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A New View of Special Types of Subgraphs with Applications on Circulation of Hepatic Portal System

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Abstract: Fuzzy graph theory has enormous range of applications in neural networks, physical, logic, algebra, topology, operations research, pattern recognition, artificial intelligence, natural and sociobiological sciences and in several other areas. We propose new concepts for generating different types of fuzzy subgraph defined as $\widetilde{\kappa}_1 \widetilde{\kappa}_2$ - fuzzy subgraph which depend on the changeable of the memberships of vertices and edges with discussing new type of paths $P^{\widetilde{\kappa}_1 \widetilde{\kappa}_2}$. Our approach is based on studying its effect of strength on different types of arcs by explaining several examples and deducing new important properties. Finally, we demonstrate all of the above on the circulation of hepatic portal system. Also, we analyze the influence of the resultant properties with medical phenomena and views with illustrating the consequences of abnormal cases. A new algorithm was created a code through MATLAP methods to calculate the membership of both the vertices and edges of whatever parts to be studied on the circulation of the hepatic portal system.

1 Introduction

Graph theory is a prosperous discipline that containing significant increase of powerful and massive theorems of wide enormous applicability that moving into the mainstream of mathematics which is functioned as an essential structure supported of modern applied mathematics in many extensive and popular branch of many fields like computer science - chemistry -science -statistical physics -biology and etc[1,2,3,4,5].

In biology, it's easy to link between the field of fuzzy graph theory and the medical field theory. For example, we can represent the human heart as graph by working to divide the heart into a set of vertices and a set of edges. The vertices represent the regions where the blood flows on it. Also, the edges represent the pathway of blood through the heart. through that we can deduce any problems or disorder may be happened in heart that affect its function and the normality of the human body[6,7,8,9].

In computer science, Graphs are used in designing the database. Graph database utilizes a modeling of a graph with vertices, edges and properties to represent and store data. This graph structure plays a vital role in the designing database because it provides quick process realization using different functions and properties of the graph[3,10,11].

In computational biochemistry, we can use graph theory in modeling chemical compound some sequences of cell samples have to prevented to resolve the discrepancy between two sequences. This is modeled in the form of graph where the nodes represent the sequences in the sample. An edge will be drawn between two nodes if and only if there is a conflict between the corresponding sequences. The aim is to remove possible nodes to clear all discrepancy[12,13,14,15].

Crisp graph is specialized case of fuzzy graph, let K be a fuzzy graph is pair of functions $K : (\sigma, \mu)$, σ is a fuzzy set includes n- non-empty set V and μ is a fuzzy relation which is symmetric on σ. The underlying graph K^* formed a crisp state of $K : (\sigma, \mu)$ where E is subset of $V \times V$[3,16].

There are some types of fuzzy graph K, such as strong fuzzy graph which satisfy that the membership of every edge is the less value of the membership of its incident vertices for all edges uv. Fuzzy graph K is a complete fuzzy graph if it?s satisfied each vertex linked

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by the other vertices and the membership of each edge in it is equal the minimum of membership of its incident vertices[2,17].

Let a path contain $(n+1)$ of distinct vertices such that each membership of consequently vertices. The strength of each path in fuzzy graph is the weakest membership of arc contain in the path. If the start vertex and the end vertex are the same is called a cycle. If it contains higher number of weakest arcs is called a fuzzy cycle[6,18,19].

To get the strength of any fuzzy graph. Firstly, by fixing two nodes in the graph and obtain all paths between them. Then find the maximum membership of all the strength of these paths. The maximum membership of the strength is the connectivity of fuzzy graph and is denoted by $CONNG(u,v)$ Where u,v any vertices in fuzzy graph[6,20,22].

Complement to the explanation of strength of connectedness[20,21]. There are three classifications of strong arcs. They are α-strong, β-strong and the remaining arcs are as δ-arcs:

–An arc $(u-v)$ is called α-strong arc, if the membership of arc $(u-v)$ is more than the maximum membership of all strength of all paths between two nodes u,v.

–An arc $(u-v)$ is called β-strong arc, if the membership of arc $(u-v)$ is equal to $CONNG-(u,v)(u,v)$.

–An arc $(u-v)$ is called δ-strong arc, if the membership of arc $(u-v)$ is smaller than the maximum membership of all strength of all paths between two nodes u,v.

2 New Result on Strong Fuzzy Subgraph

The fuzzy subgraph is a fuzzy graph that contains some of vertices and edges from the graph with its membership. This fuzzy subgraph is called induced fuzzy subgraph because it contains all vertices of the graph but it doesn?t contain all of the edges. There are many types of fuzzy subgraph such as the previous types mentioned in the introduction that depend on connectivity. Also, there is another type of fuzzy subgraph which is called partial fuzzy subgraph.

The aim of this section is to discuss how to generate different types of fuzzy subgraph by using new definition and propositions which is more general than the concepts of fuzzy subgraph and the concepts of threshold. It preserves the presence of the membership of vertices and edges for the fuzzy subgraph and not converting them into crisp graph which makes it a general state of threshold and fuzzy subgraph. Also, by using new definition and proposition of path in fuzzy graph, some properties are deduced to affect the strength of paths on different types of arc.

Definition 2.1

Let $\tilde{K} = (\tilde{V}, \tilde{E})$ be a strong fuzzy graph with fuzzy set of vertices $\tilde{V} = \{(v_1, \alpha_1), (v_2, \alpha_2)..., (v_n, \alpha_n)\}$ and set of fuzzy edges $\tilde{E} = \{(e_1, \beta_1), (e_2, \beta_2)?, (e_n, \beta_n)\}$ where α_i, β_i are membership of vertices and edges. Any fuzzy subgraph from $\tilde{K}$ is called $\tilde{\kappa}_1 \tilde{\kappa}_2$ - strong fuzzy subgraph if $\alpha(v_i) > \kappa_1$ and $\beta(e_{ij}) > \kappa_2$, for any arbitrary values of $\kappa_1 \kappa_2, 0 \leqslant \kappa_1 \leqslant 1$ and $0 \leqslant \kappa_2 \leqslant 1$.

