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Abstract- **This study mainly aims to investigate and quantify the influence of piano key weir (PKW) characteristics on its discharge coefficient. Accordingly, based on dimensional analysis, seventy-two experimental runs were carried out using sixteen PKW physical models of various geometric characteristics as well as different flow rates too to develop the target empirical discharge coefficient formula. The analysis highlights that relative length (*L/W*) and alveoli width (*Wi/Wo*) are the most studied effective PKW dimensionless ratios that significantly influence the discharge coefficient. Increasing *L/W* from 1 to 7 leads to noted discharge coefficient values increasing up to 27.15%. While variation Wi/Wo ratio from 0.5 to 2 can cause the discharge coefficient to increase up to 25.28%.The study results were statistically compared with the previously developed models and it has shown good agreement with their outcomes. Considering the proposed discharge coefficient formula, the optimum hydraulic design of the new PKW model can be successfully facilitated by the extrapolation of characteristics of the idealized scale.**

Keywords: **Dimensional Analysis, Discharge Coefficient, Flow Rate, Hydraulic Design, Relative Length, Piano Key Weir I**

1. INTRODUCTION

The Piano Key Weir (PKW) is a non-linear type of overflow weir which tries to reduce the overflowing head for a given discharge. These hydraulic structures have been utilized in recent years both as dams and as river regulation works [1]. Many researchers have conducted distinctive experimental programs to investigate PKW hydraulic behavior and its corresponding configuration to satisfy the optimum economical design [2].

Lempérière et al. [3] illustrated that the crest length magnification ratio L/W is one of the dominant ratios in determining the PKW discharge. The main significant output involved in the most efficient PKW geometric parameters determination: L/W = 5, W /W = 1, B/P = 2.4, and B /B = 0. 5. Likewise, Pfister et al. [4] investigated the keys widths ratio Wi/Wo and the overhangs positions ratio Bo/Bi as the main influencing ratios that also governing PKW target design discharge.

Kabiri-Samani et al. [5] developed a discharge coefficient formula based on an experimental program that involved PKW various operation conditions, they utilized the discharge ranges from 0.025 to 0.175m3/s. Leite Ribeiro et al. [6] conducted an experimental study of PKW models and ranked the governing parameters according to their discharge coefficient effect. Machiels et al, [7] conducted an extended experimental program to investigate the optimum PKWs discharge capacity in relation to their various main dimensionless. These contribute to the increase of comprehension regarding the hydraulic behavior of PKWs.

Ribeiro et. al. [8] conducted experimental studies on PKW with a crest length related the width of the PKW. They indicated that the PKWs discharge capacity has a noted direct proportion relationship with low heads. Guo et. al. [9] developed an empirical equation for predicting the release capacity of PKWs. This proposed formula was mainly based on multi-parameter optimization.

This research aims to propose a new formula that links the PKW governing configuration parameters to the discharge coefficient using dimensionless group ratio terms.

2. DIMENSION ANALYSIS

Dimensional analysis was carried out by using Buckingham, s pi-theorem to achieve the interrelationship between the dimensionless group terms. To describe the free flow over the PKW, the parameter that influences discharge coefficient (C_d), the determination of the different dimensionless group terms is conducted by a dimensional analysis approach. geometry parameters which can be described as shown in Fig. 1 were; the height of the piano key weir (p), the height of the foundation P_d, the width of the piano key weir (W), the Head over the piano key weir (H), total development length (L), the width of inlet key (Wi), the width of outlet key (Wo), lateral length of the piano key weir (B), Length of upstream overhang (Bi), and length of downstream overhang (Bo). Likewise, kinematic parameters can be described as kinematic viscosity (μ), and gravitational acceleration (g), and the dynamic parameters were surface tension (σ) and mass density (ρ). Therefore, the PCW discharge can be expressed as

