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Reliability and Sensitivity Analysis of the k-out-of-n:G Warm Standby Parallel Repairable System with Replacement at Common-Cause Failure using Markov Model

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Abstract: Standby redundancy is a technique that has been widely applied for improving system reliability and availability in system design. In this paper, probabilistic model for a redundant system with replacement at each common-cause failure has been developed to analyze the reliability measures using Markov models. We investigate the reliability and sensitivity analysis of k-out-of-n:G warm standby parallel repairable system. All failure and repair times of the system are exponentially distributed and when one of the operating primary units fails then it is instantaneously replaced by a warm standby unit if one is available. Comparative analysis of reliability measures between two dissimilar configurations has been developed. Configuration I is a 2-out-of-4:G warm standby parallel repairable system, while Configuration II is a 2-out-of-5:G warm standby parallel repairable system. We get a closed-form solution of the reliability measures of the system for the two configurations. Comparisons are performed for specific values of system parameter. Sensitivity analysis is also carried out to depict the effect of various parameters on the reliability function and mean time to failure of the system. Numerical example is given to illustrate the results obtained.

Keywords: k-out-of-n:G warm standby, reliability, parallel, common-cause failure, Markov model, sensitivity analysis.

1 Introduction

Standby redundancy is a technique used to improve system reliability and availability. Standby redundancy represents a situation with one unit operating and a number of units on standby. [1] Classified standby redundancy according to failure characteristics; hot standby, cold standby, and warm standby. Warm standby repairable systems have received attention by several authors. [2] analyzed the dynamic behavior of a two unit parallel system with warm standby and common-cause failure. The system is composed of three identical units; two units are operating and one unit is in warm standby state. [3] studied reliability and availability for four series configurations with both warm and cold standby and common-cause failure.[4] and [5] analyzed the reliability and availability of warm standby systems with common-cause failures and human errors. [6] considered a warm standby redundant system with (M+N) identical units, r repair facilities. The system is under common-cause failure and repair times are arbitrary distributed.

The k-out-of-n:G repairable system is one of the most popular and widely used systems in practice. The k-out-of-n:G systems have been studied in certain situations where redundancy is of importance. Redundancy is required not only to extend the functioning of the system but also to achieve a certain reliability of the system. The k-out-of-n:G systems can be classified into:

- **Active redundant systems (k-out-of-n:G system):** in which all the n units are active even though only k units are required for the proper functioning of the system;
- **Cold standby systems**: in which the n-k cold standby units will not be active and upon failure of one of the k active units, cold standby unit will instantaneously replace the failed unit;
- **Warm standby systems:** in which the n-k warm standby units will have a smaller failure rate compared to the k active ones;

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- **Hot standby systems:** in which the n-k hot standby units and the k active ones will have the same failure rate.

Due to their importance in industries and design, the k-out-of-n:G systems have received attention from different researchers. [7] analyzed the reliability and availability of a k-out-of-n:G system with three failures using Markov model. [8] presented the reliability and availability analysis of a k-out-of-(M+S):G warm standby system with time varying failure and repair rates in presence of common-cause failure. [9] presented continuous-time homogeneous Markov process to evaluate availability, reliability and MTTF for circular consecutive k-out-of-n:G system with repairman. [10] analyzed the k-out-of-n:G system model with critical human errors, common-cause failures and time dependent system repair-rate. [11] analyzed the k-out-of-(M+N):G warm standby system. In the system, not all components in standby can be treated as identical as they have different failure and repair rates. [12] presented a continuous time Markov chain (CTMC) model to obtain closed form expressions of the mean time between system failures (MTBF) for k-out-of-n:G systems subject to M exponential failure modes and repairs. [13] made a brief review on standby redundancy techniques. In their research, a general closed form equation was developed for system reliability of a k-out-of-n warm standby system. [14] analyzed the mean time to system failure of a repairable 2-out-of-4 warm standby system. [15] analyzed the mean time to system failure of 2-out-of-5 warm standby repairable system with replacement at the occurrence of each common-cause failure using Kolmogorov's forward equations method.

In recent years, it has been realized that in order to predict realistic reliability and availability of standby systems, the occurrence of common-cause failures must be considered. Common-cause failures can only occur in the system with more than one good unit. A common-cause failure is defined as any instance where multiple units or components fail due to a single cause. The concept of common-cause failure and its impact on reliability measures of system effectiveness has been introduced by several authors. [16] studied common-cause failure analysis of a non-identical unit parallel repairable system with arbitrary distributed repair times. [17] studied cost analysis of a system involving common-cause failures and preventive maintenance. [18] analyzed the reliability of redundant system with common-cause failure.

In this paper, we analyze the reliability measures of the k-out-of-n:G warm standby parallel repairable system with replacement at each common-cause failure with constant failure and repair rates of the operating primary units and warm standby units using Markov model.

The following reliability measures of the system are obtained using Markov model for two configurations. Configuration I is a 2-out-of-4:G warm standby parallel repairable system, while Configuration II is a 2-out-of-5:G warm standby parallel repairable system.

i. Availability and steady state availability of the system.

ii. Reliability and mean time to failure of the system.

We also perform sensitivity analysis for changes in the reliability and mean time to failure of the system along with changes in specific values of the system parameters.

