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A Novel Nonlinear Control Law with Trajectory Tracking Capability for Nonholonomic Mobile Robots: Closed-Form Solution Design

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Abstract: A novel nonlinear robust trajectory tracking control law for nonholonomic mobile robot is presented in this paper. This approach can be applied to generate trajectory tracking control commands on nonholonomic mobile robot movement. The design objective is to specify one nonlinear robust control law that satisfies the H_2 performance, for the nonlinear trajectory tracking control of nonholonomic mobile robot. In general, it is hard to obtain the closed-form solution from this nonlinear trajectory-tracking problem. Fortunately, because of the skew symmetric property of the trajectory tracking system of the nonholonomic mobile robot and adequate choice of state variable transformation, the H_2 trajectory-tracking problems can be reduced to solving one nonlinear time varying Riccati-like equations. Furthermore, one closed-form solution to this nonlinear time varying Riccati-like equation can be obtained with very simple forms for the preceding control design. Finally, there are two practical testing conditions: circular and square like reference trajectories are used for performance verifications.

Keywords: nonholonomic mobile robot, nonlinear robust control law, H_2, closed-form solution

1. Introduction

Recently years, wheeled mobile robots are applied in various industrial and service fields which include transportation, inspection and security etc. Hence, it is more and more important at wheeled mobile robots manipulation accurately, especially in the trajectory tracking subject. Many studies [1-7] in trajectory tracking problem, but some results can not ease and simple to implementation. In this paper, we proposed a novel nonlinear control law to practice a nonholonomic mobile robot with closed form solution design.

This paper will exhibit as following sections. The mathematical model and design objective of nonholonomic mobile robot will brief in section II. Problem formulation and controller design for robust trajectory tracking will be described in section III, and practice results of the nonholonomic mobile robot robust trajectory tracking by the proposed design are demonstrated in section IV. Finally, the conclusions are summarized in section V.

2. Mathematical Model and Design Objective

2.1. Model and dynamics of a nonholonomic mobile robot

In general, the structure of wheel mobile robot consists of two driving wheels which locate at the same axis and a passive self-adjusted supporting wheel which leads the mechanical system. Both driving wheels which are for the motion and orientation purpose are driven by two actuators (e.g. DC motors) independently.

As figure 1 shows the two driving wheels with the same radius denoted by r and separated by 2R. The location of the vehicle in the global coordinate frame$\{O,X,Y\}$is represented by the vector $P=\begin{bmatrix} x_c \; y_c \; \theta \end{bmatrix}^T$, where$x_c$,$y_c$are the coordinates of the point C in the global coordinate frame and$\theta$is the orientation of the local frame$\{C,X_c,Y_c\}$. The generalized coordinate of the vehicle is described as

$$q=\begin{bmatrix} x_c \; y_c \; \theta \end{bmatrix}^T \tag{2.1}$$

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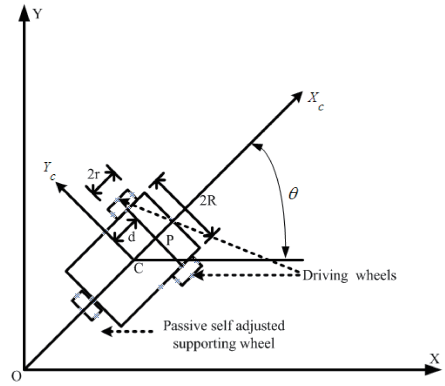

Figure 1 Nonholonomic mobile robot

For ordinary mobile robot system, the robot just can move as the direction of the axis of the driving wheels with pure rolling and nonslipping nonholonomic condition status. Consequently, the velocity of contact point with the ground and orthogonal to the plane of the wheel is zero. We can express as following [8]

$$\dot{y}_c \cos\theta - \dot{x}_c \sin\theta - d\dot{\theta} = 0 \tag{2.2}$$

and then the kinematic equation can also be described as

$$\dot{q} = \begin{bmatrix} \dot{x}_c \\ \dot{y}_c \\ \dot{\theta} \end{bmatrix} = \begin{bmatrix} \cos\theta & -d\sin\theta \\ \sin\theta & d\cos\theta \\ 0 & 1 \end{bmatrix} \begin{bmatrix} v_l \\ w \end{bmatrix} \tag{2.3}$$

