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Generalized Inverted Exponential Distribution on Optimum SS-PALT: Some Bayes Estimation

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Abstract: The present article, studied about the Bayes risks of the unknown parameters of the generalized Inverted Exponential distribution. The Optimum Step-Stress Partially Accelerated Life Test (SS-PALT) has been used under the different censoring patterns. A comparison between Bayes risks of two different asymmetric loss functions has presented. Numerical illustration has also been carried out by the help of the real and simulated data set.

Keywords: Bayes Estimator, First-Failure Progressive Censoring (FFPC) Scheme, Step Stress Partially Accelerated Life Test (SS-PALT), Generalized Inverted Exponential Distribution, General Entropy Loss Function (GELF), Invariant LINEX Loss Function (ILLF).

1 Introduction

Due to simple mathematical usage and interesting properties, the Exponential distribution is widely used model in life-testing experiments. The inverted Exponential distribution was first introduced by [15]. They study the properties of the maximum likelihood estimation, confidence limits, uniformly minimum variance unbiased estimator and reliability function for complete sample case. Prakash [20] revisited the Inverted exponential distribution in the Bayesian view point. The properties of the Bayes estimates of the reliability parameters under different loss functions and the Bayes predictive interval and shift point estimation also included by Prakash [20].

Gupta & Kundu [8], generalized the Exponential distribution by appending the shape parameter, and named the distribution as the generalized Exponential distribution. Generalized Inverted Exponential distribution was first introduced by [3]. This distribution originated from the exponentiated Frechet distribution ([16]).

The generalized Inverted Exponential distribution on a convenient structure of the distribution function, provides many practical applicabilities, including, in horse racing, queue theory, modeling wind speeds. [18] has explored the statistical properties of the Generalized Inverted Generalized Exponential distribution and its parameters were estimated at both censored and uncensored cases using the method of maximum likelihood estimation (MLE). [6] presents some estimation and prediction of unknown parameters based on progressively censored generalized Inverted Exponential data.

The probability density and cumulative density function of generalized Inverted exponential distribution with shape parameter β and scale parameter α, are given respectively as

$$f(x;\beta,\alpha) = \frac{\beta\alpha}{x^2}\, exp\left(-\frac{\alpha}{x}\right)\left(1 - exp\left(-\frac{\alpha}{x}\right)\right)^{\beta-1};\ \beta>0, \alpha>0, x>0 \tag{1}$$

and

$$F(x;\beta,\alpha) = 1 - \left(1 - exp\left(-\frac{\alpha}{x}\right)\right)^{\beta};\ \beta>0, \alpha>0, x>0. \tag{2}$$

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The article has the main objective to present a comparative analysis for the Bayes estimators based on different censoring patterns, those are the special cases of the FFPC. For this the generalized Inverted Exponential distribution has designated as the underlying model with Optimum Step-Stress Partially Accelerated Life Test situation. The Bayes estimators for the unknown parameters have obtained under two different asymmetric loss functions and compared their Bayes risks on a simulated and real data set.

2 SS-PALT under FFPC

In life testing product experiments, it is much difficult to gather lifetimes of extremely reliable products, having a very long lifespan. Because, under the normal operating conditions, a very few or even no failures may occur within a limited testing time interval. The partially accelerated life test criterion is very useful test criterion in such cases and, are provided significant reduction in the time and cost of the experiment.

In partially accelerated life test (PALT) criterion, only a few test units from all the test units are kept under severe stress condition ([2]). There are two different methods of stress loading onto PALT, named as constant-stress and step-stress. In the present article, the step-stress PALT is considered, and is permitting the test to be changed from the normal use condition to the accelerated condition at a pre-assumed time.

It is also serious to find the changing times when the test to be changed from normal stress condition to severe test condition. [27] and [28], have suggested the optimum change time in their study on step-stress partially accelerated life tests of the censored data. In the present article, we also have to determine the optimal stress change time, which minimizes the generalized asymptotic variance of the ML estimates of the parameters.