This paper is organized as follows: Section 2 is devoted to the description and basic assumptions of the system. Section 3 is devoted to the reliability and availability analysis of the k-out-of-n:G warm standby parallel repairable system. In Section 4 we make a comparative analysis of the reliability measures of two dissimilar configurations of the k-out-of-n:G warm standby parallel repairable system. Sensitivity analysis is carried out to depict the effect of various parameters on the reliability function and mean time to failure of the system. In Section 5, a numerical example is given. In Section 6, some concluding remarks are given.

2 Model Description and Assumptions

The following assumptions are associated with the system:

1. The system under consideration is a k-out-of-n:G warm standby parallel repairable system. At least k units of the system are required for the system to work.
2. The system consists of k primary units and $n-k$ warm standby units and all the units are identical.
3. The system is subject to failure of a single unit and common-cause failure of more than one unit.
4. All primary units and warm standby units are considered repairable.
5. Each of the primary units fails independent of the state of the others, according to an exponential failure time distribution with parameter λ, and the available warm standby units can also fail according to an exponential failure time distribution with parameter λ_s, $(0<\lambda_s<\lambda)$.

6. When one of the primary units fails, it is instantaneously replaced by a warm standby unit if one is available. Switching from warm standby to operative unit is perfect and instantaneous.
7. When a standby unit switches into the operating primary unit successfully, its failure characteristics will be the same as that of the operating primary units.
8. Whenever one of the operating units or warm standby units fail, it is immediately sent to a repair. After repairing, the failed unit works like a new one.
9. There is a single repairman who attends to the failed units.
10. The repairmen can repair only one failed unit at a time.
11. The failed system repair times are exponentially distributed. The units are repaired according to an exponential repair time distribution with parameter μ.
12. Common-cause failure and failure of a single unit are statistically independent.
13. The common-cause failure affects only the units in operation and the affected units are replaced instantaneously.
14. The system at any working state can completely fail due to common-cause failure with constant common-cause failure rate.
15. When the system fails, no failure will occur for other working components.

Notations:

S_i : state of the system, $i = 0,1,2,....,n-k,n$

λ : failure rate of a single primary unit

λ_s : failure rate of a single warm standby unit

μ : repair rate of a single unit

λ_{cj}/β_j : common-cause failure rate/replacement rate of j units, $j = 2,3,4,....,n$

$P_i(t)$: probability that the system is in state i at time t, $i = 0,1,2,....,n-k,n$

$P_i^*(s)$: Laplace transformation of $P_i(t)$, $i = 0,1,2,....,n-k,n$

$A_1(t)/A_2(t)$: availability of Configuration I / Configuration II

A_1/A_2 : steady state availability of the Configuration I / Configuration II

$R_1(t)/R_2(t)$: reliability of Configuration I / Configuration II

$R_1^*(s)/R_2^*(s)$: Laplace transformation of the reliability function of Configuration I / Configuration II

$MTTF_1/MTTF_2$: mean time to failure of Configuration I / Configuration II

3 Reliability and Availability Analysis of the System

With the help of the above notations and possible states of the system; the state transition diagram of the k-out-of-n:G warm standby parallel repairable system with replacement at each common-cause failure is shown in Figure 1.

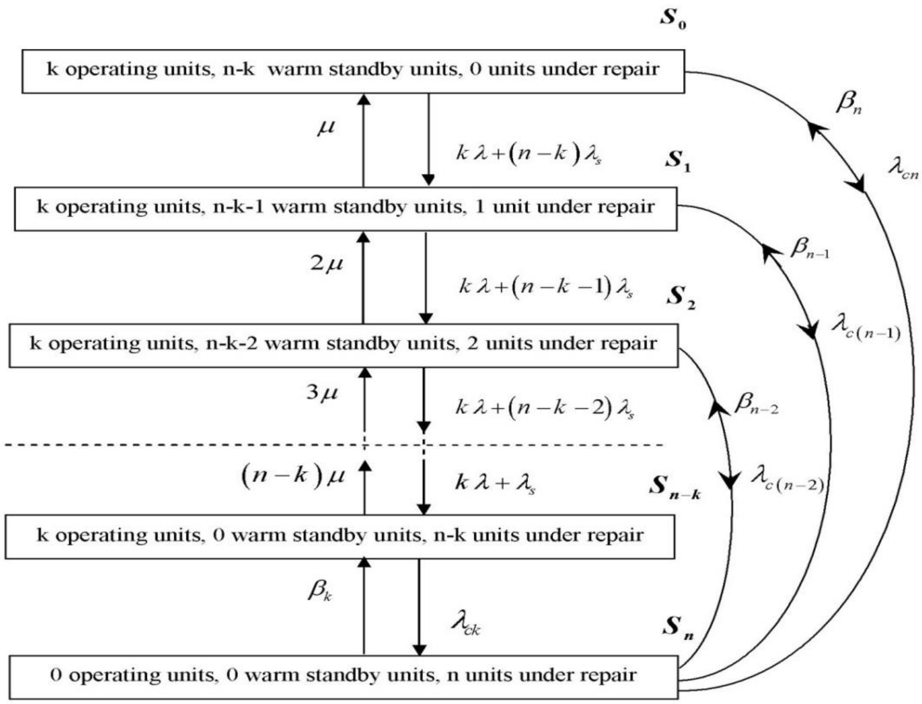

Fig. 1: State transition diagram of the k-out-of-n:G warm standby parallel repairable system