$$Q = f(H, L, W, Wi, Wo, B, Bo,, Pi, Pd, \rho, g, \mu, \sigma) \qquad (1)$$

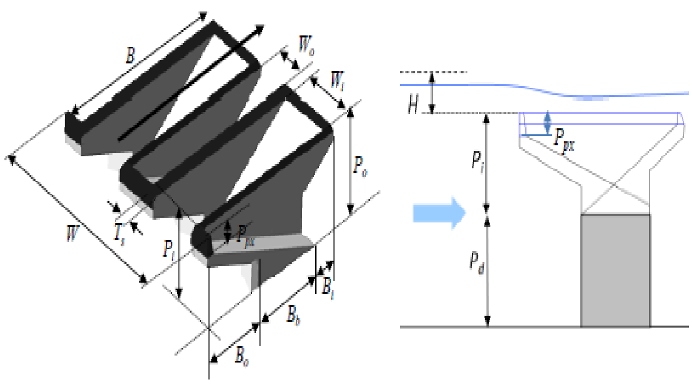

Fig. 1. Definition sketch of the used models

According to Buckingham Pi-theorem, by taking ρ, g, and H as repeating variables. Meanwhile, the viscosity (μ) and the surface tension (σ) can be considered a neglected effect due to the test heads values adequacy. The following π terms can be considered:

$$C_d = f(\frac{H}{P}, \frac{L}{W}, \frac{Wi}{Wo}, \frac{B}{Pi}, \frac{Bo}{B}, \frac{Pd}{Pi}) \qquad (2)$$

3. EXPERIMENTAL WORK

Fig. 2 illustrates the present study laboratory rectangular flume with a length of 12.5 m and a width of 30 cm. The walls of the flume are manufactured of 10-mm transparent glass and the floor is made of polished metal. Moreover, in terms of geometric specifications, Table 1 highlights the configuration of the sixteen PKWs type B physical models that were used in this study. These models were manufactured from transparent polycarbonate. In addition, seventy-two experimental runs are conducted to investigate the influence of various PKWs Type-B models' dominant parameters on their discharge coefficients'. After that, the discharge consequently changed to conduct a new experiment, Fig. 3.

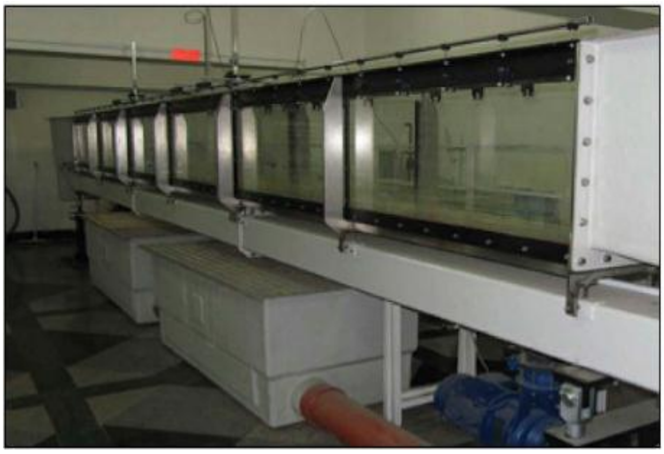

Fig. 2. Experimental flume and equipment used in the laboratory

Fig. 3. PKW models front view and downstream view

4. RESULTS

A. Relative length effect

The effect of relative length (L/W) on the C_d was analyzed in Fig. 4. This relationship was plotted by representing the various water heads to their corresponding heights of weir crests ratios as a dependent variable on x-axis and the C_d as an independent variable. Generally, the data plotted show that the increase of the ratio (H/P) decreases the discharge coefficient Cd. Moreover, the relative length (L/W) is indirect proportion to the discharge coefficient.