where v_l and w are the linear and angular velocity along the robot axis. In this paper, we develop a H_2 technique to solve this kind of problem, so the mobile robot dynamic equation can be described as [9]

$$H(q)\ddot{q} + C(q,\dot{q})\dot{q} = B(q)\tau \tag{2.4}$$

where $H(q) \in \Re^{3\times 3}$ is a symmetric positive define inertia matrix, $C(q,\dot{q}) \in \Re^{3\times 3}$ is the centripetal and coriolis matrix, $B(q) \in \Re^{3\times 2}$ is the input transformation matrix, and $\tau \in \Re^{2\times 1}$ is the input vector, where τ_r and τ_l represents right and left wheel torques, respectively.

$$H(q) = \begin{bmatrix} m & 0 & md\sin\theta \\ 0 & m & -md\cos\theta \\ md\sin\theta & -md\cos\theta & I \end{bmatrix}$$

$$C(q,\dot{q}) = \begin{bmatrix} 0 & 0 & md\dot{\theta}\cos\theta \\ 0 & 0 & md\sin\theta \\ 0 & 0 & 0 \end{bmatrix}$$

$$B(q) = \frac{1}{r}\begin{bmatrix} \cos\theta & \sin\theta \\ \sin\theta & \sin\theta \\ R & -R \end{bmatrix}, \tau = \begin{bmatrix} \tau_r \\ \tau_l \end{bmatrix} \tag{2.5}$$

2.2. Problem Formulation

We will develop a H_2 trajectory tracking control design in this paper. The desired tracking reference trajectory q_r is supposed to be existed into bounded time functions of position vector $q_r \in C^2$ which is a twice continuously differentiable function. The velocity vector and acceleration vector of q_r can be expressed as $\dot{q}_r$ and $\ddot{q}_r$ respectively. As following is the tracking error definition:

$$e = \begin{bmatrix} \dot{\hat{q}} \\ \hat{q} \end{bmatrix} = \begin{bmatrix} \dot{q} - \dot{q}_r \\ q - q_r \end{bmatrix} \tag{2.6}$$

and tracking error dynamic equation is given as

$$\begin{aligned} \dot{e} = &\begin{bmatrix} -H^{-1}(q)C(q,\dot{q}) & 0_{2\times 2} \\ I_{2\times 2} & 0_{2\times 2} \end{bmatrix} e \\ &+ \begin{bmatrix} -\ddot{q}_r - H^{-1}(q)C(q,\dot{q})\dot{q}_r \\ 0_{2\times 2} \end{bmatrix} + \begin{bmatrix} H^{-1}(q)B(q)\tau \\ 0_{2\times 2} \end{bmatrix} \end{aligned} \tag{2.7}$$

The error dynamic equation (2.7) is difficult to direct apply in this study due to complication equation. For control design purpose, as following the filtered link of tracking error $l(t)$and the state-space transformation matrix $T$are be defined to simplify the control formulation and the stability analysis.

$$l(t) = \eta\dot{\hat{q}} + \Phi\hat{q} \tag{2.8}$$

$$T = \begin{bmatrix} T_1 \\ T_2 \end{bmatrix} = \begin{bmatrix} T_{11} & T_{12} \\ 0_{2\times 2} & I_{2\times 2} \end{bmatrix} = \begin{bmatrix} \eta I_{2\times 2} & \Phi \\ 0_{2\times 2} & I_{2\times 2} \end{bmatrix} \tag{2.9}$$

Some of positive scale $\eta$and positive definite matrix $\Phi \in \Re^{2\times 2}$are constants and then the error dynamic equation (2.7) can be rewritten as a compact form:

$$\begin{aligned} \dot{e} &= T^{-1}\begin{bmatrix} \dot{l}(t) \\ \dot{\hat{q}}(t) \end{bmatrix} \\ &= M_T(e,t)e + N_T(e,t)\left[\eta(-S(e,t) + B(q)\tau)\right] + N_T(e,t)d \end{aligned} \tag{2.10}$$