Probability considerations gives the following set of differential difference equations associated with the state transition diagram of the k-out-of-n:G warm standby parallel repairable system:

$$\frac{d}{dt}P_0(t) = -\left(k\lambda + (n-k)\lambda_s + \lambda_{cn}\right)P_0(t) + \mu P_1(t) + \beta_n P_n(t) \tag{1}$$

$$\begin{aligned}\frac{d}{dt}P_i(t) = &-\left(k\lambda + (n-k-i)\lambda_s + \lambda_{c(n-i)} + i\mu\right)P_i(t) + \left(k\lambda + (n-k-i+1)\lambda_s\right)P_{i-1}(t) \\ &+ (i+1)\mu P_{i+1}(t) + \beta_{n-i}P_n(t) \qquad ,0 < i < n-k\end{aligned} \tag{2}$$

$$\frac{d}{dt}P_{n-k}(t) = -\left(\lambda_{ck} + (n-k)\mu\right)P_{n-k}(t) + \left(k\lambda + \lambda_s\right)P_{n-k-1}(t) + \beta_k P_n(t) \qquad ,n-k \ge 2 \tag{3}$$

$$\frac{d}{dt}P_n(t) = -\sum_{i=0}^{n-k}\beta_{n-i}P_n(t) + \sum_{i=0}^{n-k}\lambda_{c(n-i)}P_i(t) \qquad ,n-k \ge 2 \tag{4}$$

The system availability is given by:

$$A(t) = \sum_{i=0}^{n-k} P_i(t) \tag{5}$$

The initial conditions of the system are:

$$\begin{aligned} &P_0(0) = 1 \\ &P_i(0) = 0, \qquad i = 1,2...,n-k,n \end{aligned} \tag{6}$$

To obtain the reliability function of the system, we assume that the failed states are absorbing states and set all transition rates from these states equal to zero. Now let $P_i(t) \to \tilde{P}_i(t)$ $,i = 0,1,....,n-k,n$ in equations (1-4).

The system reliability is given by:

$$R(t) = \sum_{i=0}^{n-k} \tilde{P}_i(t) \tag{7}$$

4 Comparative Analysis of Reliability Measures

On the basis of the above description and assumptions, we investigate the reliability measures of two dissimilar configurations. Configuration I is a 2-out-of-4:G warm standby parallel repairable system, while Configuration II is a 2-out-of-5:G warm standby parallel repairable system.

4.1 Configuration I

Configuration I consists of 2 operating primary units and 2 warm standby units and all the units are identical and at least 2 units are required for the system to work.

4.1.1 Availability Analysis of the System

The state transition diagram of the 2-out-of-4:G warm standby parallel repairable system is shown in Figure 2.

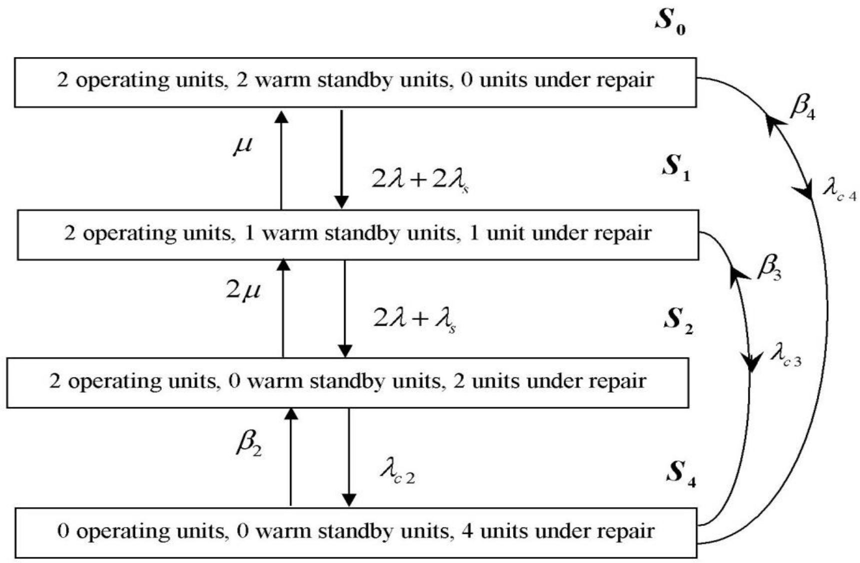

Fig 2: State transition diagram of 2-out-of-4:G warm standby parallel repairable system

The system of differential difference equations associated with the state transition diagram of the system are given by:

$$\frac{d}{dt}P_0(t) = -(2\lambda + 2\lambda_s + \lambda_{c4})P_0(t) + \mu P_1(t) + \beta_4 P_4(t) \tag{8}$$

$$\frac{d}{dt}P_1(t) = -(2\lambda + \lambda_s + \lambda_{c3} + \mu)P_1(t) + (2\lambda + 2\lambda_s)P_0(t) + 2\mu P_2(t) + \beta_3 P_4(t) \tag{9}$$

$$\frac{d}{dt}P_2(t) = -(\lambda_{c2} + 2\mu)P_2(t) + (2\lambda + \lambda_s)P_1(t) + \beta_2 P_4(t) \tag{10}$$

$$\frac{d}{dt}P_4(t) = -(\beta_2 + \beta_3 + \beta_4)P_4(t) + \lambda_{c4}P_0(t) + \lambda_{c3}P_1(t) + \lambda_{c2}P_2(t) \tag{11}$$