The Partially accelerated life tests of step-stress scheme has studied by several authors, a little few of them are [5], [1], [12], [11], [10], [13], [26], [9] and [22]. In SS-PALT, all the test units are run first at normal condition on given stress change time and if they do not fail, then the test is changed from the accelerated condition and retained the test until all the units fail. The tampered random variable model under SS-PALT for the lifetime of a unit Y is defined for stress change time $(=\varepsilon)$ and the acceleration factor $(=\lambda)$ as

$$X=\begin{cases} Y & ;\ 0<Y\leq\varepsilon \\ \varepsilon+\frac{Y-\varepsilon}{\lambda} & ;\quad Y>\varepsilon \end{cases}. \tag{3}$$

Using Eq. (1) in Eq. (3), we get

$$f\left(x_i^R;\beta,\alpha\right)=\begin{cases} f_1\left(x\right)=\frac{\beta\alpha}{x^2}\,e^{-\frac{\alpha}{x}}\left(1-e^{-\frac{\alpha}{x}}\right)^{\beta-1} \\ f_2\left(x\right)=\frac{\beta\alpha\lambda}{\tilde{X}^2}\,e^{-\frac{\alpha}{\tilde{X}}}\left(1-e^{-\frac{\alpha}{\tilde{X}}}\right)^{\beta-1} \end{cases} \tag{4}$$

where $\tilde{X}=\varepsilon+\lambda\left(x-\varepsilon\right)$.

In the first-failure life test procedures, the researcher divided all the test units into a number of groups and run all the units simultaneously by first-failure occurred in each group. This procedure now combines with the Progressive censoring scheme and named as first-failure progressive (FFP) censoring, with the advantages in term of reducing test time, in which more items are used but only a few items are failed. See [23] for more details on the FFP censoring.

The joint probability density function under FFP censoring is defined, when the total test units divided into k independent groups, each have n units within each group, all are putting on a life test. If $x_1^R<x_2^R<\ldots<x_m^R$ are the progressively first-failure censored order statistic of size m, with pre assumed progressive censoring scheme $R=\left(R_1,R_2,\ldots,R_m\right)$, then

$$L\propto\prod_{i=1}^{m}f\left(x_i^R,\beta,\alpha\right)\,\left(1-F\left(x_i^R,\beta,\alpha\right)\right)^{k(R_i+1)-1}. \tag{5}$$

In the present article the FFP censoring technique has combined with the SS-PALT, therefore, the Eq. (5) is now re-written as

$$L \propto \prod_{i=1}^{m_1} f_1(x)\,(1-F_1(x))^{k(R_i+1)-1} \times \prod_{i=m_1+1}^{m} f_2(x)\,(1-F_2(x))^{k(R_i+1)-1} \tag{6}$$

Here, m_1 test units are kept on normal test conditions whereas remaining are on severe stress condition. Using Eq. (4) in Eq. (6), the required joint density function based on FFPC on SS-PALT is given as

$$L \propto \prod_{i=1}^{m_1}\left\{\frac{\beta\alpha}{x_i^2}\,e^{-\frac{\alpha}{x_i}}\left(1-e^{-\frac{\alpha}{x_i}}\right)^{\beta k(R_i+1)-1}\right\} \times \prod_{i=m_1+1}^{m}\left\{\frac{\beta\alpha\lambda}{\tilde{X}_i^2}\,e^{-\frac{\alpha}{\tilde{X}_i}}\left(1-e^{-\frac{\alpha}{\tilde{X}_i}}\right)^{\beta k(R_i+1)-1}\right\}$$

$$\Rightarrow L \propto \beta^m\,\alpha^m\,\lambda^{m-m_1}\,T_0(\underline{x},\lambda)\times e^{-\alpha(T_1(\underline{x}))+T_2(\underline{x},\lambda)}\,e^{T_3(\underline{x},\beta,\alpha)+T_4(\underline{x},\lambda,\beta,\alpha)}; \tag{7}$$

where $T_0(\underline{x},\lambda) = \prod_{i=m_1+1}^{m}\left(\tilde{X}_i^{-2}\right)$, $T_1(\underline{x}) = \sum_{i=1}^{m_1}\left(x_i^{-1}\right)$, $T_2(\underline{x},\lambda) = \sum_{i=m_i+1}^{m}\left(\tilde{X}_i^{-1}\right)$, $T_3(\underline{x},\beta,\alpha) = \sum_{i=1}^{m_1}(\beta k(R_i+1)-1)\ \log\ \left(1-exp\left(-\frac{\alpha}{x_i}\right)\right)$, $T_4(\underline{x},\lambda,\beta,\alpha) = \sum_{i=m_i+1}^{m}(\beta k(R_i+1)-1)\ \log\ \left(1-exp\left(-\frac{\alpha}{\tilde{X}_i}\right)\right)$ and $\tilde{X}_i = \varepsilon+\lambda(x_i-\varepsilon)$.