Definition 2.2

Let $\tilde{K} = (\tilde{V}, \tilde{E})$ be a strong fuzzy graph with fuzzy set of vertices $\tilde{V} = \{(v_1, \alpha_1), (v_2, \alpha_2)..., (v_n, \alpha_n)\}$ and set of fuzzy edges $\tilde{E} = \{(e_1, \beta_1), (e_2, \beta_2)?, (e_n, \beta_n)\}$ where α_i, β_i are membership of vertices and edges. Any fuzzy subgraph from $\tilde{K}$ is called $\tilde{\kappa}_1 \tilde{\kappa}_2$ - weak fuzzy subgraph if $\alpha(v_i) \geqslant \kappa_1$ and $\beta(e_{ij}) \geqslant \kappa_2$, for any arbitrary values of $\kappa_1 \kappa_2, 0 \leqslant \kappa_1 \leqslant 1$ and $0 \leqslant \kappa_2 \leqslant 1$.

Definition 2.3

Let $\tilde{K} = (\tilde{V}, \tilde{E})$ be a strong fuzzy graph, $P^{\kappa_1 \kappa_2}$ path is $\tilde{\kappa}_1 \tilde{\kappa}_2$ fuzzy subgraph linked paths through every edge and vertices once and linked two fuzzy nodes.

Proposition 2.1

Let $\tilde{K} = (\tilde{V}, \tilde{E})$ be a strong fuzzy graph, then the following properties holds:

a) If $\tilde{\kappa}_1 \tilde{\kappa}_2$ fuzzy subgraph has value (κ_1, κ_2) where $\kappa_1 = min\ \alpha(v_i)$ and $\kappa_2 = min\ \beta(e_{ij})$, then $\tilde{\kappa}_1 \tilde{\kappa}_2$ - fuzzy subgraph isa strong subgraph.

b) If $\tilde{\kappa}_1 \tilde{\kappa}_2$ fuzzy subgraph has value (κ_1, κ_2) where $\kappa_1 > max\ \alpha(v_i)$ and $\kappa_2 > max\ \beta(e_{ij})$, then there is no induced $\tilde{\kappa}_1 \tilde{\kappa}_2$ - weak fuzzy subgraph.

Proof

a) Let $\tilde{K} = (\tilde{V}, \tilde{E})$ be a strong fuzzy graph, if we take $\kappa_1 = min\ \alpha(v_i)$ and $\kappa_2 = min\ \beta(e_{ij})$. From the new definition of (κ_1, κ_2) - fuzzy subgraph. There is a path that goes through all vertices and edges whose membership is greater than the minimum membership for vertices and edges of the graph. This will result in the path having all vertices and edges that its memberships in the fuzzy graph will be strong.

b) Let $\tilde{K} = (\tilde{V}, \tilde{E})$ be a strong fuzzy graph, if $\kappa_1 > max\ \alpha(v_i)$ and $\kappa_2 > max\ \beta(e_{ij})$. From the new definition of (κ_1, κ_2) - fuzzy subgraph, all the edges and vertices that have membership less than the memberships of κ_1 and κ_2 will be deleted that will be effect on the strength of the path.

Proposition 2.2

Let $\tilde{K} = (\tilde{V}, \tilde{E})$ be a strong fuzzy graph, the induced $\tilde{\kappa}_1\tilde{\kappa}_2$ - fuzzy subgraph doesn?t contain any δ-arc. If (κ_1, κ_2) - fuzzy subgraph has values (κ_1, κ_2) where $\kappa_1 = min \ \alpha(v_i)$ and $\kappa_2 > min \ \beta(e_{ij})$. For all graph, there is not contained any isolated point.
Proof. Obliviously

Proposition 2.3

Let $\tilde{K} = (\tilde{V}, \tilde{E})$ be a strong fuzzy graph, if $\tilde{\kappa}_1\tilde{\kappa}_2$ subgraph has values (κ_1, κ_2) where $\kappa_1 > min \ \alpha(v_i)$ and $\kappa_2 = min \ \beta(e_{ij})$. Then the resultant (κ_1, κ_2) - fuzzy subgraph contains α - strong arcs.
Proof. Obliviously

Proposition 2.4

Let $\tilde{K} = (\tilde{V}, \tilde{E})$ be a connected fuzzy graph. There are values of $\tilde{\kappa}_1\tilde{\kappa}_2$ - fuzzy subgraph $i = 1, 2, ..., m$, then the induced fuzzy subgraph
$\tilde{K}_{ij} = \{(\tilde{V}_{ij}, \tilde{E}_{ij}) : \ \tilde{\kappa}_1 \geq \tilde{\kappa}_{1i}, \ \tilde{\kappa}_2 \geq \tilde{\kappa}_{2i}\}$ is satisfied in
$\tilde{K}_{x_{11}x_2} \supseteq \tilde{K}_{x_{12}x_2} \supseteq \tilde{K}_{x_{13}x_2} \supseteq ...\tilde{K}_{x_{1n}x_2}$, for
$\tilde{\kappa}_1 : \tilde{\kappa}_{11}, \ \tilde{\kappa}_{12} < \tilde{\kappa}_{13} < ... << \tilde{\kappa}_{1n}$
And
$\tilde{K}_{x_1x_{21}} \supseteq \tilde{K}_{x_1x_{22}} \supseteq \tilde{K}_{x_1x_{23}} \supseteq ...\tilde{K}_{x_1x_{2m}}$, for
$\tilde{\kappa}_2 : \tilde{\kappa}_{21}, \ \tilde{\kappa}_{22} < \tilde{\kappa}_{23} < ... << \tilde{\kappa}_{2m}$
Proof. Obliviously