The system availability is given by:

$$A_1(t) = P_0(t) + P_1(t) + P_2(t) \tag{12}$$

The initial conditions of the system are given by:

$$P_0(0)=1$$
$$P_1(0)=P_2(0)=P_4(0)=0 \quad (13)$$

The steady state equations of the system are then:

$$0=-(2\lambda+2\lambda_s+\lambda_{c4})P_0+\mu P_1+\beta_4 P_4 \quad (14)$$

$$0=-(2\lambda+\lambda_s+\lambda_{c3}+\mu)P_1+(2\lambda+2\lambda_s)P_0+2\mu P_2+\beta_3 P_4 \quad (15)$$

$$0=-(\lambda_{c2}+2\mu)P_2+(2\lambda+\lambda_s)P_1+\beta_2 P_4 \quad (16)$$

$$0=-(\beta_2+\beta_3+\beta_4)P_4+\lambda_{c4}P_0+\lambda_{c3}P_1+\lambda_{c2}P_2 \quad (17)$$

Solving the system of linear equations (14-17) using Maple program, we get the state probabilities determining the steady state availability of the system:

The steady state availability of the system is given by:

$$A_1=P_0+P_1+P_2 \quad (18)$$

4.1.2 System Reliability and Mean Time to Failure

We assume that the failed states are absorbing states and set all transition rates from these states equal to zero. Now let $P_i(t)\rightarrow\tilde{P}_i(t)\quad ,i=0,1,2,4$ in equations (8-11).

The set of differential equations associated with the system are given by:

$$\frac{d}{dt}\tilde{P}_0(t)=-(2\lambda+2\lambda_s+\lambda_{c4})\tilde{P}_0(t)+\mu\tilde{P}_1(t) \quad (19)$$

$$\frac{d}{dt}\tilde{P}_1(t)=-(2\lambda+\lambda_s+\lambda_{c3}+\mu)\tilde{P}_1(t)+(2\lambda+2\lambda_s)\tilde{P}_0(t)+2\mu\tilde{P}_2(t) \quad (20)$$

$$\frac{d}{dt}\tilde{P}_2(t)=-(\lambda_{c2}+2\mu)\tilde{P}_2(t)+(2\lambda+\lambda_s)\tilde{P}_1(t) \quad (21)$$

$$\frac{d}{dt}\tilde{P}_4(t)=\lambda_{c4}\tilde{P}_0(t)+\lambda_{c3}\tilde{P}_1(t)+\lambda_{c2}\tilde{P}_2(t) \quad (22)$$

The system reliability is given by:

$$R_1(t)=\tilde{P}_0(t)+\tilde{P}_1(t)+\tilde{P}_2(t) \quad (23)$$

Taking Laplace transformation of equations (19-22) using the initial conditions equation (13), we obtain:

$$(s+2\lambda+2\lambda_s+\lambda_{c4})\tilde{P}_0^*(s)-\mu\tilde{P}_1^*(s)=1 \quad (24)$$

$$(s+2\lambda+\lambda_s+\lambda_{c3}+\mu)\tilde{P}_1^*(s)-(2\lambda+2\lambda_s)\tilde{P}_0^*(s)-2\mu\tilde{P}_2^*(s)=0 \quad (25)$$

$$(s+\lambda_{c2}+2\mu)\tilde{P}_2^*(s)-(2\lambda+\lambda_s)\tilde{P}_1^*(s)=0 \quad (26)$$

$$s\tilde{P}_4^*(s)-\lambda_{c4}\tilde{P}_0^*(s)-\lambda_{c3}\tilde{P}_1^*(s)-\lambda_{c2}\tilde{P}_2^*(s)=0 \quad (27)$$

On solving equations (24-27), we obtain the Laplace transformations $P_i^*(s),\ i=0,1,2,4$.

The Laplace transformation of the reliability function of the system is given by:

$$R_1^*(s)=\tilde{P}_0^*(s)+\tilde{P}_1^*(s)+\tilde{P}_2^*(s) \quad (28)$$

The mean time to system failure ($MTTF_1$) is obtained using:

$$MTTF_1 = \int_0^\infty R_1(t)dt = \lim_{s \to 0} R_1^*(s) = R_1^*(0) \quad (29)$$

4.1.3 Sensitivity Analysis of the Reliability and Mean Time to Failure of the System

The objective of reliability sensitivity analysis is to determine input variables that mostly contribute to the variability of the failure probability.

The results which can be obtained from any model are sensitive to manyfactors. Inthis paper, we concentrate our attention on parametricsensitivity analysis**.** Parametric sensitivity analysis helps in identifying the model parameters that could produce significant modeling errors.

One approach to parametric sensitivity analysis is to use upper and lower bounds on each parameter in the model to compute optimistic and conservative bounds on system reliability ([19]). Our approach is to compute the derivative of the measures of interest with respect to the model parameters ([20] and [21]).

We perform sensitivity analysis for changes in the system reliability $R_1(t)$ resulting from changes in parameters $\lambda, \lambda_{c2}, \lambda_{c3}, \lambda_{c4}$ and μ. We obtain the derivative of equation (23) with respect to the parameters $\lambda, \lambda_{c2}, \lambda_{c3}, \lambda_{c4}$ and μ.

Also we perform sensitivity analysis for changes in the mean time to failure $MTTF_1$ of the system resulting from changes in parameters $\lambda, \lambda_s, \lambda_{c2}, \lambda_{c3}, \lambda_{c4}$ and μ. We obtain the derivative of equation (29) with respect to the parameters $\lambda, \lambda_s, \lambda_{c2}, \lambda_{c3}, \lambda_{c4}$ and μ.

4.2 Configuration II

Configuration II consists of 2 operating primary units and 3 warm standby units and all the units are identical and at least 2 units are required for the system to work.