3 Bayes Estimation

The present section has studied about the Bayes estimators and their Bayes risks under two different asymmetric loss functions by using the FFP censoring in SS-PALT. It may be noted that, no joint conjugate prior exists, if both parameters α and β are considered to be unknown. In such condition, the piecewise independent priors is assumed. The Gamma prior have been selected for these parameters and given as

$$\pi_\alpha \propto \alpha^{a-1}\,e^{-\alpha};\ a>0 \tag{8}$$

and

$$\pi_\beta \propto \beta^{b-1}\,e^{-\beta};\ b>0. \tag{9}$$

A vague prior do not play any significant role in the analyses, hence for the acceleration factor λ, the vague prior is assumed and defined as

$$\pi_\lambda \propto \lambda^{-1};\ \lambda>0. \tag{10}$$

Hence, the marginal posterior densities corresponding to the parameters α, β and λ are obtained and given respectively as

$$\pi^*_\alpha = \Omega\frac{\alpha^{m+a-1}}{e^{\alpha(T_1(\underline{x})+1)}}\int_\beta \beta^{m+b-1}\,e^{T_3(\underline{x},\beta,\alpha)-\beta}\times\int_\lambda \lambda^{m-m_1-1}T_0(\underline{x},\lambda)\,e^{T_4(\underline{x},\lambda,\beta,\alpha)-\alpha T_2(\underline{x},\lambda)}\,d\lambda\,d\beta \tag{11}$$

$$\pi^*_\beta = \Omega\frac{\beta^{m+b+1}}{e^{\beta}}\int_\alpha \alpha^{m+a-1}\,e^{-\alpha(T_1(\underline{x})+1)}e^{T_3(\underline{x},\beta,\alpha)}\times\int_\lambda \lambda^{m-m_1-1}T_0(\underline{x},\lambda)\,\frac{e^{T_4(\underline{x},\lambda,\beta,\alpha)}}{e^{\alpha T_2(\underline{x},\lambda)}}\,d\lambda\,d\alpha \tag{12}$$

and

$$\pi^*_\lambda = \Omega\frac{T_0(\underline{x},\lambda)}{\lambda^{m_1-m+1}}\int_\alpha \alpha^{m+a-1}e^{-\alpha(T_1(\underline{x})+T_2(\underline{x},\lambda)+1)}\times\int_\beta \beta^{m+b-1}\,e^{T_3(\underline{x},\beta,\alpha)+T_4(\underline{x},\lambda,\beta,\alpha)-\beta}d\beta\,d\alpha \tag{13}$$

where $\Omega = \left\{\int_\alpha \frac{\alpha^{m+a-1}}{e^{\alpha(T_1(\underline{x})+1)}}\int_\beta \beta^{m+b-1}\,e^{T_3(\underline{x},\beta,\alpha)-\beta}\ \times\int_\lambda \frac{T_0(\underline{x},\lambda)}{\lambda^{m_1-m+1}}e^{T_4(\underline{x},\lambda,\beta,\alpha)-\alpha T_2(\underline{x},\lambda)}d\lambda\,d\beta\,d\alpha\right\}^{-1}$.

The most of the Bayesian inference procedures have been developed with the usual symmetric loss function named as squared error loss function. The squared error loss is symmetrical and associates equal importance to the losses due to overestimation and underestimation. [24] discussed about the unfeasible of the squared error loss in most of the situations of practical importance. Hence, due to practical importance of the underlying distribution, we are going with two different asymmetric loss functions.

Following, Prakash [21], a useful and flexible class of asymmetric loss function, named as invariant LINEX loss function (ILLF), and defined for any estimate $\hat{\theta}$ as

$$L_I(\partial) = e^{c\,\partial} - c\,\partial - 1\,;\, \partial = \Phi\left(= \frac{\hat{\theta}}{\theta}\right) - 1\,;\, c \neq 0. \tag{14}$$

