\tilde{y}_i(t|t+N-\tau_i)=y_i(t)-\hat{y}_i(t|t+N-\tau_i)$ is given by

$$\tilde{y}_i(t|t+N-\tau_i)=F^{(i)}_{-(N-\tau_i)}(q^{-1})\varepsilon_i(t) \tag{45}$$

Introducing $\bar{F}^{(i)}_{-(N-\tau_i)}(q)=F^{(i)}_{-(N-\tau_i)}(q^{-1})q^{-(N-\tau_i)-1}$ with the coefficient matrices $\bar{F}^{(i)}_{-(N-\tau_i),r}$ yields another form of $\tilde{y}_i(t|t+N-\tau_i)$ as

$$\begin{aligned}&\tilde{y}_i(t|t+N-\tau_i)=\\&\sum_{r=0}^{-(N-\tau_i)-1}\bar{F}^{(i)}_{-(N-\tau_i),r}\varepsilon_i(t+N-\tau_i+r+1)\end{aligned} \tag{46}$$

The steady-state error variance matrices $P_{yi}(N-\tau_i)=\mathrm{E}[\tilde{y}_i(t|t+N-\tau_i)\tilde{y}_i^{\mathrm{T}}(t|t+N-\tau_i)]$ with $\tilde{y}_i(t|t+N-\tau_i)=y_i(t)-\hat{y}_i(t|t+N-\tau_i)$ are given by

$$\begin{cases}P_{yi}(N-\tau_i)=\sum\limits_{r=0}^{-(N-\tau_i)-1}\sum\limits_{r=0}^{-(N-\tau_i)-1}F^{(i)}_{-(N-\tau_i),r}\times\\ Q_{\varepsilon i}F^{(i)\mathrm{T}}_{-(N-\tau_i),s}, N<\tau_i\\ P_{yi}(N-\tau_i)=0, N\geq\tau_i\end{cases} \tag{47}$$

or

$$\begin{cases}P_{yi}(N-\tau_i)=\sum\limits_{r=0}^{-(N-\tau_i)-1}\sum\limits_{r=0}^{-(N-\tau_i)-1}\bar{F}^{(i)}_{-(N-\tau_i),r}\times\\ Q_{\varepsilon i}\bar{F}^{(i)\mathrm{T}}_{-(N-\tau_i),s}, N<\tau_i\\ P_{yi}(N-\tau_i)=0, N\geq\tau_i\end{cases} \tag{48}$$

Proof. From (40) and (43) we have

$$\begin{aligned}&y_i(t)=\Lambda_i^{-1}(q^{-1})(\psi_i(q^{-1})I_m)\varepsilon_i(t)\\&=(\psi_i(q^{-1})I_m)\Lambda_i^{-1}(q^{-1})\varepsilon_i(t)\\&=F^{(i)}_{-(N-\tau_i)}(q^{-1})\varepsilon_i(t)+\\&G^{(i)}_{-(N-\tau_i)}(q^{-1})\Lambda_i^{-1}(q^{-1})\varepsilon_i(t+N-\tau_i)\\&=F^{(i)}_{-(N-\tau_i)}(q^{-1})\varepsilon_i(t)+\\&(\psi_i^{-1}(q^{-1})I_m)G^{(i)}_{-(N-\tau_i)}(q^{-1})y_i(t+N-\tau_i)\end{aligned} \tag{49}$$

where for $N<\tau_i$, $F^{(i)}_{-(N-\tau_i)}(q^{-1})\varepsilon_i(t)$ is the linear combination of $\varepsilon_i(t),\varepsilon_i(t-1),\cdots,\varepsilon_i(t+N-\tau_i+1)$, which is uncorrelated with $\psi_i^{-1}(q^{-1})G^{(i)}_{-(N-\tau_i)}(q^{-1})y_i(t+N-\tau_i)$. This yields the steady-state optimal filter (42) and the filtering error (45), and further from (46) we obtain (48). When $N\geq\tau_i$, we have $\hat{y}_i(t|t+N-\tau_i)=y_i(t)$, which yields (44) and (48). The proof is completed.

Theorem 3 For the two-sensor multichannel ARMA signal (1) and (2) with measurement delays, under the assumptions 1 and 2, the local Wiener signal filters $\hat{s}_i^z(t|t+N)$ are given by

$$\begin{aligned}&\psi_i(q^{-1})\hat{s}_i^z(t|t+N)=K_{i,N-\tau_i}(q^{-1})z_i(t+N),\\&N\geq 0, N<0, , i=1,2\end{aligned} \tag{50}$$

where $N<\tau_i$ or $N\geq\tau_i$, and we define

$$\begin{aligned}&K_{i,N-\tau_i}(q^{-1})=G^{(i)}_{-(N-\tau_i)}(q^{-1})-\\&L^v_{i,N-\tau_i}(q^{-1})\Lambda_i(q^{-1})\end{aligned} \tag{51}$$

where $G^{(i)}_{-(N-\tau_i)}(q^{-1})$ and $L^v_{i,N-\tau_i}(q^{-1})$ are determined in (28) and (44).

The filtering error variance matrices are given by

$$\begin{cases}P^z_{ii}(N)=Q_{vi}-\sum\limits_{j=0}^{N-\tau_i}M^{(i)}_jQ_{\varepsilon i}M^{(i)\mathrm{T}}_j, N\geq\tau_i\\ P^z_{ii}(N)=\sum\limits_{j=0}^{\tau_i-N-1}F^{(i)}_{\tau_i-N,j}Q_{\varepsilon i}F^{(i)\mathrm{T}}_{\tau_i-N,j}, N<\tau_i\end{cases} \tag{52}$$

Proof. Substituting (29) and (42) into (20) yields

$$\psi_i(q^{-1})\hat{s}_i(t|t+N-\tau_i)=K_{i,N-\tau_i}(q^{-1})y(t+N-\tau_i) \tag{53}$$

with $K_{i,N-\tau_i}(q^{-1})$ defined in (51).

Substituting (10) and (13) into (53) yields (50).