4.2.1 Availability Analysis of the System

The state transition diagram of the 2-out-of-5:G warm standby parallel repairable system is shown in Figure 3.

The system of differential difference equations associated with the state transition diagram of the system are given by:

$$\frac{d}{dt}P_0(t) = -(2\lambda + 3\lambda_s + \lambda_{c5})P_0(t) + \mu P_1(t) + \beta_5 P_5(t) \quad (30)$$

$$\frac{d}{dt}P_1(t) = -(2\lambda + 2\lambda_s + \lambda_{c4} + \mu)P_1(t) + (2\lambda + 3\lambda_s)P_0(t) + 2\mu P_2(t) + \beta_4 P_5(t) \quad (31)$$

$$\frac{d}{dt}P_2(t) = -(2\lambda + \lambda_s + \lambda_{c3} + 2\mu)P_2(t) + (2\lambda + 2\lambda_s)P_1(t) + 3\mu P_3(t) + \beta_3 P_5(t) \quad (32)$$

$$\frac{d}{dt}P_3(t) = -(\lambda_{c2} + 3\mu)P_3(t) + (2\lambda + \lambda_s)P_2(t) + \beta_2 P_5(t) \quad (33)$$

$$\frac{d}{dt}P_5(t) = -(\beta_2 + \beta_3 + \beta_4 + \beta_5)P_5(t) + \lambda_{c5}P_0(t) + \lambda_{c4}P_1(t) + \lambda_{c3}P_2(t) + \lambda_{c2}P_3(t) \quad (34)$$

The system availability is given by:

$$A_2(t) = P_0(t) + P_1(t) + P_2(t) + P_3(t) \quad (35)$$

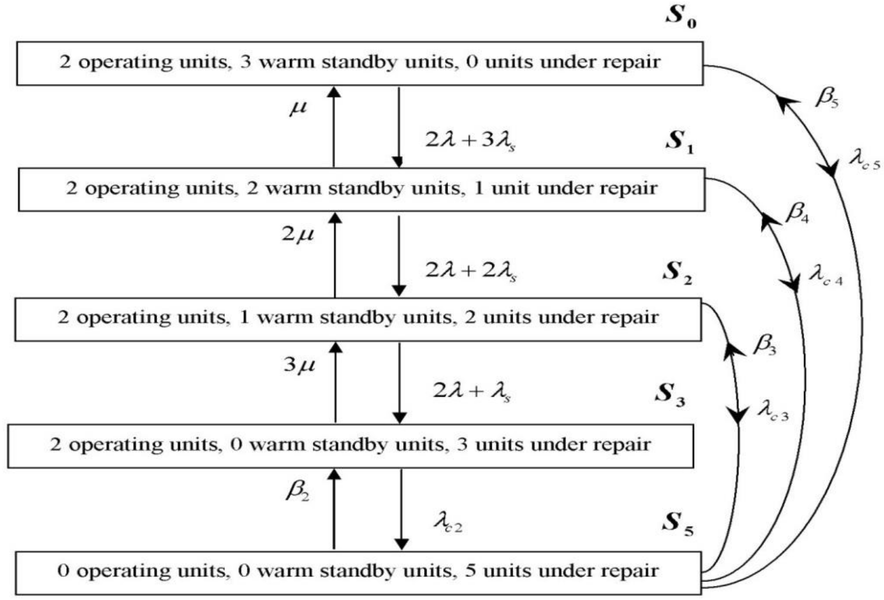

Fig. 3: State transition diagram of 2-out-of-5:G warm standby parallel repairable system

The initial conditions of the system are given by:

$$P_0(0)=1 \qquad P_1(0)=P_2(0)=P_3(t)=P_5(t)=0 \tag{36}$$

Usually we are mainly concerned with systems running for a long time. The steady state availability of the system is the availability function as time approaches infinity. This can be obtained mathematically by taking $\frac{d}{dt}\rightarrow 0$ as $t\rightarrow\infty$ in the system of equations (30-34) therefore, the system of equations (30-34) reduces to the following system of linear equations:

$$0=-(2\lambda+3\lambda_s+\lambda_{c5})P_0+\mu P_1+\beta_5 P_5 \tag{37}$$

$$0=-(2\lambda+2\lambda_s+\lambda_{c4}+\mu)P_1+(2\lambda+3\lambda_s)P_0+2\mu P_2+\beta_4 P_5 \tag{38}$$

$$0=-(2\lambda+\lambda_s+\lambda_{c3}+2\mu)P_2+(2\lambda+2\lambda_s)P_1+3\mu P_3+\beta_3 P_5 \tag{39}$$

$$0=-(\lambda_{c2}+3\mu)P_3+(2\lambda+\lambda_s)P_2+\beta_2 P_5 \tag{40}$$

$$0=-(\beta_2+\beta_3+\beta_4+\beta_5)P_5+\lambda_{c5}P_0+\lambda_{c4}P_1+\lambda_{c3}P_2+\lambda_{c2}P_3 \tag{41}$$

Solving the system of linear equations (37-41) using Maple program, we get the state probabilities determining the steady state availability of the system.

The steady state availability of the system is given by:

$$A_2=P_0+P_1+P_2+P_3 \tag{42}$$

4.2.2 System Reliability and Mean Time to Failure

To obtain the reliability function of the system, we assume that the set of failed states are absorbing states and set all transition rates from these states equal to zero. Now let $P_i(t)\rightarrow\tilde{P}_i(t)$, $i=0,1,2,3,5$ in equations (37-41).