where

$$M_T(e,t) = T^{-1}\begin{bmatrix} -H^{-1}(q)C(q,\dot{q}) & 0_{2\times 2} \\ \frac{1}{\eta}I_{2\times 2} & -\frac{1}{\eta}\Phi \end{bmatrix} T$$

$$N_T(e,t) = T^{-1}VH^{-1}(q)$$

$$S(e,t) = H(q)(\ddot{q}_r - \frac{1}{\lambda}\Phi\dot{\hat{q}}) + C(q,\dot{q})(\dot{q}_r - \frac{1}{\lambda}\Phi\hat{q})$$

with

$$V = \begin{bmatrix} I_{2\times 2} \\ 0_{2\times 2} \end{bmatrix} \tag{2.11}$$

If the following applied torque input is selected

$$B(q)\tau = S(e,t) + \frac{1}{\eta}u \tag{2.12}$$

then the tracking error dynamic equation drives by the control input$u$ will become

$$\dot{e} = M_T(e,t)e + N_T(e,t)u \tag{2.13}$$

3. Problem Formulation and Controller Design

3.1. The nonlinear H_2 trajectory tracking control problem

To consider the nonlinear nonholonomic mobile robot system of the form equation (2.13) that the unknown disturbance modeling error and the effect of the constraint force are not considered. To given some weighting matrices G_2andU_2, the nonlinear H_2 trajectory tracking control problem can be solved by solving the following tracking control problem.

$$\begin{aligned} J(\dot{u}_2) &= \min_{u_2} J(u_2,) \\ &= \min_{u_2} \Big[e^T(t_f) G_{2f} e(t_f) \\ &\quad + \int_0^{t_f} \left[e^T(t) G_2 e(t) + u_2^T(t) U_2 u_2(t) \right] dt \Big] \\ &= e^T(0) J_2(e(0),0) e(0) \end{aligned} \tag{3.1}$$

Supposing above exists an optimal control law u_2^* which satisfies [10] for all $t_f \in [0,\infty)$and for some positive definite matrix

$$G_{2f} = G_{2f}^T > 0 \tag{3.2}$$

3.2. Control Design of H_2

In this section, we will solve the nonlinear nonholonomic mobile robot tracking control problems that are formulated in the above section. For this purpose, a novel nonlinear control law with trajectory tracking capability for nonholonomic mobile robot will be described by nonlinear H_2 control theorem and the optimal control law shows as below.

$$u_2^*(e,t) = -U_2^{-1} N_T^T(e,t) J_2(e,t) e(t) \tag{3.3}$$

If $J_2(e,t)$satisfies the following time-varying Riccati-like equation, and then it is the same to solve the nonlinear H_2 trajectory tracking problem in equation (3.1)

$$\begin{aligned} &\dot{J}_2(e,t) + J_2(e,t) M_T(e,t) + M_T(e,t)^T J_2(e,t) + G_2 - \\ &\quad J_2(e,t) N_T(e,t) U_2^{-1} N_T^T(e,t) J_2(e,t) = 0 \end{aligned} \tag{3.4}$$

with $J_2(e,t) = J_2^T(e,t) \geq 0$ and $G_{2f} = J_2(e(t_f),t_f)$.

3.3. Solutions of the nonlinear time-varying Riccati equations of H_2 problem

The solution $J_2(e,t)$ the nonlinear Riccati-like equation (3.4) can be put in more explicit form because the state transformation equation (2.9) has been involved in the process of design and describes as following.

$$J_2(e,t) = T_2^T \begin{bmatrix} H(e,t) & 0_{2\times2} \\ 0_{2\times2} & X_2 \end{bmatrix} \tag{3.5}$$

where $X_2$is some positive definite symmetric constant matrix. Besides, the matrices $T_2^T$and X_2 can also be solved from a pair of algebraic Riccati-like equation, and consider the second and third terms on the left-hand side of the time varying Riccati-like equation (3.4). We can obtain

$$\begin{aligned} &J_2(e,t) M_T(e,t) + M_T^T(e,t) J_2(e,t) \\ &= \begin{bmatrix} 0_{2\times2} & X_2 \\ X_2 & 0_{2\times2} \end{bmatrix} + T_2^T \begin{bmatrix} -\dot{H}(e,t) & 0_{2\times2} \\ 0_{2\times2} & 0_{2\times2} \end{bmatrix} T_2 \end{aligned} \tag{3.6}$$