When $N \geq \tau_i$, from (19) and (20), and $\hat{y}_i(t|t+N-\tau_i) = y_i(t)$, we have

$$\tilde{s}_i(t|t+N-\tau_i) = -\tilde{v}_i(t|t+N-\tau_i) \tag{54}$$

Applying (17) and (18) yields $P^z_{ii}(N) = P_{ii}(N-\tau_i)$, so from (32) we obtain the first equation of (52).

When $N < \tau_i$, we have $\hat{v}_i(t|t+N-\tau_i) = 0$. Hence from (19) and (20), we have

$$\tilde{y}_i(t|t+N-\tau_i) = \tilde{s}_i(t|t+N-\tau_i) + v_i(t) \tag{55}$$

Notice that $v_i(t)$ is uncorrelated with $\tilde{s}_i(t|t+N-\tau_i)$. Hence applying (15)$\sim$(18) and (55) yields $P^z_{ii}(N) = P_{ii}(N-\tau_i, N-\tau_i) = P_{yi}(N-\tau_i) - Q_{vi}$, and then applying (47) yields the second equation of (52).

Theorem 4 For the two-sensor multichannel ARMA signal (1) and (2) with measurement delays, under the assumptions 1 and 2, when $N \geq \tau_i$ and $N \geq \tau_j$, the cross-covariances between the local filtering errors are given by

$$P^z_{ij}(N) = \sum_{r=0}^{N-\tau_i}\sum_{s=0}^{N-\tau_j} M_r^{(i)} E_{ij}(r,s) M_s^{(i)\mathrm{T}}, i \neq j \tag{56}$$

where $E_{ij}(r,s)$ is given by

$$E_{ij}(r,s) = H\Psi^r_{pi}\Sigma_{ij}\Psi^{s\mathrm{T}}_{pj}H^{\mathrm{T}} + \sum_{k=1}^{\min(r,s)} H\Psi^{r-k}_{pi}\Gamma Q_w \Gamma^{\mathrm{T}}\Psi^{(s-k)\mathrm{T}}_{pj}H^{\mathrm{T}}, \min(r,s) \neq 0 \tag{57}$$

when $\min(r,s) = 0$, we define that the second term of (57) is equal to zeros.

Proof. When $N \geq \tau_i$, $N \geq \tau_j$, then $\hat{y}_i(t|t+N-\tau_i) = y_i(t)$, $\hat{y}_j(t|t+N-\tau_j) = y_j(t)$, which yields $\tilde{y}_i(t|t+N-\tau_i) = 0$, $\tilde{y}_j(t|t+N-\tau_j) = 0$, hence from (19) and (20), we have

$$\tilde{s}_i(t|t+N-\tau_i) = -\tilde{v}_i(t|t+N-\tau_i) = -[v_i(t) - \sum_{r=0}^{N-\tau_i} M_r^{(i)}\varepsilon_i(t+r)] \tag{58}$$

$$\tilde{s}_j(t|t+N-\tau_j) = -\tilde{v}_j(t|t+N-\tau_j) = -[v_j(t) - \sum_{s=0}^{N-\tau_i} M_s^{(j)}\varepsilon_j(t+s)] \tag{59}$$

From (36), we know that $v_j(t)$ and $\varepsilon_i(t+k)$ are uncorrelated, and $v_i(t)$ and $\varepsilon_j(t+k)$ are uncorrelated.

Applying (17), (18), (58) and (59) yields (56), with the definitions

$$E_{ij}(r,s) = \mathrm{E}[\varepsilon_i(t+r)\varepsilon_j^{\mathrm{T}}(t+s)] \tag{60}$$

From (11) and (23) it follows that

$$\varepsilon_i(t+j) = H\tilde{x}_i(t+j|t+j-1) + v_i(t+j) \tag{61}$$

Substituting (36) into (60), and then substituting (61) into (60), and applying assumption 1, we obtain (57). The proof is completed.

Theorem 5 For the two-sensor multichannel ARMA signal (1) and (2) with measurement delays, under the assumptions 1 and 2, when $N < \tau_i$ and $N < \tau_j$, the cross-covariances between the local filtering errors are given by

$$P^z_{ij}(N) = \sum_{r=0}^{\tau_i-N-1}\sum_{s=0}^{\tau_j-N-1} \bar{F}^{(i)}_{-(N-\tau_i),r} \times \Sigma_{ij}(r,s)\bar{F}^{(j)\mathrm{T}}_{-(N-\tau_i),s}, i \neq j \tag{62}$$

where $\bar{F}^{(i)}_{-(N-\tau_i),i}$ is determined in (46). $\Sigma_{ij}(r,s)$ is given by

$$\Sigma_{ij}(r,s) = H\Psi^r_{pi}\Sigma_{ij}\Psi^{(\tau_i-\tau_j+s)\mathrm{T}}_{pj}H^{\mathrm{T}} + \sum_{k=1}^{\min(r,\tau_i-\tau_j+s)} H\Psi^{r-k}_{pi}\Gamma Q_w\Gamma^{\mathrm{T}}\Psi^{(s+\tau_i-\tau_j-k)\mathrm{T}}_{pj}H^{\mathrm{T}}, \tau_i \geq \tau_j \tag{63}$$

$$\Sigma_{ij}(r,s) = H\Psi^{\tau_j-\tau_i+r}_{pi}\Sigma_{ij}\Psi^{s\mathrm{T}}_{pj}H^{\mathrm{T}} + \sum_{k=1}^{\min(\tau_j-\tau_i+r,s)} H\Psi^{r+\tau_j-\tau_i-k}_{pi}\Gamma Q_w\Gamma^{\mathrm{T}}\Psi^{(s-k)\mathrm{T}}_{pj}H^{\mathrm{T}}, \tau_i < \tau_j \tag{64}$$

where Σ_{ij} is defined by (26).