Example 2.1:

Consider the fuzzy graph in Fig. (1), $\tilde{K} = (\tilde{V}, \tilde{E})$ be a connected fuzzy graph with vertices $(a, 0.8)$, $(b, 0.6)$, $(c, 1)$ and $(d, 0.9)$ and edges $ab = 0.3$, $bc = 0.5$, $cd = 0.6$, $da = 0.4$, $ac = 0.7$ and $bd = 0.5$. Here, (a, b) and (a, d) are δ-arc, (b, c) and (b, d) are β-strong arcs and (c, d) and (a, c) are α-strong arcs.

Example 2.2:

For example 2.1, let $\tilde{K}_1\tilde{K}_2$ subgraph has value (κ_1, κ_2) where $\kappa_1 = min \ \alpha(v_i)$ and $\kappa_2 > min \ \beta(e_{ij})$.

Then from fig. (2) at $\kappa_2 = 0.4$, we will notice that all arcs that its membership is less than κ_2 will be deleted such as arc (a, b) Here (a, d) are δ-arc and both of (c, d) and (a, c) are α- strong arcs.

Also, at $\kappa_2 = 0.5$, An arc (a, d) is deleted. Here (c, d) and (a, c) are α- strong arcs. By repeating the previous steps by maximize the value of κ_2, we will deduce $\tilde{\kappa}_1\tilde{\kappa}_2$ - fuzzy subgraph that doesn?t contain any δ-arc as shown in Fig. (3).
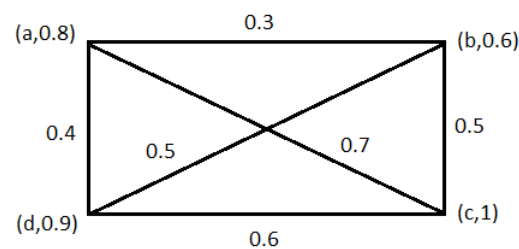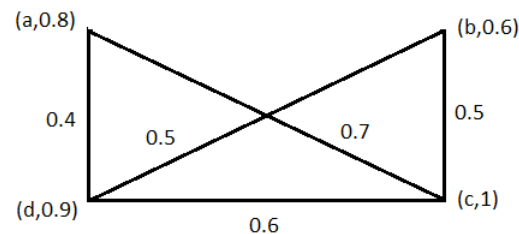

Fig. 1: Example 2.1

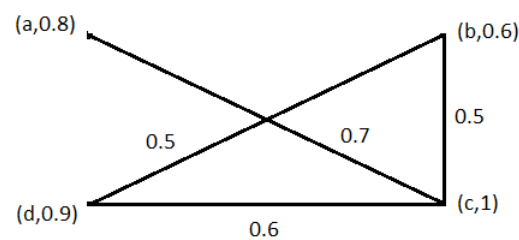**Fig. 2:** Example 2.2 at $\kappa_2 = 0.4$

Fig. 3: Example 2.2 at $\kappa_2 = 0.5$

Example 2.3:

From example 2.1: The idea in the example is our dependence on the degree of the vertices κ_1 so that by increasing the membership of the vertices in each step, it

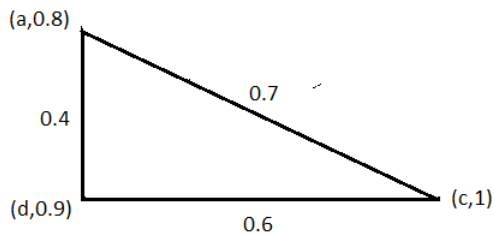will be found that the vertices are omitted and all the edges connected to them.

Fig. 4: Example 2.3 at $\kappa_1 = 0.8$

As shown in Fig. (4), when $\kappa_1 = 0.8$, we can observe the vertex which be deleted and its incident edges.

Finally, after many steps, we will find that the resultant $\tilde{\kappa}_1 \tilde{\kappa}_2$ - fuzzy subgraph contain are α- strong arcs as proposition 2.3.

3 Fuzzy Graph Properties VS Medical Application on Circulation of Hepatic Portal System

The aim of this section is to link graph theory with fuzzy graph on some of medical application by representing the circulation of hepatic portal system in human body as a strong fuzzy in human body by applying all of above on it to know the benefit of studying this application on medical phenomena.

Section 3.1 gives a simple survey about the circulatory system of the human body and its function. Also, it?s mentioned that how the hepatic portal circulation in the human body work with describing its multi functions in the blood. Also, it presents a simple survey about the important functions of liver.

The purpose of section 3.2 is to represent the hepatic portal system as a graph which is classified into set of vertices that represent the regions of the organs of hepatic portal system and set of edges that represent the pathway of blood through it.

The aim of this section 3.3 is to explain a new algorithm using MATLAB methods to help us calculate the effect of each vertex in the circulatory system that represents the organs of system and calculates the effect of each edge in the hepatic circulation that represents the oxidized arteries or the non-oxidized veins and by illustrating them all in steps.

Last section 3.4, is to illustrate the different types of induced fuzzy subgraphs and its properties by different methods and its effect on strength of fuzzy graph, and also comparing that with the medical application.