We can easily verifed that

$$N_T^T(e,t) J_2(e,t) = N^T T_2 \tag{3.7}$$

equation (3.4) can be rewritten to the following algebraic Riccati-like equation with the results of equations (3.6) and (3.7)

$$\begin{bmatrix} 0_{2\times2} & X_2 \\ X_2 & 0_{2\times2} \end{bmatrix} + G_2 - T_2^T N U_2^{-1} N^T T_2 = 0 \tag{3.8}$$

Suppose, the optimal control law $u_2^*(e,t)$ can be expressed as

$$u_2^*(e,t) = -U_2^{-1} N^T T_2 e \tag{3.9}$$

By choosing

$$U_2 = \alpha_2^2 I_{2\times2} \tag{3.10}$$

where $\alpha_2 > 0$and the positive definite symmetric matrix G_2can also be factorized by Cholesky factorization as

$$G_2 = \begin{bmatrix} G_{211}^T G_{11} & G_{212} \\ G_{212}^T & G_{222}^T G_{222} \end{bmatrix} \tag{3.11}$$

Applying the definitions of $T_2$and Nin equations (2.8) and (2.10) together with assumptions equations (3.10) and (3.11), the Riccati-like equation (3.6) can be separated into the following equations

$$G_{211}^T G_{211} - \frac{1}{\alpha_2^2} T_{211}^T T_{211} = 0 \tag{3.12}$$

$$X_2 + G_{212} - \frac{1}{\alpha_2^2} T_{211}^T T_{212} = 0 \tag{3.13}$$

$$X_2 + G_{212}^T - \frac{1}{\alpha_2^2} T_{212}^T T_{211} = 0 \tag{3.14}$$

$$G_{222}^T G_{222} - \frac{1}{\alpha_2^2} T_{212}^T T_{212} = 0 \tag{3.15}$$

From equations (3.12) and (3.15), the sub-matrices $T_{211}$and T_{212}can be represented as

$$T_{211} = \alpha_2 G_{211}, \quad T_{212} = \alpha_2 G_{222} \tag{3.16}$$

Then, we get

$$T_2 \begin{bmatrix} \alpha_2 G_{211} & \alpha_2 G_{222} \\ 0_{2\times 2} & I_{2\times 2} \end{bmatrix} \quad (3.17)$$

For satisfying with $T_{211} = \eta I_{2\times 2}$ in equation (2.8), the weighting matrix G_{211} in equation (3.11) must be selected as the following diagonal form

$$G_{211} = g_{211} I_{2\times 2} \quad (3.18)$$

by choosing a positive scale q_{211}, and a scale η for the H_2 tracking problem, we have the following relationship as

$$\eta = \alpha_2 g_{211} \quad (3.19)$$

Let the weighting matrix U_2 is given as the form shown in equation (3.10) for any bounded $\alpha_2 > 0$ and allow the weighting matrix $G_2 > 0$ be taken as

$$G_2 = \begin{bmatrix} G_{211}^T G_{211} & G_{212} \\ G_{212}^T & G_{222}^T G_{222} \end{bmatrix} \quad (3.20)$$

with G_{211}, G_{222} and G_{212} satisfying the requirements in equations (3.17) and (3.18). Then the nonlinear H_2 nonholonomic mobile robot trajectory tracking control problem is solved by the following optimal controller

$$u_2^* = -\frac{1}{\alpha_2} \begin{bmatrix} G_{211} & G_{222} \end{bmatrix} e \quad (3.21)$$

3.4. Actuator dynamics

The wheels of nonholonomic mobile robot are driven by DC motors. The figure 2 shows simplified circuit model for a DC motor. By applying Kirchoff voltage law to the circuit, we can get

$$u = iR_a + e_{emf} \quad (3.22)$$

and

$$\tau_L = \frac{NK_T}{R_a} u - \frac{N^2 K_T}{R_a} K_b w_L. \quad (3.23)$$

where u is the input voltage, i is the motor current, R_a is the motor resistance, e_{emf} is the back emf (electromotive force), $N = r_2/r_1$ the gear ratio, τ_m and τ_L are the torque triggered by motor and on load, w_m (before gears) and w_L (after gears) are the angular velocities of the motor, θ_m (before gears) and θ_L (after gears) are the angles of the motor, K_T and K_b are torque and velocity constant, respectively. The mobile robot dynamics can be rewritten as following.