Proof. When $N < \tau_i$ and $N < \tau_j$, from (27) and (28), we have $\hat{v}_i(t|t+N-\tau_i) = 0$, $\hat{v}_j(t|t+N-\tau_j) = 0$, so from (20), we have

$$\hat{s}_i(t|t+N-\tau_i) = \hat{y}_i(t|t+N-\tau_i), \hat{s}_j(t|t+N-\tau_j) = \hat{y}_j(t|t+N-\tau_j) \tag{65}$$

Hence from (19) yields

$$\tilde{s}_i(t|t+N-\tau_i) = \tilde{y}_i(t|t+N-\tau_i) - v_i(t), \tilde{s}_j(t|t+N-\tau_j) = \tilde{y}_j(t|t+N-\tau_j) - v_j(t) \tag{66}$$

Since $v_i(t)$ and $v_j(t)$ $(i \neq j)$ are uncorrelated, then

$$P^z_{ij}(N) = P_{ij}(N-\tau_i, N-\tau_j) = \mathrm{E}[\tilde{y}_i(t|t+N-\tau_i)\tilde{y}_j^{\mathrm{T}}(t|t+N-\tau_j)] \tag{67}$$

Substituting (46) into (67) yields (62) with the definition

$$\Sigma_{ij}(r,s) = \mathrm{E}[\varepsilon_i(t+N-\tau_i+1+r) \times \varepsilon_j^{\mathrm{T}}(t+N-\tau_j+1+r)] \tag{68}$$

When $\tau_i \geq \tau_j$, we have

$$\Sigma_{ij}(r,s) = \mathrm{E}[\varepsilon_i(t+N-\tau_i+1+r) \times \varepsilon_j^{\mathrm{T}}(t+N-\tau_i+1+r+(\tau_i-\tau_j))] \tag{69}$$

Setting $r = t+N-\tau_i+1+r, s = t+N-\tau_i+1+r+(\tau_i-\tau_j)$, applying (57) into (69) yields (63). Similarly, we can obtained (64). The proof is completed.

Theorem 6 For the two-sensor multichannel ARMA signal (1) and (2) with measurement delays, under the assumptions 1 and 2, when $N \geq \tau_i$ and $N < \tau_j$, the cross-covariances between the local filtering errors are given by

$$P_{ij}^{z}(N) = \sum_{r=0}^{N-\tau_i} \sum_{s=0}^{\tau_j-N-1} M_r^{(i)} \Delta_{ij}^{(2,1)}(r,s) \bar{F}_{-(N-\tau_j),s}^{(j)\mathrm{T}} \quad (70)$$

$$i \neq j$$

where $\Delta_{ij}^{(2,1)}(r,s)$ is given by

$$\Delta_{ij}^{(2,1)}(r,s) = H\Psi_{pi}^{\tau_j-N-1+r}\Sigma_{ij}\Psi_{pj}^{s\mathrm{T}}H^{\mathrm{T}} + \sum_{k=1}^{\min(\tau_j-N-1+r,s)} H\Psi_{pi}^{\tau_j-N-1+r-k}\Gamma Q_w \Gamma^{\mathrm{T}}\Psi_{pj}^{(s-k)\mathrm{T}}H^{\mathrm{T}} \quad (71)$$

when $N < \tau_i$ and $N \geq \tau_j$, the cross-covariances between the local smoothing errors are given by

$$P_{ij}^{z}(N) = \sum_{r=0}^{\tau_i-N-1} \sum_{s=0}^{N-\tau_j} \bar{F}_{-(N-\tau_i),r}^{(i)\mathrm{T}} \times \Delta_{ij}^{(2,2)}(r,s) M_s^{(j)\mathrm{T}}, i \neq j \quad (72)$$

where $\Delta_{ij}^{(2,2)}(r,s)$ is given by

$$\Delta_{ij}^{(2,2)}(r,s) = H\Psi_{pi}^{r}\Sigma_{ij}\Psi_{pj}^{(\tau_i-N-1+s)\mathrm{T}}H^{\mathrm{T}} + \sum_{k=1}^{\min(r,\tau_i-N-1+s)} H\Psi_{pi}^{r-k}\Gamma Q_w \Gamma^{\mathrm{T}}\Psi_{pj}^{(\tau_i-N-1+s-k)\mathrm{T}}H^{\mathrm{T}} \quad (73)$$

Proof. Applying (18), (27) and (46), and noting that

$$\tilde{s}_i(t|t+N-\tau_i) = \tilde{v}_i(t|t+N-\tau_i), N > \tau_i \quad (74)$$

$$\tilde{s}_j(t|t+N-\tau_j) = \tilde{y}_j(t|t+N-\tau_j) - v_j(t), N < \tau_j \quad (75)$$

which yields (70), with the definition

$$\Delta_{ij}^{(1)}(r,s) = \mathrm{E}[\varepsilon_i(t+r)\varepsilon_j^{\mathrm{T}}(t+N-\tau_j+1+s)] \quad (76)$$

From the definition (57), we have the relation

$$\Delta_{ij}^{(1)}(r,s) = \mathrm{E}[\varepsilon_i(t+N-\tau_j+1-N+\tau_j -1+r)\varepsilon_j^{\mathrm{T}}(t+N-\tau_j+1+s)] = E_{ij}(\tau_j-N-1+r,s) \quad (77)$$

Hence setting $r = \tau_j - N - 1 + r$ in (57) yields (71). Similarly, we can obtain (72) and (73). The proof is completed.