3.1 The Circulation of Hepatic Portal System:

The circulatory system [23] is known as many scientific names like the cardiovascular system or the vascular system, is a closed system that allow blood in continuous and controlled movement to circulate and transport nutrients acids and oxygen, carbon dioxide and hormones within either the heart or blood vessels at all times. The blood vessels that are part of the closed circulatory system of humans form a vast network to help keep the flow of the blood in one direction that form this network are the arteries, capillaries, and veins [24, 25] as shown in Fig. (5).
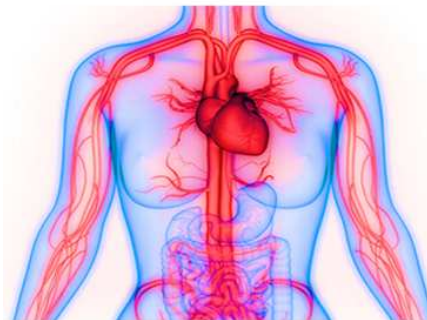

Fig. 5: Circulatory system in human body

The arteries ae portion of the circulatory system, which is responsible for the delivery of oxygenated blood and nutrients that takes away from the human heart to all parts of the body cells. The largest artery in a human body is an aorta in heart[26].

The veins are another part of the circulatory system, which are inelastic blood vessel that transports deoxygenated blood from several regions of the body to the human heart. Also, it circulates blood to provide nutrients to the cells of the body. Venae cava are the largest veins in the human body which enter the right atrium of the human heart from above and below[26].

The smallest and most numerous forms of a blood vessel in the body are capillaries which are tiny blood-containing structures that linked the arteries to the veins. They are small enough to break through the body tissues to permit oxygen, nutrients, and waste products to be exchanged between tissues and the blood[27].

After the pulmonary circuit[28] is complete in the human heart which is responsible for returning the blood relatively high in oxygen concentration into the aorta to

all the body tissues and organs[24,29]. The aorta emerges from the heart as the ascending aorta, turns to the left and curved over the heart through the aortic arch then passes downward as the descending aorta. Then The descending aorta runs down through an opening in the diaphragm at the aortic hiatus and into the abdominal cavity. The continuance of descending thoracic aorta is called the abdominal aorta. The abdominal aorta gives rise to several three major arteries which form an extensive network supplying blood to the gastrointestinal tract are the celiac trunk, superior mesenteric, and inferior mesenteric arteries[30].

The celiac trunk and its three branches supply arterial oxygenated blood to the derivatives of the foregut that involve the left gastric artery to supply blood to the stomach and esophagus. The splenic artery is very tortuous artery that has some branches supplying he superior part of the pancreas by the pancreatic artery as well as the greater curvature of the stomach and its ends at the spleen. finally, the common hepatic artery is the last branch of the celiac trunk, it will branch off to the right and has many actually branches, it becomes the hepatic proper artery which will supply the liver with oxygen. the other branch is the right gastric artery to supply blood along the lesser curvature of stomach and gastroduodenal artery[30] as shown in Fig. (6).
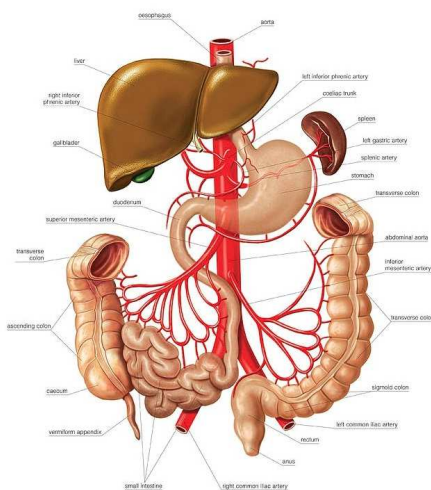

Fig. 6: Blood supply of gastrointestinal tract

The superior mesenteric artery branches from the abdominal aorta inferior arises approximately 2.5 cm after the celiac trunk which branches into several major vessels that supplies blood to the small intestine (duodenum, jejunum, and ileum), the pancreas, and a majority of the large intestine[30].

The inferior mesenteric artery originates from the front of abdominal aorta about 4 cm. It divides into three

branches to supply blood rich with oxygen to the distal segment of the large intestine, including the rectum. It includes the left colic artery of the descending colon, the sigmoid artery and the superior rectal artery that responsible for supplying the blood to different parts of the large intestine and rectum[32].

The venous drainage of the gastrointestinal trunk is the hepatic portal system which is a unique system that connects the capillaries of the gastrointestinal tract with the capillaries in the liver that responsible for processing and storing nutrients in the body like produces plasma proteins, clotting factors, and bile, It is also responsible for cleansing of the bacteria and detoxifying the blood that are picked up by the blood while it is being perfused through the intestines as shown in Fig. (7)[33].
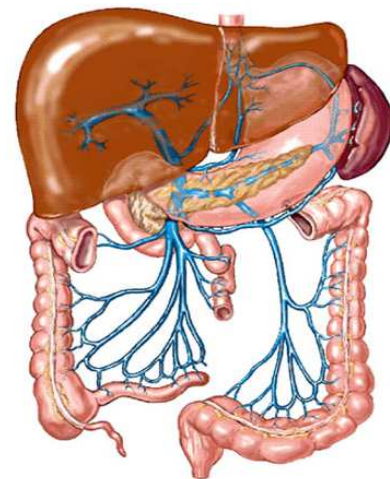

Fig. 7: Hepatic portal circulation

The hepatic portal system consists of giant blood vessel called the hepatic portal vein and the veins that drain into it filters the blood directly into the liver and provides it with 70% of its blood supply. The hepatic portal vein is formed by the merger of the splenic vein is responsible for blood flow from the spleen while the superior mesenteric vein is responsible for the blood flow coming from the small intestine and is remarkably high in nutrients. The pancreas drains into the pancreatic vein while the gastric veins provide drainage for the lesser curvature of the stomach and the gallbladder is drained by the cystic vein. Finally, the inferior mesenteric vein is responsible for the blood flow out of the large intestine[33].