The set of differential equations associated with the system are given by:

$$\frac{d}{dt}\tilde{P}_0(t) = -(2\lambda + 3\lambda_s + \lambda_{c5})\tilde{P}_0(t) + \mu\tilde{P}_1(t) \tag{43}$$

$$\frac{d}{dt}\tilde{P}_1(t) = -(2\lambda + 2\lambda_s + \lambda_{c4} + \mu)\tilde{P}_1(t) + (2\lambda + 3\lambda_s)\tilde{P}_0(t) + 2\mu\tilde{P}_2(t) \tag{44}$$

$$\frac{d}{dt}\tilde{P}_2(t) = -(2\lambda + \lambda_s + \lambda_{c3} + 2\mu)\tilde{P}_2(t) + (2\lambda + 2\lambda_s)\tilde{P}_1(t) + 3\mu\tilde{P}_3(t) \tag{45}$$

$$\frac{d}{dt}\tilde{P}_3(t) = -(\lambda_{c2} + 3\mu)\tilde{P}_3(t) + (2\lambda + \lambda_s)\tilde{P}_2(t) \tag{46}$$

$$\frac{d}{dt}\tilde{P}_5(t) = \lambda_{c5}\tilde{P}_0(t) + \lambda_{c4}\tilde{P}_1(t) + \lambda_{c3}\tilde{P}_2(t) + \lambda_{c2}\tilde{P}_3(t) \tag{47}$$

The system reliability is given by:

$$R_2(t) = \tilde{P}_0(t) + \tilde{P}_1(t) + \tilde{P}_2(t) + \tilde{P}_3(t) \tag{48}$$

Taking Laplace transformation of equations (43-47) using the initial conditions equation (36), we obtain:

$$(s + 2\lambda + 3\lambda_s + \lambda_{c5})\tilde{P}_0^*(s) - \mu\tilde{P}_1^*(s) = 1 \tag{49}$$

$$(s + 2\lambda + 2\lambda_s + \lambda_{c4} + \mu)\tilde{P}_1^*(s) - (2\lambda + 3\lambda_s)\tilde{P}_0^*(s) - 2\mu\tilde{P}_2^*(s) = 0 \tag{50}$$

$$(s + 2\lambda + \lambda_s + \lambda_{c3} + 2\mu)\tilde{P}_2^*(s) - (2\lambda + 2\lambda_s)\tilde{P}_1^*(s) - 3\mu\tilde{P}_3^*(s) = 0 \tag{51}$$

$$(s + \lambda_{c2} + 3\mu)\tilde{P}_3^*(s) - (2\lambda + \lambda_s)\tilde{P}_2^*(s) = 0 \tag{52}$$

$$s\tilde{P}_5^*(s) - \lambda_{c5}\tilde{P}_0^*(s) - \lambda_{c4}\tilde{P}_1^*(s) - \lambda_{c3}\tilde{P}_2^*(s) - \lambda_{c2}\tilde{P}_3^*(s) = 0 \tag{53}$$

On solving equations (49-53), we obtain the Laplace transformations $P_i^*(s),\ i = 0,1,2,3,5$.

The Laplace transformation of the reliability function of the system is given by:

$$R_2^*(s) = \tilde{P}_0^*(s) + \tilde{P}_1^*(s) + \tilde{P}_2^*(s) + \tilde{P}_3^*(s) \tag{54}$$

The mean time to system failure ($MTTF_2$) is obtained using:

$$MTTF_2 = \int_0^\infty R_2(t)dt = \lim_{s\to 0} R_2^*(s) = R_2^*(0) \tag{55}$$

4.2.3 Sensitivity Analysis of the Reliability and Mean Time to Failure of the System

We perform sensitivity analysis for changes in the reliability of the system $R_2(t)$ resulting from changes in parameters $\lambda, \lambda_{c2}, \lambda_{c3}, \lambda_{c4}, \lambda_{c5}$ and μ. We obtain the derivative of equation (48) with respect to the parameters $\lambda, \lambda_{c2}, \lambda_{c3}, \lambda_{c4}, \lambda_{c5}$ and μ.

Also we perform sensitivity analysis for changes in the mean time to failure $MTTF_2$ of the system resulting from changes in parameters $\lambda, \lambda_s, \lambda_{c2}, \lambda_{c3}, \lambda_{c4}, \lambda_{c5}$ and μ. We obtain the derivative of equation (55) with respect to the parameters $\lambda, \lambda_s, \lambda_{c2}, \lambda_{c3}, \lambda_{c4}, \lambda_{c5}$ and μ.

5 Numerical Example

For comparative analysis of reliability measures between configuration I and configuration II; the failure, repair, common-cause failure and replacement rates are given by:

$$\lambda = 0.2, \quad \lambda_s = 0.1, \quad \mu = 0.4, \quad \lambda_{c5} = 0.05, \quad \lambda_{c4} = 0.1, \quad \lambda_{c3} = 0.15$$
$$\lambda_{c2} = 0.2, \quad \beta_5 = 0.25, \quad \beta_4 = 0.5, \quad \beta_3 = 0.6, \quad \beta_2 = 0.7$$

Figures 4–5 show the availability and reliability for configuration I and II versus time. We conclude that the availability and reliability of configuration II is greater than the availability and reliability of configuration I.