4. Three Weighting Wiener signal fusers and CI Wiener signal fuser

The steady-state optimal Wiener signal fuser $\hat{s}_m^z(t|t+N)$ weighted by matrices is given by [28,29]

$$\hat{s}_m^z(t|t+N) = \Omega_1\hat{s}_1^z(t|t+N) + \Omega_2\hat{s}_2^z(t|t+N) \quad (78)$$

with the optimal weighting matrices[28]

$$\Omega_1 = (P_{22}^z(N) - P_{21}^z(N)) \times (P_{11}^z(N) + P_{22}^z(N) - P_{12}^z(N) - P_{21}^z(N))^{-1} \quad (79)$$

$$\Omega_2 = (P_{11}^z(N) - P_{12}^z(N)) \times (P_{11}^z(N) + P_{22}^z(N) - P_{12}^z(N) - P_{21}^z(N))^{-1} \quad (80)$$

The fused error variance matrix $P_m^z(N)$ is given by

$$P_m^z(N) = P_{11}^z(N) - (P_{11}^z(N) - P_{12}^z(N)) \times (P_{11}^z(N) + P_{22}^z(N) - P_{12}^z(N) - P_{21}^z(N))^{-1} \times (P_{11}^z(N) - P_{12}^z(N))^{\mathrm{T}} \quad (81)$$

The steady-state optimal Wiener signal fuser with the scalar weights is given by [28,29]

$$\hat{s}_s^z(t|t+N) = \omega_1\hat{s}_1^z(t|t+N) + \omega_2\hat{s}_2^z(t|t+N) \quad (82)$$

where the optimal weights is given by

$$[\omega_1, \omega_2] = (e^{\mathrm{T}}P_{tr}^{-1}(N)e)^{-1}e^{\mathrm{T}}P_{tr}^{-1}(N) \quad (83)$$

where $P_{tr}(N) = (trP_{ij}^z(N))_{2\times 2}$, the notation tr denotes the trace of matrix, $e^{\mathrm{T}} = [1,1]$.

The fused error variance is given by

$$P_s^z(N) = \sum_{i=1}^{2}\sum_{j=1}^{2}\omega_i\omega_j P_{ij}^z(N) \quad (84)$$

The steady-state optimal Wiener signal fuser with the diagonal matrices is given by[28,29]

$$\hat{s}_d^z(t|t+N) = \Omega_1^d\hat{s}_1^z(t|t+N) + \Omega_2^d\hat{s}_2^z(t|t+N) \quad (85)$$

$$\Omega_i^d = diag(a_{i1}, \cdots, a_{in}), i = 1,2 \quad (86)$$

The optimal diagonal weighting vectors are given by

$$[a_{1l}, a_{2l}] = [e^{\mathrm{T}}(P^{ll}(N))^{-1}e]^{-1}e^{\mathrm{T}}(P^{ll}(N))^{-1}, \quad l = 1, \cdots, n \quad (87)$$

where $e^{\mathrm{T}} = [1,1]$, $P^{ll}(N) = (P_{ks}^{(ll)}(N))_{2\times 2}$, $P_{ks}^{(ll)}(N)$ is the (l,l) diagonal element of $P_{ks}^z(N)$. The fused error variance weighted by diagonal matrix is given by

$$P_d^z(N) = \sum_{i=1}^{2}\sum_{j=1}^{2}\Omega_i^d P_{ij}^z(N)\Omega_j^{d\mathrm{T}} \quad (88)$$

Based on covariance intersection (CI) method, the CI fusion Wiener signal filter is given as

$$\hat{s}_{CI}^z(t|t+N) = P_{CI}^z(N)[\omega(P_{11}^z(N))^{-1}\hat{s}_1^z(t|t+N) +(1-\omega)(P_{22}^z(N))^{-1}\hat{s}_2^z(t|t+N)] \quad (89)$$

$$(P_{CI}^z(N))^{-1} = \omega(P_{11}^z(N))^{-1} + (1-\omega)(P_{22}^z(N))^{-1} \quad (90)$$

where $\omega \in [0,1]$, and minimizes the performance index

$$\min_{\omega} trP_{CI}^z(N) = \min_{\omega\in[0,1]} tr\{[\omega(P_{11}^z(N))^{-1} + (1-\omega)(P_{22}^z(N))^{-1}]^{-1}\} \quad (91)$$

Remark 1. From (89) we see that the CI Kalman fuser (89) is also a weighting fuser with the matrix weights

$$\Omega_1 = \omega P^z_{CI}(N)(P^z_{11}(N))^{-1}, \quad \Omega_2 = (1-\omega)P^z_{CI}(N)(P^z_{22}(N))^{-1} \tag{92}$$

Remark 2. Based on (91), to find the optimal weighting coefficient ω is an optimization problem for nonlinear function

$$\min_{\omega} J(\omega) = tr\{[\omega(P^z_{11}(N))^{-1} + (1-\omega)(P^z_{22}(N))^{-1}]^{-1}\} \tag{93}$$

with constraint condition $\omega \in [0,1]$, the optimal ω can fast be obtained by the gold section method or Fibonacci method [31].

5. Accuracy comparison of CI Wiener signal fuser with three weighting Wiener signal fusers

Theorem 7. When $P^z_{ii}(N)$ and $P^z_{ij}(N)$ are exactly known, the CI fuser has the actual filtering error variance matrix

$$\bar{P}^z_{CI}(N) = P^z_{CI}(N)[\omega^2(P^z_{11}(N))^{-1} + \omega(1-\omega)(P^z_{11}(N))^{-1}P^z_{12}(N)(P^z_{22}(N))^{-1} + \omega(1-\omega)(P^z_{22}(N))^{-1}P^z_{21}(N)(P^z_{11}(N))^{-1} + (1-\omega)^2(P^z_{22}(N))^{-1}]P^z_{CI}(N) \tag{94}$$

where the actual CI fusion filtering error variance $\bar{P}^z_{CI}(N) = \mathrm{E}[\tilde{s}^z_{CI}(t|t+N)\tilde{s}^{z\mathrm{T}}_{CI}(t|t+N)]$, $\tilde{s}^z_{CI}(t|t+N) = s(t) - \hat{s}^z_{CI}(t|t+N)$, and E denotes the mathematical expectation, from[20] the CI fuser is consistent, i.e.

$$\bar{P}^z_{CI}(N) \le P^z_{CI}(N) \tag{95}$$

where $P^z_{CI}(N)$ is an upper bound of $\bar{P}^z_{CI}(N)$, which is determined by (94). The matrix inequality (95) means that $P^z_{CI}(N) - \bar{P}^z_{CI}(N) \ge 0$ is positive semidefinite.