The largest visceral tissue mass in the human body is the liver. The liver receives blood fully oxygenated blood from the hepatic artery (30%) and partially oxygenated blood from the hepatic portal vein (70%). It has multi functions in the body. The liver's main job is to filter the blood coming from the digestive tract, before passing it to

the rest of the body. it breaks down and detoxifies substances in the body, and it also acts as a storage unit. it?s responsible for making many of the proteins in the body, including blood clotting factors, and albumin, required to maintain fluid within the circulation system. The liver is also responsible for manufacturing cholesterol and triglycerides. The liver is also storing vitamins and chemicals that the body requires as building blocks.

Following processing of the blood by the functional cells of the liver, the blood collects via the three hepatic veins which consist of an accumulation of central veins. deoxygenated hepatic blood will ultimately converge in the right and left hepatic veins, which exit the superior surface of the liver and empty into the inferior vena cava to be distributed to the rest of the body.

3.2 Representing Hepatic Portal System by Graph:

Now, through the medical application, we can able to classify the hepatic portal system into set of vertices and set of edges. The vertices represent the regions of the organs of hepatic portal system which receive and send the blood through them. The edges represent the pathway of blood through the hepatic portal system as shown in Fig. (8).
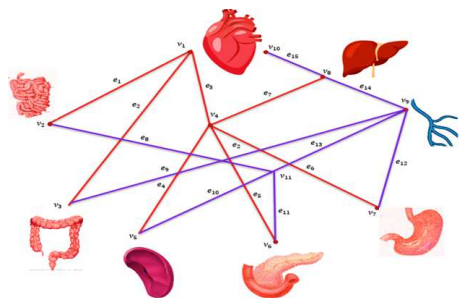

Fig. 8: Circulation of hepatic portal system as a graph

The vertices are $v_1, v_2, v_3, v_4, v_5, v_6, v_7, v_8, v_9, v_{10}$ that represent Abdominal Aorta , Small Intestine, Large Intestine, Coeliac Trunk, Spleen, Pancreas, Stomach, Portal Vein , Liver and Inferior Vena Cava, respectively.

The edges are $e_1, e_2, e_3, e_4, e_5, e_6, e_7, e_8, e_9, e_{10}, e_{11}, e_{12}, e_{13}, e_{14}$ that represent Superior Mesenteric Artery, Inferior Mesenteric Artery, Celiac Artery, Splenic Artery, Pancreatic Artery, Gastric Artery, Hepatic Artery, Superior Mesenteric Vein, Inferior Mesenteric Vein, Splenic Vein ,Pancreatic Vein , Gastric Vein, Portal Vein, Hepatic Portal Vein, Splenic-Mesenteric Confluence, respectively.

3.3 New Result:

Now, we must know that the blood volume of any human body differs from one person to another, blood volume indicates the total amount of fluid circulating inside the arteries, capillaries, veins and the most important part in human body is the heart at any time. Many organs communicate with each other to participatory in producing blood and organizing the blood volume and optimally controlling the blood volume[34, 35].

The amount of blood circulating within the human body depends on the size and weight of each one. For example, Women head for having a lower blood volume than men. However, during pregnancy woman's blood volume increases by nearly 50%[35, 36]. So, it's difficult to calculate the blood volume in our medical application if only we take many vital cases of persons.

Wherefore, we obtain new algorithm to calculate the effect of each vertex in the circulatory system that represent the organs of system on the blood pathways that exit from the heart and return it to the heart after termination of its function and Calculate the effect of each edge in the hepatic circulation that represents the oxidized arteries or the non-oxidized veins to ensure delivered blood in one direction through different paths until it returns to the heart again.

Step 1: input the graph of hepatic portal system.

Step 2: finding all the paths that start from abdominal aorta v_1 and ending in inferior vena cava v_{10} (back to the heart).

$P_1 = \{v_1, e_1, v_2, e_8, v_{11}, e_{13}, v_9, e_{14}, v_8, e_{15}, v_{10}\}$,
where $v(p_1) = \{v_1, v_2, v_{11}, v_9, v_{10}, v_8\}$
$P_2 = \{v_1, e_2, v_3, e_9, v_9, e_{14}, v_8, e_{15}, v_{10}\}$,
where $v(p_2) = \{v_1, v_3, v_9, v_8, v_{10}\}$
$P_3 = \{v_1, e_3, v_4, e_4, v_5, e_{10}, v_{11}, e_{13}, v_9, e_{14}, v_8, e_{15}, v_{10}\}$,
where $v(p_3) = \{v_1, v_4, v_5, v_{11}, v_9, v_8, v_{10}\}$
$P_4 = \{v_1, e_3, v_4, e_5, v_6, e_{11}, v_{11}, e_{13}, v_9, e_{14}, v_8, e_{15}, v_{10}\}$,
where $v(p_4) = \{v_1, v_4, v_6, v_{11}, v_9, v_8, v_{10}\}$
$P_5 = \{v_1, e_3, v_4, e_6, v_7, e_{12}, v_9, e_{14}, v_8, e_{15}, v_{10}\}$,
where $v(p_5) = \{v_1, v_4, v_7, v_9, v_8, v_{10}\}$
$P_6 = \{v_1, e_3, v_4, e_7, v_8, e_{15}, v_{10}\}$,
where $v(p_6) = \{v_1, v_4, v_8, v_{10}\}$

Step 3: divide the graph into connected subgraph: oxygenated subgraph represented by $L_{1-OXY} = \{v_1, v_2, v_3, v_4, v_5, v_6, v_7, v_8\}$ and $M_{1-OXY} = \{e_1, e_2, e_3, e_4, e_5, e_6, e_7\}$, the other, de-oxygenated subgraph represented by $L_{2-D-OXY} = \{v_2, v_3, v_5, v_6, v_7, v_8, v_9, v_{10}, v_{11}\}$ and $M_{2-D-OXY} = \{e_8, e_9, e_{10}, e_{11}, e_{12}, e_{13}, e_{14}, e_{15}\}$.