$$u_2(e,t) = \frac{NK_T}{R_a} u_{input} - \frac{N^2 K_T}{R_a} K_b w_L \quad (3.24)$$

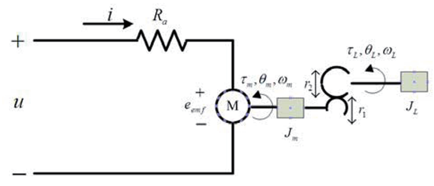

Figure 2 Simplified circuit model for a DC motor

4. Implementation results

4.1. Control system architecture

The novel proposed control method is practiced on two driving wheels and one passive self adjusted supporting wheel mobile robot system as figure 3A small passive self adjusted support wheel is attached in the back of vehicle to carry the framework of mobile robot system. The driving wheels are driven by individual DC motor. The nonholonomic mobile robot system is implemented by computer side and mobile robot side. The computer side consists of a personal computer and a Bluetooth USB Dongle. It provides an operation interface to issue control commands. The mobile robot side consists of a Bluetooth RS232 adapter, a controller main board, a DC motor controller component, two DC motors, a vehicle and an INS component. The overall control flowchart of architecture shows in figure 4 and the red block is computer side and blue block is mobile robot side.

The PC interacts through Bluetooth USB Dongle with Bluetooth RS232 adapter on the mobile robot side. The controller main board consists of dsPIC 30F4011 microchip and performs proposed algorithm by MPLAB IDE and C30 C complier and WinPIC tool. The DC motor controller receives the control command to trigger DC motor and then rotates the wheels of vehicle to move as designate trajectory.

4.2. Experimental results

In this section, the proposed control law will be verified and presented by implemented mobile robot system by figure 3. As follows we will demonstrate the results of real practice by tracking circular and square like reference trajectories. First, we need to set a desired tracking trajectory and transmit the control commands to vehicle by control interface on computer side and then the vehicle will draw a circular or square like as desired tracking trajectory. The results of real practice show as figure 5 and 6. Additionally, there is a slow turn at each corner of the square which causes by the desired orientation angle sudden change. Above results with the proposed control algorithm are capable of tracking for continuous and noncontinuous situation.

Figure 3 Mobile robot system

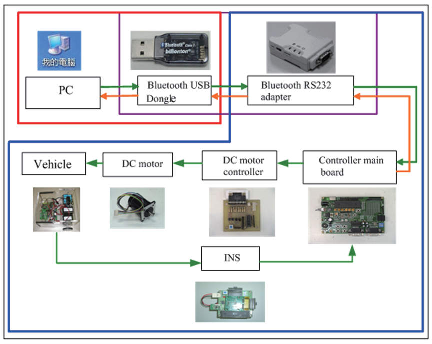

Figure 4 Control flowchart of the mobile robot system

Figure 5 Circle real time tracking performance

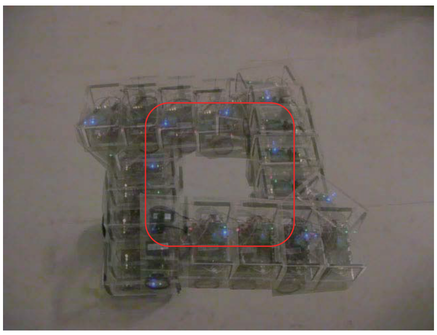

Figure 6 Square like real time tracking performance

5. Conclusions

A nonlinear control law is successfully developed for improving the maneuverability of the nonlinear nonholonomic mobile robot system in this paper. We can obtain general solutions of the H_2 control problem of nonholonomic mobile robot by using the skew-symmetric property and transformation techniques. The proposed method achieves satisfactory simulation and experimental results for tracking the desired trajectory in circular and square like reference scenarios. From the experimental results, it is easy to implement and find out the H_2 nonholonomic mobile robot-tracking control exhibits an excellent tracking capability by the proposed method in this paper.

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