Proof. From (90) we have

$$(P^z_{CI}(N))^{-1}s(t) = \omega(P^z_{11}(N))^{-1}s(t) + (1-\omega)(P^z_{22}(N))^{-1}s(t) \tag{96}$$

Subtracting (89) from (96) yields

$$(P^z_{CI}(N))^{-1}\tilde{s}^z_{CI}(t|t+N) = \omega(P^z_{11}(N))^{-1}\tilde{s}^z_1(t|t+N) + (1-\omega)(P^z_{22}(N))^{-1}\tilde{s}^z_2(t|t+N) \tag{97}$$

with $\tilde{s}^z_i(t|t+N) = s(t) - \tilde{s}^z_i(t|t+N),\ i=1,2.$

Hence we have

$$\tilde{s}^z_{CI}(t|t+N) = P^z_{CI}(N)[\omega(P^z_{11}(N))^{-1}\tilde{s}^z_1(t|t+N) + (1-\omega)(P^z_{22}(N))^{-1}\tilde{s}^z_2(t|t+N)] \tag{98}$$

Applying the definitions $P^z_{12}(N) = \mathrm{E}[\tilde{s}^z_1(t|t+N)\tilde{s}^{z\mathrm{T}}_2(t|t+N)]$, $P^z_{21}(N) = \mathrm{E}[\tilde{s}^z_2(t|t+N)\tilde{s}^{z\mathrm{T}}_1(t|t+N)]$, and $P^z_{ii}(N) = \mathrm{E}[\tilde{s}^z_i(t|t+N)\tilde{s}^{z\mathrm{T}}_i(t|t+N)]$, substituting (98) into $\bar{P}^z_{CI}(N) = \mathrm{E}[\tilde{s}^z_{CI}(t|t+N)\tilde{s}^{z\mathrm{T}}_{CI}(t|t+N)]$, we obtain (94). The consistency of CI fuser has been proven in [20, 22]. The proof is completed.

Theorem 8. For the local Wiener signal filters, the three weighting and CI Wiener signal fusers, the following accuracy relations hold

$$P^z_m(N) \le P^z_d(N), P^z_m(N) \le P^z_s(N), \quad P^z_m(N) \le \bar{P}^z_{CI}(N), P^z_m(N) \le P^z_{ii}(N), i=1,2 \tag{99}$$

Proof. from (78), $\hat{s}^z_m(t|t+N)$ is the ULMV estimate of $s(t)$ based on the linear space $L_m = L_m(\hat{s}^z_1(t|t+N), \hat{s}^z_2(t|t+N))$ spanned by $(\hat{s}^z_1(t|t+N), \hat{s}^z_2(t|t+N))$. From (82), (85), (89), we have $\hat{s}^z_\theta(t|t+N) \in L_m, \theta = s,d,CI,i,$ which yields (99). The proof is completed.

Theorem 9. The accuracy relation among the CI Wiener signal fuser, the three weighting Wiener signal fuser and the local Wiener signal filters is given by

$$trP^z_m(N) \le tr\bar{P}^z_{CI}(N) \le trP^z_{CI}(N) \le trP^z_{ii}(N), \quad i=1,2 \tag{100}$$

$$trP^z_m(N) \le trP^z_d(N) \le trP^z_s(N) \le trP^z_{ii}(N), \quad i=1,2 \tag{101}$$

Proof. Taking the trace operation for (95) and (99) yields $trP^z_m(N) \le tr\bar{P}^z_{CI}(N) \le trP^z_{CI}(N)$, noting that trP^z_{CI} minimizes (93) for $\omega \in [0,1]$. Taking $\omega = 0$, we have $trP^z_{CI}(N) = trP^z_{22}(N)$, taking $\omega = 1$ yields $trP^z_{CI}(N) = trP^z_{11}(N)$. Hence we have $trP^z_{CI}(N) \le trP^z_{ii}(N),\ i = 1,2$, so the (100) holds. Because the fusers with matrix weights contain the fusers with diagonal matrix weights, and the fuser with diagonal matrix weights contain the fusers with scalar weights and the fusers with scalar weights contain the local filters, then the accuracy relation (101) holds. The proof is completed.

6. Simulation example

Consider the two-sensor, multichannel ARMA signal system

$$A(q^{-1})s(t) = C(q^{-1})w(t) \tag{102}$$

$$z_i(t) = s(t-\tau_i) + \xi_i(t) i = 1,2 \tag{103}$$

where $\tau_i = 1, \tau_j = 2$ are the time delays, $A(q^{-1}) = I_m + A_1q^{-1} + A_2q^{-2}C(q^{-1}) = C_1q^{-1}$. The aim is to compare the accuracy among the local optimal steady-state Wiener signal filters $\hat{s}^z_i(t|t+N), N \ge 0, N < 0$ and the CI fuser $\hat{s}^z_{CI}(t|t+N)$ and three weighting fusers $\hat{s}^z_m(t|t+N)$,$\hat{s}^z_s(t|t+N)$,$\hat{s}^z_d(t|t+N)$.

In order to give a powerful geometric interpretation with respect to accuracy relations among local and fused Wiener signal filters, the concept on the covariance ellipse was introduced in [23]. The covariance ellipse for a variance matrix P is defined as the locus of points $\{x : x^{\mathrm{T}}P^{-1}x = c\}$ where c is a constant. In the sequel, $c = 1$ will be assumed without loss of generality. The following properties were proven in [23,33] for the two-sensor system: $P_1 \leq P_2$ is equivalent to that the covariance ellipse of P_1 is enclosed in the covariance ellipse of P_2. Hence from (99), (100) and (101), the ellipse of the fused variance $P_m^z(N)$ should lie within the intersection of covariance ellipses for $P_{11}^z(N)$ and $P_{22}^z(N)$, and be enclosed in the ellipses of $P_d^z(N)$, $P_s^z(N)$ and $\bar{P}_{CI}^z(N)$. The covariance ellipse for the upper bound $P_{CI}^z(N)$ of the CI fused variance encloses the intersection region of covariance ellipses $P_{11}^z(N)$ and $P_{22}^z(N)$, and passes through the four points of intersection of covariance ellipses for $P_{11}^z(N)$ and $P_{22}^z(N)$.