Step 4: calculate the membership for each vertex by $\alpha(v_i) = \frac{|v(P_i) \cap L_j|}{|v(P_i)|}$, where L_j are vertices of connected subgraph $j = 1, 2$ then find the median.

Step 5: calculate the membership of edges by

$$\mu(e_{ij}) = \begin{cases} \sum_{i=1}^{n} \frac{|E(P_i) \bigcap M_i|}{|E(P_i)|} & if\ \mu(e_{ij}) \leq min(\mu(v_i), \mu(v_i)) \\ min(\mu(v_i), \mu(v_i)) & if\ \mu(e_{ij}) > min(\mu(v_i), \mu(v_i)) \end{cases}$$

Step 5: end.

In the previous part, we were able to create a code through MATLAB and presented the following results that are illustrated through Figure 9, which is the calculation of the membership of both the vertices and edges of whatever parts to be studied on the circulation of the hepatic portal system.

3.4 Fuzzy Properties VS Circulation of Hepatic Portal System:

Now, so it's easy to discuss the difference forms of induced fuzzy subgraphs and its properties that mentioned previously in section 2 by the changeable of the membership of vertices or edges by different methods and its effect on strength of fuzzy graph, and also comparing the resultant graphs with medical phenomena and views. We will show that in the following section.
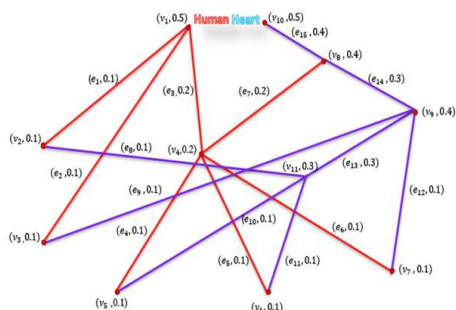

Fig. 9: Strong fuzzy subgraph of hepatic portal circulation

Initially, we were able in the fuzzy graph to determine the membership of each of vertices and edges by using a new algorithm that we mentioned it earlier. We noticed that a resultant graph is a strong fuzzy graph that satisfied all conditions that maintain the strength of fuzzy graph and does not contain any isolated points to keep the normality of circulation of flowing the blood between the organs of the hepatic system.

a) If we take $\kappa_1 = min\ \alpha(v_i)$ and $\kappa_2 = min\ \beta(e_{ij})$. We will notice that the $P^{\kappa_1 \kappa_2}$ path passes through all the vertices and edges whose membership is greater than minimum of membership of vertices and edges of the graph. It will result in that the $P^{\kappa_1 \kappa_2}$ path contain all vertices and edges of the graph. So, we will conclude that the resulting graph is a strong fuzzy subgraph that same

as the original fuzzy graph. This indicates medically that the blood circulation in the hepatic portal system is proceeding normally since the exit of oxygenated blood from the heart to the different organs of the gastrointestinal system through the abdominal aorta so that each organ achieved its function. Then the deoxygenated blood and various substances is returned through the hepatic vein to supply the liver with metabolites and ensures that ingested substances are processed in the liver before returning to the heart and passing it on to the rest of the body.

b) If we take $\kappa_1 = min\ \alpha(v_i)$ and $\kappa_2 > min\ \beta(e_{ij})$. We will observe that in each step when enlarging the value of membership of κ_2, edges have membership less than κ_2 will be deleted. So, this led to that there is no $P^{\kappa_1 \kappa_2}$ path passes through the vertices of the graph and the resulting fuzzy graph contain only the set of vertices only, and there are no edges connecting them. But this is a contradiction for our medical application. except in abnormal cases such as medical problems and chronic diseases. For example, it has become a major phenomenon widespread among the elderly and it spreads among young people who suffer from health problems such as Arteriosclerosis, which is one of the most dangerous diseases, which is the accumulation of fat and cholesterol on the walls of the arteries, which leads to their hardening and narrowing, which limits blood flow to the organs of the body, and this Leading over time and not avoiding the increase of the causes of the disease to chronic blockage in the blood vessels, and as a result of these many diseases, for example, causing cardiovascular disease such as heart attacks, strokes, and peripheral vascular disease.

c) If we take $\kappa_1 > min\ \alpha(v_i)$ and $\kappa_2 = min\ \beta(e_{ij})$. We will notice that by enlarging the value of membership of $kappa_1$ in successive steps, each vertex that has membership less than $kappa_1$ will be omitted. Medically, this indicates to the disorder of the functions of some organs in human body and there are many reasons that lead to that. But currently one of the most dangerous diseases that threaten human life is "COVID 19" and its serious consequences for human health.

Researchers in different health science centers explained in their recent study that the "COVID 19" could have severe consequences on heart health, so very important information was revealed about the direct effect of the "COVID 19" virus on the heart, especially the heart muscle. Due to the increased levels of protein, which is usually found in low blood concentrations, it causes damaging effects to the heart cells in the form of inflammation in the heart muscle, similar to what happens when suffering from coronary artery failure or cardiac arrest. Thus, the heart muscle weakens and increases the pain of heart patients, threatening their lives and may lead to death. It also proved an indirect effect of the "COVID 19" virus on heart health, resulting from its main effect on the lungs and respiratory system as a result of pneumonia caused by the virus, which causes a burden on the heart.