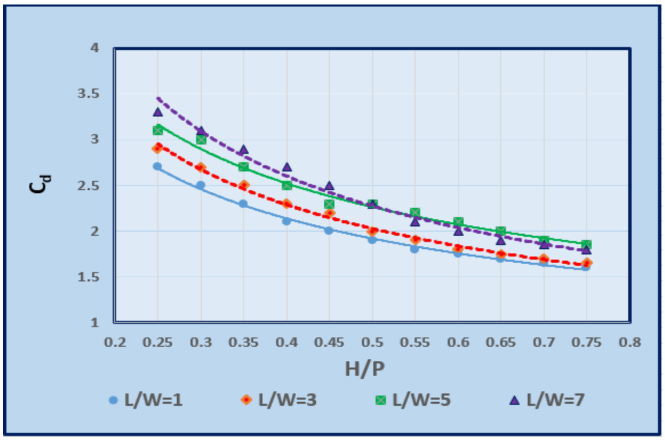

Fig. 4. Variation of relative length with a discharge coefficient

Table 1. PKWs Physical Models Dimensions

Model	(L/W)	(W/Wo)	(B/Pi)	(Bo/B)	(Pd/Pi)
PKW1	1.00	0.50	1.08	0.30	0
PKW2	3.00	1.00	1.65	0.50	0.40
PKW3	5.00	1.50	2.45	0.70	0.60
PKW4	7.00	2.50	3.52	0.90	1.00
PKW5	1.00	0.50	1.08	0.30	0
PKW6	3.00	1.00	1.65	0.50	0.40
PKW7	5.00	1.50	2.45	0.70	0.60
PKW8	7.00	2.50	3.52	0.90	1.00
PKW9	1.00	0.50	1.08	0.30	0
PKW10	3.00	1.00	1.65	0.50	0.40
PKW11	5.00	1.50	2.45	0.70	0.60
PKW12	7.00	2.50	3.52	0.90	1.00
PKW13	1.00	0.50	1.08	0.30	0
PKW14	3.00	1.00	1.65	0.50	0.40
PKW15	5.00	1.50	2.45	0.70	0.60
PKW16	7.00	2.50	3.52	0.90	1.00

Additionally, when the ratio (H/P) increases from 0.25 to 0.45, the discharge coefficient decreases by 22.43%, 25.32, and 26.11 27.15% for L/W equal 1, 3, 5, and 7 respectively. While for 0.45≤ (H/P) ≤ 0.75, a relatively limited decrease in discharge coefficient by 8.25%, 9.76%, 10.32%, and 11.04% for the same mentioned L/W experiment's values.

B. Alveoli Width Effect

Fig. 5 indicates the alveoli width effect on the discharge coefficient and its relationship with for different (H/P) ratios. The figure shows how this discharge coefficient decreases with increasing the H/P ratio. For the H/P ratio ranging from 0.25 to 0.75, the discharge coefficient increases by a variable percentage from 19.37% to 25.28% when Wi/Wo ratio increases from 0.50 to 2.00. The maximum discharge coefficient values occurred when Wi/Wo ratio equals 1.00.

C. PKW Height Effect

Fig. 6 describes the relationship between (H/P) and discharge coefficient for different side wall lengths to weir crest heights (B/P) ratios. It is clear that the discharge coefficient has an inverse proportion to the H/P ratio. For various experiment B/P values, a relatively high decrease in discharge coefficient was noted when H/P values increased from 0.25 to 0.45. While at relatively high head values (0.45<H/P≤0.75), a slightly decreasing in discharge coefficient by 12.23% and 14.76% for B/P= 1.08 and B/P= 3.52.

D. Overhangs Length Effect

Figure 7 presents the relation between H/P and discharge coefficient for different overhangs length (Bo/B). From Fig. 7 it can be noticed that the discharge coefficient decreased with the increasing (H/P) ratio for all overhang lengths (Bo/B) test values. The cause for these decreases can be explained due to enlargement of wetted perimeters leading to inlet velocities limitations and consequently energy loss. Additionally, it was found that increasing Bo/B from 0.3 to 0.9 causes a discharge coefficient average decreasing percent of 11.12% and 13.23% in cases of H/P < 0.45 and 0.45 ≤H/P ≤ 0.75 respectively.

E. WEIR Height Effect

The dam height can be considered one of the important dominant parameters that influence PKW discharge capacity. Fig. 8 shows that increasing the (Pd/P) ratio results in an increasing discharge coefficient. Moreover, the effect of the said (Pd/P) ratio can be observed when (H/P < 0.45). On the other hand, with H/P increases from 0.25 to 0.45, the average discharge coefficient decreased by13.34%, 14.33%, 15.75%, and 16.65% for (Pd/P) ratios equal 0, 0.4, 0.6, and 1.0 respectively. On the other hand, under high heads (0.45 ≤H/P ≤ 0.75) the discharge coefficient has a relatively gradual decrease. It is obvious that the discharge coefficients are larger at low heads (H/P < 0.45) and smaller at higher head conditions (0.45 ≤H/P ≤ 0.75) for various experimental Pd/P values.