In order to verify the above theoretical results for accuracy relations, the mean square error (MSE) value at time t for local and fused Wiener signal filters is defined as sampled average for $trP_{ii}^z = tr\mathrm{E}[(\hat{s}_i^z(t|t) - s(t))(\hat{s}_i^z(t|t) - s(t))^{\mathrm{T}}]$, i.e.,

$$MSE_i(t) = \frac{1}{M}\sum_{j=1}^{M}((\hat{s}_i^{z(j)}(t|t) - s^{(j)}(t))^T \times \\ (\hat{s}_i^{z(j)}(t|t) - s^{(j)}(t)),\ i = 1, 2, m, d, s, CI \tag{104}$$

where $t = 1, \cdots, 200$, $\hat{s}_i^{z(j)}(t|t)$ or $s^{(j)}(t)$ denotes the jth realization of $\hat{s}_i^z(t|t)$ or $s(t)$, respectively. M is the number of Monte-Carlo runs. According to the ergodicity[34], we have

$$MSE_i(t) \to trP_{ii}^z, \quad as \quad t \to \infty, M \to \infty, \\ i = 1, 2, m, d, s, CI \tag{105}$$

6.1. Example 1

For the system (100) and (101), in simulation, we take $N = 0$, $A_1 = \begin{bmatrix} 0.6 & -0.1 \\ 0.1 & -1.5 \end{bmatrix}$, $A_2 = \begin{bmatrix} -0.16 & -0.08 \\ -0.02 & 0.55 \end{bmatrix}$, $C_1 = \begin{bmatrix} -0.7 & 0.2 \\ 0.5 & -0.3 \end{bmatrix}$, $Q_w = \begin{bmatrix} 4 & 0 \\ 0 & 2 \end{bmatrix}$, $Q_{\xi 1} = \begin{bmatrix} 4 & 0 \\ 0 & 16 \end{bmatrix}$, $Q_{\xi 2} = \begin{bmatrix} 0.25 & 0 \\ 0 & 0.16 \end{bmatrix}$.

The simulation results and the $MSE_i(t)$ curves are shown in Figure 1 and Figure 2.

6.2. Example 2

For the system (100) and (101), in simulation, we take $N = 2$, $A_1 = \begin{bmatrix} 0.6 & -0.1 \\ 0.1 & -1.5 \end{bmatrix}$, $A_2 = \begin{bmatrix} -0.16 & -0.08 \\ -0.02 & 0.55 \end{bmatrix}$, $C_1 = \begin{bmatrix} -0.7 & 0.2 \\ 0.5 & -0.3 \end{bmatrix}$, $Q_w = \begin{bmatrix} 4 & 0 \\ 0 & 1 \end{bmatrix}$, $Q_{\xi 1} = \begin{bmatrix} 1 & 0 \\ 0 & 0.49 \end{bmatrix}$, $Q_{\xi 2} = \begin{bmatrix} 0.25 & 0 \\ 0 & 2 \end{bmatrix}$.

The simulation results and the $MSE_i(t)$ curves are shown in Figure 3 and Figure 4.

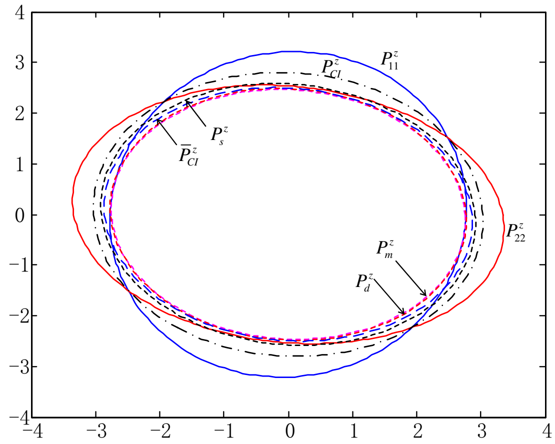

Figure 1 Comparison of accuracy relations for Wiener filters based on covariance ellipses

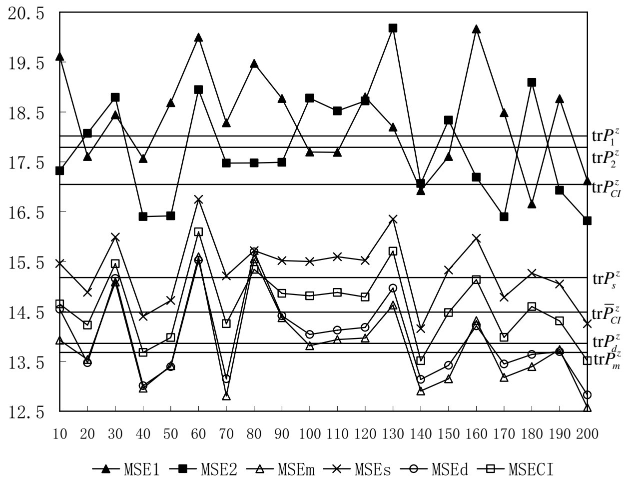

Figure 2 Comparison of MSE curves for local and fused Wiener filters

6.3. Example 3

For the system (100) and (101), in simulation, we take $N = -1$, $A_1 = \begin{bmatrix} 0.7 & 0.2 \\ 0.4 & -1.5 \end{bmatrix}$, $A_2 = \begin{bmatrix} -0.08 & 0.16 \\ -0.04 & -0.46 \end{bmatrix}$, $C_1 = \begin{bmatrix} -0.7 & 0.2 \\ 0.5 & -0.3 \end{bmatrix}$, $Q_w = \begin{bmatrix} 8 & 0 \\ 0 & 1 \end{bmatrix}$, $Q_{\xi 1} = \begin{bmatrix} 4 & 0 \\ 0 & 25 \end{bmatrix}$, $Q_{\xi 2} = \begin{bmatrix} 0.25 & 0 \\ 0 & 0.36 \end{bmatrix}$.