This effect is due to a viral infection of the myocardium, which affects the heart muscle, especially what is known as the electrical system, which reduces the ability of the heart to pump blood, and causes disturbances in the heart rhythm, and strokes can occur. Which leads to a stroke or heart attack.

4 Conclusion

The future suggestion of our work is to take a specific disease and study its effect on the blood circulation of the human body. If some parts of the body become defective or stop working, it can appear in the graph. Also, if a failure occurs in a part of the blood circulation, we can predict some diseases in the blood circulation before they occur.

For example, "COVID 19" cause increase in coagulation process and may cause thrombosis or embolism that could affect blood flow to lungs and other vital organs. By calculating blood flow to different organs, we can predict prognosis and mortality rate for "COVID 19" patients by different methods that related to fuzzy graph theory.

5 Matlab

a) Detecting the memberships of vertices by

$$\alpha(v_i) = \frac{|v(P_i) \cap L_j|}{|v(P_i)|}$$

```
clear;
clc;
l1=[1:8];
l2=[2 3 5 6 7 8 9 10 11];
p1=[1 2 11 9 8 10];
p2=[1 3 9 8 10];
p3=[1 4 5 11 9 8 10];
p4=[1 4 6 11 9 8 10];
p5=[1 4 7 9 8 10];
p6=[1 4 8 10];
n(1)=length(p1);
n(2)=length(p2);
n(3)=length(p3);
n(4)=length(p4);
n(5)=length(p5);
n(6)=length(p6);
%m of 1 to 11 with l1
mean_v_lo(1:11)=0;
mean_v_ld(1:11)=0;
%m(v1,l1)
z=intersect(p1,l1);
z(z==8)=[];
mv1lo(1)=length(z)/n(1);
z=intersect(p2,l1);
z(z==8)=[];
```

```
mv1lo(2)=length(z)/n(2);
z=intersect(p3,l1);
z(z==8)=[];
mv1lo(3)=length(z)/n(3);
z=intersect(p4,l1);
z(z==8)=[];
mv1lo(4)=length(z)/n(4);
z=intersect(p5,l1);
z(z==8)=[];
mv1lo(5)=length(z)/n(5);
z=intersect(p6,l1);
mv1lo(6)=length(z)/n(6);
mean_v_lo(1)=mean(mv1lo);
%m of v4 with l1
mv4lo(1:2)=0;
z=intersect(p3,l1);
z(z==1)=[];
z(z==8)=[];
mv4lo(3)=length(z)/n(3);
z=intersect(p4,l1);
z(z==1)=[];
z(z==8)=[];
mv4lo(4)=length(z)/n(4);
z=intersect(p5,l1);
z(z==1)=[];
z(z==8)=[];
mv4lo(5)=length(z)/n(5);
z=intersect(p6,l1);
z(z==1)=[];
mv4lo(6)=length(z)/n(6);
mean_v_lo(4)=mean(mv4lo);
%%%%%%%%%%%
mean_v_ld(1)=0;
%m(v2,l2)
mv2ld(1:6)=0;
z=intersect(p1,l2);
mv2ld(1)=length(z)/n(1);
mean_v_ld(2)=mean(mv2ld);
%m(v3,l2)
mv3ld(1:6)=0;
z=intersect(p2,l2);
mv3ld(2)=length(z)/n(2);
mean_v_ld(3)=mean(mv3ld);
%m(v4,l2)
mean_v_ld(4)=0;
%m(v5,l2)
mv5ld(1:6)=0;
z=intersect(p3,l2);
mv5ld(4)=length(z)/n(3);
mean_v_ld(5)=mean(mv5ld);
%m(v8,l2)
z=intersect(p1,l2);
z(z==2)=[];
z(z==11)=[];
z(z==9)=[];
mv8ld(1)=length(z)/n(1);
z=intersect(p2,l2);
z(z==3)=[];
```

```
z(z==9)=[];
mv8ld(2)=length(z)/n(2);
z=intersect(p3,l2);
z(z==5)=[];
z(z==9)=[];
z(z==11)=[];
mv8ld(3)=length(z)/n(3);
z=intersect(p4,l2);
z(z==6)=[];
z(z==11)=[];
z(z==9)=[];
mv8ld(4)=length(z)/n(4);
z=intersect(p5,l2);
z(z==7)=[];
z(z==9)=[];
mv8ld(5)=length(z)/n(5);
z=intersect(p6,l2);
mv8ld(6)=length(z)/n(6);
mean_v_ld(8)=mean(mv8ld);
%m(v9,l2)
z=intersect(p1,l2);
z(z==2)=[];
z(z==11)=[];
mv9ld(1)=length(z)/n(1);
%m(v6,l2)
mv6ld(1:6)=0;
z=intersect(p4,l2);
mv6ld(4)=length(z)/n(4);
mean_v_ld(6)=mean(mv6ld);
%m(v7,l2)
mv7ld(1:6)=0;
z=intersect(p5,l2);
mv7ld(4)=length(z)/n(5);
mean_v_ld(7)=mean(mv7ld);
z=intersect(p2,l2);
z(z==3)=[];
mv9ld(2)=length(z)/n(2);
z=intersect(p3,l2);
z(z==5)=[];
z(z==11)=[];
mv9ld(3)=length(z)/n(3);
z=intersect(p4,l2);
z(z==6)=[];
z(z==11)=[];
mv9ld(4)=length(z)/n(4);
z=intersect(p5,l2);
z(z==7)=[];
mv9ld(5)=length(z)/n(5);
mv9ld(6)=0;
mean_v_ld(9)=mean(mv9ld);
%m(v11,l2)
mv11ld(1:6)=0;
z=intersect(p1,l2);
z(z==2)=[];
mv11ld(1)=length(z)/n(1);
z=intersect(p3,l2);
z(z==5)=[];
mv11ld(3)=length(z)/n(3);
```

```
z=intersect(p4,l2);
z(z==6)=[];
mv11ld(4)=length(z)/n(4);
mean_v_ld(11)=mean(mv11l;
clear z;
```