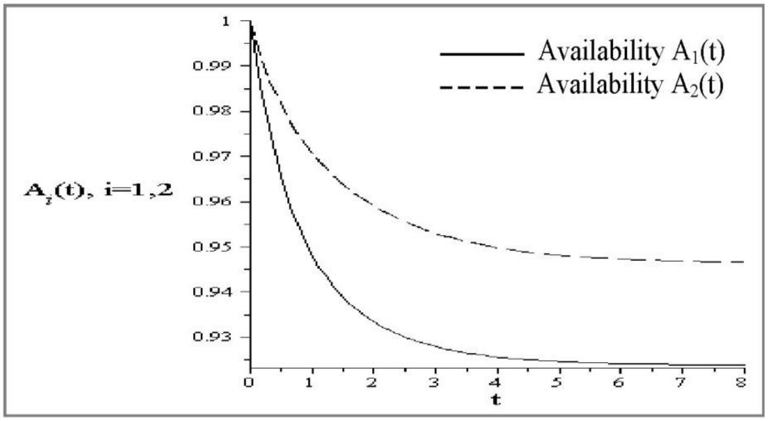

Fig. 4: Availability $A_i(t)$ versus time t, i=1,2

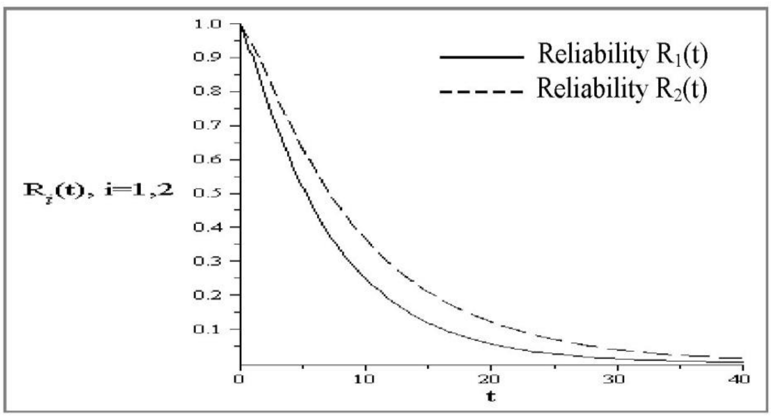

Fig. 5: Reliability $R_i(t)$ versus time t, i=1,2

Figures 6–7 show the steady state availability of configuration I and II versus failure and repair rates. It can be observed that the steady state availability configuration II is greater than that of configuration I and the steady state availability of the two configurations decreases with the increase in the failure rate λ and increases with the increase in the repair rate μ.

Sensitivity analysis for changes in the reliability functions $R_1(t)$ and $R_2(t)$ resulting from changes in specific values of the system parameters $\lambda, \lambda_{c2}, \lambda_{c3}, \lambda_{c4}, \lambda_{c5}$ and μ are shown in Figures 8–9. We can easily observe that the system parameters $\lambda, \lambda_{c2}, \lambda_{c3}, \lambda_{c4}, \lambda_{c5}$ have big impact on the reliability functions $R_1(t)$ and $R_2(t)$ of configuration I and II at the same time.

The numerical results of the sensitivity analysis of the mean time to failure of configuration I and II resulting from changes in system parameters $\lambda, \lambda_s, \lambda_{c2}, \lambda_{c3}, \lambda_{c4}, \lambda_{c5}$ and μ are shown in Tables 1–2. It can be seen from Table 1 that the order of impacts of the system parameters on $MTTF_1$ are: $\lambda_{c4} > \lambda_{c3} > \lambda_{c2} > \lambda > \lambda_s > \mu$.