The simulation results and the $MSE_i(t)$ curves are shown in Figure 5 and Figure 6.

Synthesizing the examples 1-3, the error variances of local and fused Kalman estimators for $N = 0$, $N = 2$, $N = -1$ are

$$trP_{11}^z(N) = 18.018, trP_{22}^z(N) = 17.79, \tag{106}$$

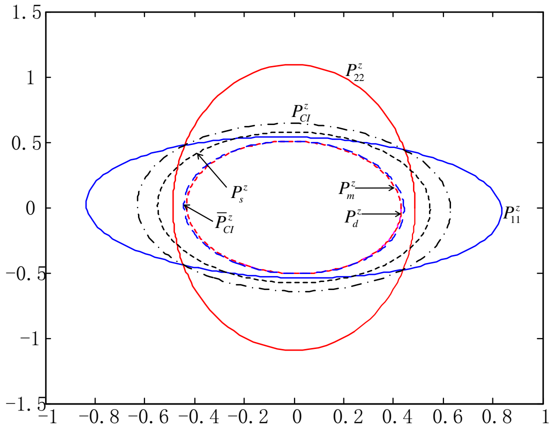

Figure 3 Comparison of accuracy relations for Wiener smoothers based on covariance ellipses

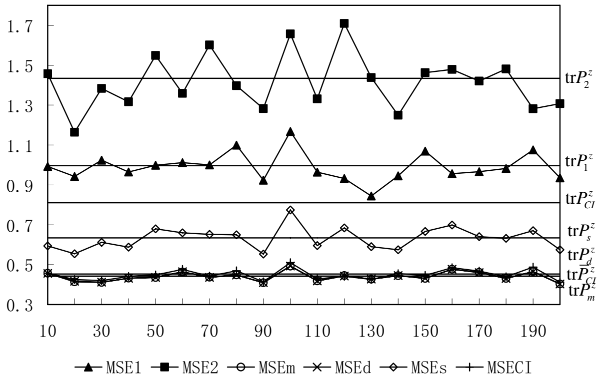

Figure 4 Comparison of MSE curves for local and fused Wiener smoothers

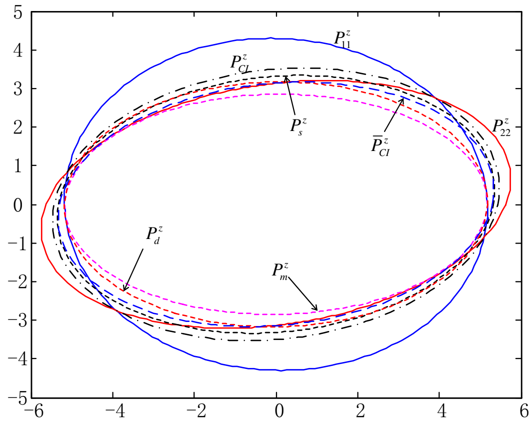

Figure 5 Comparison of accuracy relations for Wiener predictors based on covariance ellipses

$$trP_m^z(N) = 13.678, trP_d^z(N) = 13.859,$$

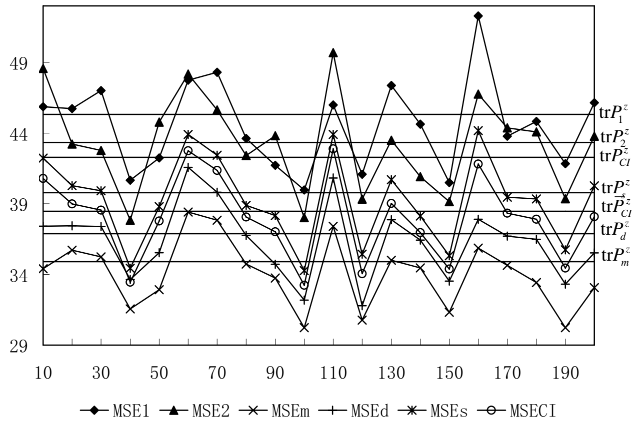

Figure 6 Comparison of MSE curves for local and fused Wiener predictors

$$trP_s^z(N) = 15.18, trP_{CI}^z(N) = 17.046,$$
$$tr\bar{P}_{11}^z(N) = 14.485,$$
$$N = 0$$

$$trP_{11}^z(N) = 0.99263, trP_{22}^z(N) = 1.4362, \quad (107)$$
$$trP_m^z(N) = 0.4424, trP_d^z(N) = 0.4428,$$
$$trP_s^z(N) = 0.63423, trP_{CI}^z(N) = 0.81086,$$
$$tr\bar{P}_{11}^z(N) = 0.45359,$$
$$N = 2$$

$$trP_{11}^z(N) = 45.319, trP_{22}^z(N) = 44.342, \quad (108)$$
$$trP_m^z(N) = 39.902, trP_d^z(N) = 36.884,$$
$$trP_s^z(N) = 39.783, trP_{CI}^z(N) = 42.28,$$
$$tr\bar{P}_{11}^z(N) = 38.467,$$
$$N = 2$$

From the ellipses relations of figures 1,3 and 5, we can obtain that the accuracy relations (99) hold. According to the MSE curves of figures 2,4 and 6, the accuracy relations among the trace of CI fuser, the three weighting fuser and the local filters, i.e., (100) and (101) hold. The values of (106),(107) and (108) show that the accuracy of CI fuser is higher than that of each local Wiener signal filter, and is close to that of the optimal matrix weighted fusion Wiener signal filter.

7. Conclusions

For the two-sensor multichannel ARMA signal systems with measurement delays and unknown cross-covariance, using CI fusion method, based on Riccati equation, the CI fusion Wiener signal filter is presented. It is rigorously proved that the accuracy of the presented CI fuser is higher than that of each local Wiener signal filter, and a little less than that of the optimal matrix weighted fusion Wiener signal filter. So it has good performance.