b) Detecting the memberships of edges by

$$
\mu(e_{ij}) = \begin{cases} \sum_{i=1}^{n} \frac{|E(P_i) \bigcap M_i|}{|E(P_i)|} & if \ \mu(e_{ij}) \leq min(\mu(v_i), \mu(v_i)) \\ min(\mu(v_i), \mu(v_i)) & if \ \mu(e_{ij}) > min(\mu(v_i), \mu(v_i)) \end{cases}
$$

```
clear;
clc;
l1=[1:15];
p1=[1 8 13 14 15];
p2=[2 9 14 15];
p3=[3 4 10 13 14 15];
p4=[3 5 11 13 14 15];
p5=[3 6 12 14 15];
p6=[3 7 15];
n(1)=length(p1);
n(2)=length(p2);
n(3)=length(p3);
n(4)=length(p4);
n(5)=length(p5);
n(6)=length(p6);
%m(e1,l1)
me1lo(1:6)=0;
z=intersect(p1,l1)
me1lo(1)=length(z)/n(1)
mean_e_lo(1)=mean(me1lo)
%m(e2,l1)
me2lo(1:6)=0;
z=intersect(p2,l1);
me2lo(2)=length(z)/n(2);
mean_e_lo(2)=mean(me2lo);
%m(e3,l1)
me3lo(1:6)=0;
z=intersect(p3,l1);
me3lo(3)=length(z)/n(3);
z=intersect(p4,l1);
me3lo(4)=length(z)/n(4);
z=intersect(p5,l1);
me3lo(5)=length(z)/n(5);
z=intersect(p6,l1);
me3lo(6)=length(z)/n(6);
mean_e_lo(3)=mean(me3lo);
%m(e4,l1)
me4lo(1:6)=0;
z=intersect(p3,l1);
z(z==3)=[];
me4lo(3)=length(z)/n(3);
mean_e_lo(4)=mean(me4lo);
%m(e5,l1)
me5lo(1:6)=0;
z=intersect(p4,l1);
z(z==3)=[];
me5lo(4)=length(z)/n(4);
```

```
mean_e_lo(5)=mean(me5lo);
%m(e6,l1)
me6lo(1:6)=0;
z=intersect(p5,l1);
z(z==3)=[];
me6lo(5)=length(z)/n(5);
mean_e_lo(6)=mean(me6lo);
%m(e7,l1)
me7lo(1:6)=0;
z=intersect(p6,l1);
z(z==3)=[];
me7lo(6)=length(z)/n(6);

 mean_e_lo(7)=mean(me7lo);
%m(e8,l1)
me8lo(1:6)=0;
z=intersect(p1,l1);
z(z==1)=[];
me8lo(1)=length(z)/n(1);
mean_e_lo(8)=mean(me8lo);
%m(e9,l1)
me9lo(1:6)=0;
z=intersect(p2,l1);
z(z==2)=[];
me9lo(2)=length(z)/n(2);
mean_e_lo(9)=mean(me9lo);
me12lo(5)=length(z)/n(5);
mean_e_lo(12)=mean(me12lo);
%m(e13,l1)
me13lo(1:6)=0;
z=intersect(p1,l1);
z(z==1)=[];
z(z==8)=[];
me13lo(1)=length(z)/n(1);
z=intersect(p3,l1);
z(z==3)=[];
z(z==4)=[];
z(z==10)=[];
me13lo(3)=length(z)/n(3);
z=intersect(p4,l1);
z(z==3)=[];
z(z==5)=[];
z(z==11)=[];
me13lo(4)=length(z)/n(4);
mean_e_lo(13)=mean(me13lo);
%m(e14,l1)
me14lo(1:6)=0;
z=intersect(p1,l1);
z(z==1)=[];
z(z==8)=[];
z(z==13)=[];
me14lo(1)=length(z)/n(1);
z=intersect(p2,l1);
z(z==2)=[];
%m(e10,l1)
me10lo(1:6)=0;
z=intersect(p3,l1);
z(z==3)=[];
```

```
z(z==4)=[];
me10lo(3)=length(z)/n(3);
mean_e_lo(10)=mean(me10lo);
%m(e11,l1)
me11lo(1:6)=0;
z=intersect(p4,l1);
z(z==3)=[];
z(z==5)=[];
me11lo(4)=length(z)/n(4);
mean_e_lo(11)=mean(me11lo);
%m(e12,l1)
me12lo(1:6)=0;
z=intersect(p5,l1);
z(z==3)=[];
z(z==6)=[];z(z==9)=[];
me14lo(2)=length(z)/n(2);
z=intersect(p3,l1);
z(z==3)=[];
z(z==4)=[];
z(z==10)=[];
z(z==13)=[];
me14lo(3)=length(z)/n(3);
z=intersect(p4,l1);
z(z==3)=[];
z(z==5)=[];
z(z==11)=[];
z(z==13)=[];
me14lo(4)=length(z)/n(4);
z=intersect(p5,l1);
z(z==3)=[];
z(z==6)=[];
z(z==12)=[];
me14lo(5)=length(z)/n(5);
mean_e_lo(14)=mean(me14lo);
mean_e_lo(15)=0;
clear z;
```

Conflicts of Interests

The authors declare that they have no conflicts of interests.

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