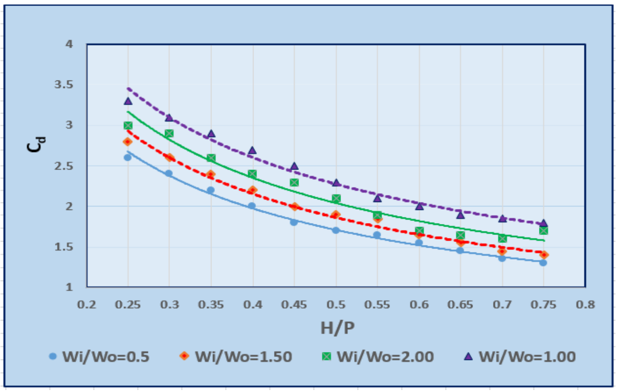

Fig. 5. Variation of alveoli width with discharge coefficient

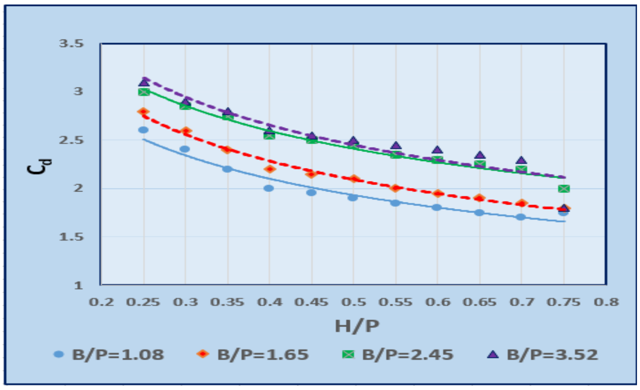

Fig. 6. Variation of PKW height with discharge coefficient

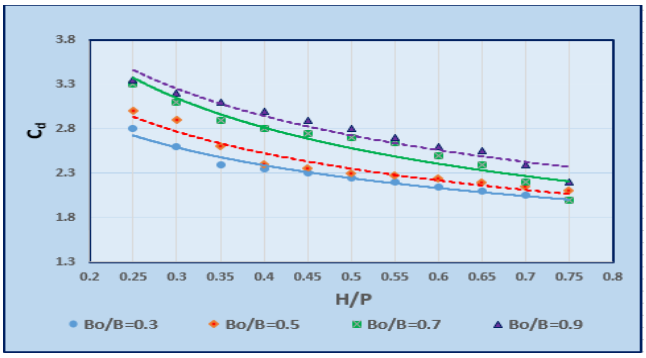

Fig. 7. Variation of overhangs length with discharge coefficient

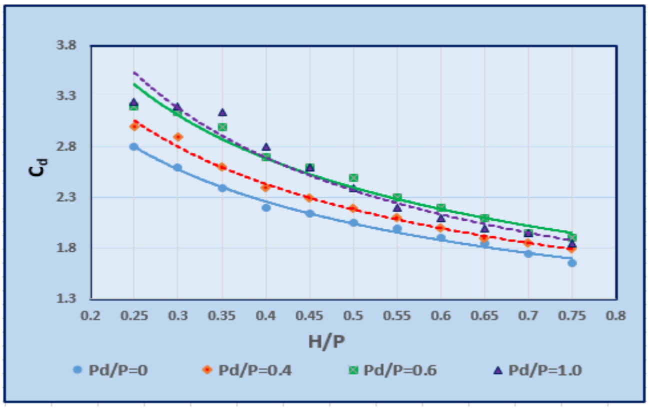

Fig. 8. Variation of overhangs length with discharge coefficient

Table 2. Comparing with Previous Studies Statistics

Compared Researchers	Equation	MAPE	RMSE	NSE
Kabri et al., (2012) [5]	C_d =2/3 {0.606+ [0.202 $(H/P)^{-0.675}$ $(L/W)^{0.377}$ $(Wi/Wo)^{0.426}$ $(B/P)^{0.306}$} * exp [1.504Bo/B) + 0.093 (B_i/B)]	0.10	.009	0.93
Leite, et al., (2012) [6]	C_d =0.42 [0.8+ [0.34 $(L/W)^{0.70}$ $(Wi/Wo)^{0.08}$ $(Pi/H)^{0.82]}$	0.07	0.12	0.95
Guo, et al., (2019) [9]	C_d =0.1+ 0.285 [$(L/W)^{0.45}$ $(B/P)^{0.10}$ $(Wi/Wo)^{0.05}$ $(H/P)^{-0.465}$]	0.08	0.07	.094

F. Developing Discharge Coefficient Formula

Because geometric parameters configuration can be considered one of the main challenges for optimum PKW discharge capacity, in this study, the specific conducted experimental data from 16 PKWs physical models were utilized to develop the discharge coefficient formula.