Acknowledgement

The authors acknowledge the financial support National Natural Science Foundation of China, project No. 60874063, Support Program for Young Professional in Regular Higher Education Institutions of Heilongjiang Province, project No.1251G012. The author is grateful to the anonymous referee for a careful checking of the details and for helpful comments that improved this paper.

References

[1] Y. Bar-shalom and, X. R. Li and T.Kirubarajan, Estimation With Applications to Tracking and Navigation (John Wiley & Son,Inc. 1999)
[2] X. R. Li, Y. M. Zhu, J. Wang and C. J. Han, IEEE Trans. Information Theory. 49. **2192**,2208 (2003).
[3] Q. Gan and C. J. Harris, IEEE Trans. Aerospace and Electronic Systems. 37. **273**,280 (2001).
[4] T. Kailath, A. H. Sayed and B. Hassibi, Linear Estimation (Upper Saddle River, Prentice-Hall, New Jersey, 2000).
[5] B. D. O. Anderson and J. B. Moore, Optimal Filtering, Englewood Cliffs (Prentice-Hall, New Jersey, 1979).
[6] A. Ahlen and M. Sternad, IEEE Trans. Acoustics, Speech, Signal Processing. 37. **217**,226 (1989).
[7] A. Ahlen and M. Sternad, IEEE Trans. Signal Processing. 39. **2387**,2399 (1991).
[8] M. J. Gramble, International Journal of Control. 41. **1545**,1564 (1985).
[9] Z. L. Deng, H. S. Zhang, S.J . Liu and L. Zhou, Automatica. 32. **199**,216 (1996).
[10] Z. L. Deng, Proc. Sixth IFAC Symposium on Identification and System Parameter Estimation, Washington, DC. 377(1982).
[11] Z. L. Deng and L. Mao, Systems Engineering and Electronics. 27.**1971**,1974 (2005).
[12] Z. L. Deng and Y. Gao, Control Theory & Applications. 22. **641**,644 (2005).
[13] H. S. Zhang and L .H. Xie, Control and Estimation of Systems with Input/Output Delays (Springer, Berlin, 2007).
[14] X. Lu, H. S. Zhang, W. Wang and K. L. Teo, Automatica. 41. **1455**,1461 (2005).
[15] S. L. Sun and N, Lv, Acta Automatic Sinica. 35. **46**,53 (2009).
[16] X. J. Sun and Z. L. Deng, IET Signal Processing. 3. **403**,415 (2009).
[17] Z. L. Deng, Y. Gao, C. B. Li and G. Hao, Automatica.44. **685**,695 (2008).
[18] C. J. Ran and Z. L. Deng, Proc. 8th IEEE International Conference on Control and Automation,645 (2010).
[19] X. J. Sun, Y. Gao, Z. L. Deng, C. Li and J. W. Wang, Information Fusion. 11. **163**,173 (2009).
[20] S. J. Julier and J. K. Uhlmann, Proc. 1999 IEEE American Control Conference, Albuquerque, NM, USA. 2369 (1997).
[21] J. K. Uhlmann, Proc. the SPIE Aerosence Conference, SPIE. 536 (1996).
[22] S. J. Julier and J. K. Uhlmann, In: Handbook of Multisensor Data Fusion, M.E. Liggins, D.L. Hall and J. Llinas (Eds.), 319 (CRC Press, 2009).
[23] L. Chen, P. O. Arambel and R. K. Mehra, IEEE Trans, Automatic Control. 47. **1879**,1882 (2002).
[24] P. O. Arambel, C. Rago and R. K. Mehra, Proc. the American Control Conference, Arington, 4398 (2001).
[25] S. J. Julier and J. K. Uhlmann, Robotics and Autonomous Systems. 55. **3**, 20 (2007).
[26] Q. Guo, S. Y. Chen, H. R. Leung and S. T. Liu, Information Sciences. 180. **3434**,3443 (2010).
[27] J. C. Bolzani, C. Ferreira and J. Waldmann, Control Engineering Practice. 15. **389**,409 (2007).
[28] Z. L. Deng, Y. Gao, L. Mao, Y. Li and G. Hao, Automatica. 41. **1695**,1707 (2005).
[29] S. L. Sun and Z. L. Deng, Automatica. 40. **1017**,1023 (2004).
[30] C. Z. Han, H. Y. Zhu and Z. S. Duan, Multi-source information fusion (Tsinghua University Press, Beijing, 2006).
[31] Y. X. Yuan and W. Y. Sun, Optimization Theory and Methods (Science Press, Beijing, 2003).
[32] Y. Bar, Shalom and L. Campo, IEEE Trans. Aerospace and Electronic Systems. 22. **803**,805 (1986).
[33] Z. L. Deng, Peng Zhang, W. J. Qi, J. F. Liu and Y. Gao, Information Sciences. 189. **293**,309 (2012).
[34] L. Ljung, System Identification, Theory for the User (Prentice-Hall PTR, New Jersey, 1999).

Peng Zhang received her B.Sc. degree and M.Sc. degree in department of Automation, Heilongjiang University in 2004 and in 2007, respectively. Currently she is a Ph.D. Candidate in Department of Automation, Heilongjiang University. And she is a university lecturer in Harbin Deqiang College of Commerce. Her research interests include multisensor information fusion and self-tuning filtering.

Zili Deng is a Professor at the Department of Automation, Heilongjiang University. He has published more than 400 papers and 8 books in the fields of optimal filtering, self-tuning filtering, time series analysis, and multisensor information fusion. He is the author the books ”Kalman Filtering and Wiener Filtering-Modern Time Series Analysis Method”(2001), ”Self-tuning Filtering Theory with Applications-Modern Time Series Analysis Method”(2003), ”Optimal Estimation Theory with Applications-Modeling, Filtering, Information Fusion Estimation”(2005), and ”Information Fusion Filtering Theory and Applications”(2007). His research interests include optimal and self-tuning filtering, deconvolution, state estimation, signal processing, estimation theory, identification, time-series analysis, and multisensor information fusion.