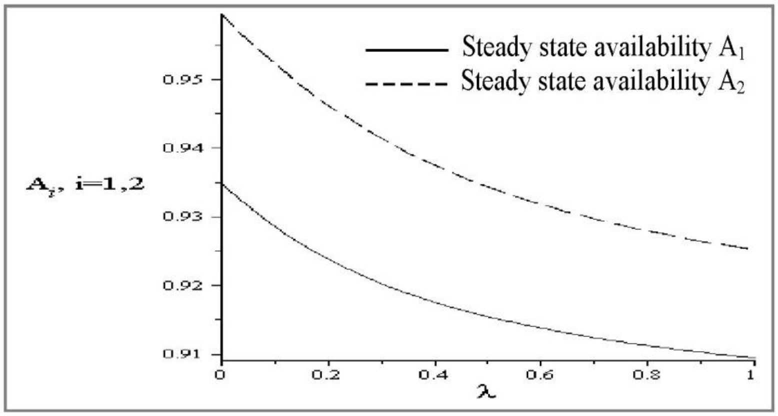

Fig. 6: Steady state availability A_i versus failure rate λ , i=1,2

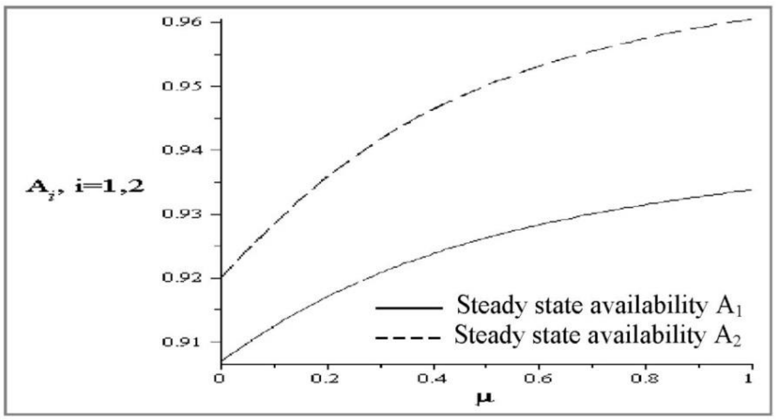

Fig. 7: Steady state availability A_i versus repair rate μ , i=1,2

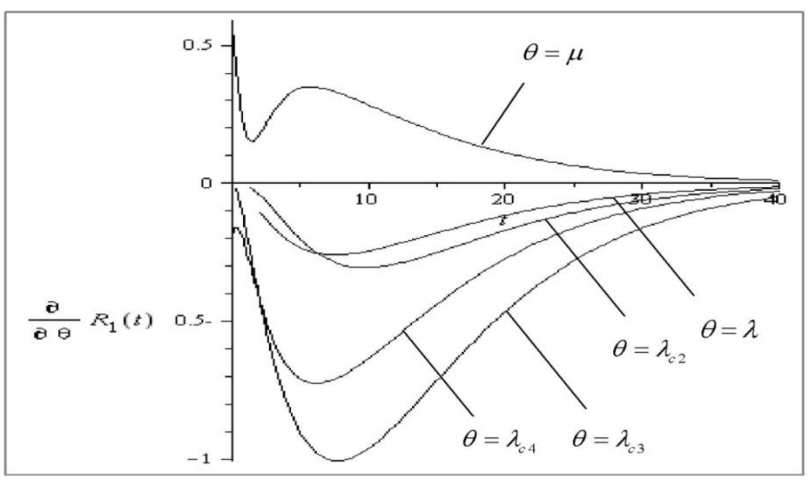

Fig. 8: Sensitivity of the reliability of configuration I with respect to system parameters.

From Table 2 the order of impacts of the system parameters on $MTTF_2$ are: $\lambda_{c4} > \lambda_{c5} > \lambda_{c3} > \lambda_s > \lambda > \mu > \lambda_{c2}$ and the mean time to failure of the two configurations are not sensitive to the replacement rates.

Finally we perform sensitivity analysis for changes in the mean time to failure $MTTF_i$ $, i = 1, 2$ along with changes in specific values of the system parameters $\lambda, \lambda_s, \lambda_{c2}, \lambda_{c3}, \lambda_{c4}, \lambda_{c5}$ and μ.

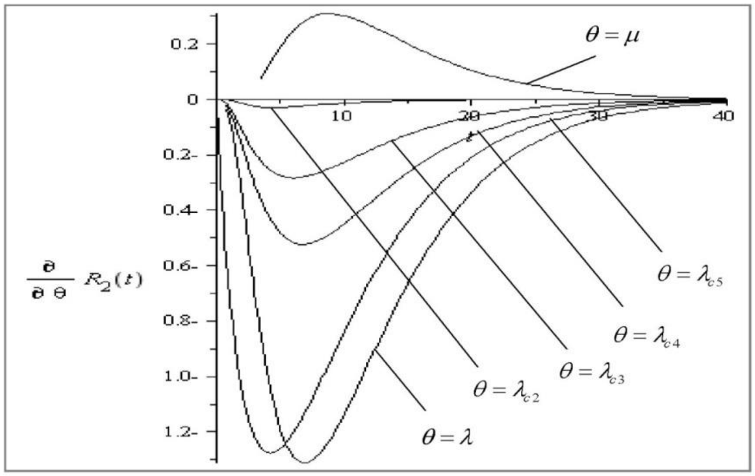

Fig. 9: Sensitivity of the reliability of configuration II with respect to system parameters

Table 1: Sensitivity analysis for $MTTF_1$

	$\theta=\lambda$	$\theta=\lambda_s$	$\theta=\mu$	$\theta=\lambda_{c2}$	$\theta=\lambda_{c3}$	$\theta=\lambda_{c4}$
$\frac{\partial MTTF_1}{\partial\theta}$	-4.76	-3.78	2.27	-8.92	-18.82	-21.8

Table 2: Sensitivity analysis for $MTTF_2$

	$\theta=\lambda$	$\theta=\lambda_s$	$\theta=\mu$	$\theta=\lambda_{c2}$	$\theta=\lambda_{c3}$	$\theta=\lambda_{c4}$	$\theta=\lambda_{c5}$
$\frac{\partial MTTF_2}{\partial\theta}$	-11.51	-12.84	6.5	-6.06	-18	-33	-32.08

It should be noted that these conclusions are only valid for the given values of system parameters. We may reach other conclusions for other values of the system parameters.

6 Conclusion

In this paper we utilized the Markov model to develop the reliability measures of the k-out-of-n:G warm standby parallel repairable system**.** All failure and repair rates of the system are constant. Comparative analysis of reliability measures between two dissimilar configurations has been developed. Configuration I is a 2-out-of-4:G warm standby parallel repairable system, while Configuration II is a 2-out-of-5:G warm standby parallel repairable system. The system of differential equations with the initial conditions has been solved numerically using Laplace transformation by the aid of Maple program. Graphical representation of the reliability and availability of the two configurations versus time are made. Sensitivity analysis is also carried out to depict the effect of various parameters on the reliability function and mean time to failure of the system.

Numerical example is given to illustrate the results obtained, and the results are shown graphically by the aid of Maple program. Results indicate that the reliability and availability of the system increase by increasing of the number of warm standby units. And the reliability and mean time to failure of the two configurations are sensitive to the failure and repair rates of the system and are not sensitive to the replacement rates.

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