However, the empirical formula for discharge coefficient determination was obtained using a large number of tests. Moreover, the multiple regression analysis was applied to the experimental results to associate PKW geometric configurations that influence the C_d into one general formula, the following shows the deduced empirical formula.

$$C_d = 1.243 + \{0.142\ (H/P)\ {-0.502}\ (L/W)\ 1.987\ (Wi/Wo)\ 0.901\ (B/P)\ 2.483\} * \exp[-2.114(H/P) - 0.271(L/W) - 0.612\ (Wi/Wo)\ -0.681\ (B/P) + 0.913(B0/B) + 0.064\ (Pd/P)] \quad R^2 = 0.90 \qquad (3)$$

From equation (3), it can be easily inferred that the discharge coefficient of a PKW is primarily close to a power-exponential function. The PKW discharge capacity is efficient for relatively lower heads, and its efficiency decreases rapidly as the head increases.

G. Comparing with Previous Studies

Many previous empirical formulas have been investigated for estimating the PKW discharge coefficients. To ensure the overall accuracy of the present study's developed formula, various statistical measures are applied to compare the present study's discharge coefficients proposed equation with the corresponding previous PKW discharge coefficients determination equations, Table 2.

However, three statistical measures: Mean Absolute Percentage Error (MAPE); Root Mean Square Error (RMSE), and Nash-Sutcliffe Efficiency (NSE) are selected for implementing this required comparison and evaluation between the current study and other researchers' PKWs discharge coefficients equations. According to evaluating statistical measures results, it is obvious that all the compared research results with the current study tend to their recommended fitness values (zero) for MAPE and RMSE. Likewise, compared with previous research, NSE results also tend to have the optimum compliance value (1.00). However, a good agreement between the current study-developed formulas and the mentioned researchers' results is attained.

5. CONCLUSION

The finding from this research may have practical applications, especially when performing PKW Type-B hydraulic design that is based on its geometric characteristics to increase discharge capacity. The following main conclusions may be drawn:

- The experimental results showed that when the ratio (H/P) increases from 0.25 to 0.45, the discharge coefficient (Cd) decreases by 22.43%, 25.32, 26.11, and 27.15% for L/W equal to 1, 3, 5, and 7 respectively. While for 0.45≤ (H/P) ≤ 0.75, a relatively limited decrease was noted in discharge coefficient by 8.25%, 9.76%, 10.32%, and 11.04% for the same mentioned L/W experiment's values.
- For various experiment B/P values, a relatively high decrease in discharge coefficient was noted when H/P values increased from 0.25 to 0.45. While at relatively high head values (0.45<H/P≤0.75), a slightly decreasing in discharge coefficient by 12.23% and 14.76% for B/P= 1.08 and B/P= 3.52 respectively.
- The discharge coefficient decreased with the increasing (H/P) ratio for all overhang lengths (Bo/B) test values. On the other hand, increasing Bo/B from 0.3 to 0.9 causes a discharge coefficient average decreasing percent of 11.12% and 13.23% in cases of H/P < 0.45 and 0.45 ≤H/P ≤ 0.75 respectively.
- By applying the study's C_d coefficient determination formula to graphical and design charts, the water authorities can easily evaluate the discharge capacity through various PKW for each particular situation.

NOMENCLATURE

B:	Length of the side wall
Bo:	Length of the outlet key
B/Pi:	Upstream-downstream length to the height
Cd:	Coefficient of discharge
H:	Water head
L:	Weir Crest
L/W:	Relative Length
P:	Height of weir crest
Pd:	Height of the foundation
Pi:	Height of the inlet key
Pd/Pi:	Weir height ratio
PKW:	Piano key side weir
Q:	Flow rate

V:	Flow velocity
W:	Total width of the weir crest
Wi:	Width of the inlet key
Wo:	Width of the outlet key
Wi/Wo	Alveoli Width

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