In ILLF, parameter c is termed as the shape parameter. The negative value of c, gives more weight to overestimation and vice versa, whereas its magnitude reflects the degree of asymmetry. The function is quite asymmetric with overestimation being more costly than underestimation for $c = 1$, and for small values of $|c|$, the ILLF is almost symmetric and is not far from the squared error loss function. The Bayes estimators corresponding to the parameters α, β and λ under the ILLF are obtained by solving following equality for each parameters respectively

$$\int_{\Theta} \left\{\frac{1}{\Theta} exp\left(-c\frac{\hat{\Theta}_L}{\Theta}\right)\right\} \pi^*_{\Theta}\, d\Theta = e^c \int_{\Theta} \frac{1}{\Theta}\, \pi^*_{\Theta}\, d\Theta\,;\, \Theta = \alpha, \beta, \lambda \tag{15}$$

The invariant LINEX loss function rises approximately exponentially on one side of zero and approximately linearly on other side. A suitable alternative of ILLF, named as general Entropy loss function (GELF) was discussed by [25] and, is defined for any estimate $\hat{\theta}$ as

$$L_E(\Phi) = (\Phi)^d - d\,log(\Phi) - 1\,;\, d \neq 0. \tag{16}$$

Here, the parameter d is called as the shape parameter of GELF ([19]). The positive error $\left(\hat{\theta} > \theta\right)$ causes more serious consequences than a negative error and vice versa. Further, the positive magnitude of shape parameter causes more serious penalties than a negative one.

Now, the Bayes estimators corresponding to the parameters α, β and λ under the GELF are obtained by solving following equality for each parameters respectively

$$\hat{\Theta}_E = \left\{\int_{\Theta} \Theta^{-d}\, \pi^*_{\Theta}\, d\Theta\right\}^{-\frac{1}{d}}\,;\, \Theta = \alpha, \beta, \lambda. \tag{17}$$

It is clear from above that, no close forms of the Bayes estimators and their corresponding Bayes risks occurred. For the numerical findings, some numerical technique is applied here.

4 Optimum Stress Change Time

One of the major issue in Step-Stress PALT is that, when the experimental items will be put on from normal stress condition to the severe stress condition. For this, the stress change time ε is optimized, based on the determinant of the Fisher's information matrix. Maximizing the determinant is equivalent to minimizing the generalized asymptotic variance of the maximum likelihood estimation of the model parameters α, β and the acceleration factor λ. For obtaining the optimum stress change time ε, the logarithm of Eq. (7) is given as

$$\begin{aligned} LogL = m\,log\beta + m\,log\alpha + (m - m_1)\,log\lambda + log\,T_0(\underline{x}, \lambda) - \alpha\left(T_1(\underline{x}) + T_2(\underline{x}, \lambda)\right) \\ + T_3(\underline{x}, \beta, \alpha) + T_4(\underline{x}, \lambda, \beta, \alpha)\,. \end{aligned} \tag{18}$$

Differentiating Eq. (18) with respect to the parameters α, β and the acceleration factor λ respectively, we get

$$\frac{\partial}{\partial \alpha} LogL = \frac{m}{\alpha} - T_1(\underline{x}) - T_2(\underline{x}, \lambda) + \sum_{i=1}^{m_1} \left\{\frac{(\beta k (R_i + 1) - 1)\, exp\left(-\frac{\alpha}{x_i}\right)}{x_i\left(1 - exp\left(-\frac{\alpha}{x_i}\right)\right)}\right\} + \sum_{i=m_1+1}^{m} \left\{\frac{(\beta k (R_i + 1) - 1)\, exp\left(-\frac{\alpha}{\tilde{X}_i}\right)}{\tilde{X}_i\left(1 - exp\left(-\frac{\alpha}{\tilde{X}_i}\right)\right)}\right\},$$

$$\frac{\partial}{\partial\beta}LogL=\frac{m}{\beta}+\sum_{i=1}^{m_1}(k(R_i+1)-1)\times log\left(1-exp\left(-\frac{\alpha}{x_i}\right)\right)+\sum_{i=m_1+1}^{m}(k(R_i+1)-1)\times log\left(1-exp\left(-\frac{\alpha}{\tilde{X}_i}\right)\right)$$

and

$$\frac{\partial}{\partial\lambda}LogL=\frac{m-m_1}{\lambda}+\sum_{i=m_1+1}^{m}\left\{\frac{x_i-\varepsilon}{\tilde{X}_i}\left(\left(\frac{\alpha}{\tilde{X}_i}\right)\left(1+\frac{\beta k(R_i+1)-1}{1-exp\left(\frac{\alpha}{\tilde{X}_i}\right)}\right)-2\right)\right\}.$$

Similarly, the second derivatives corresponding to the parameters α, β and λ are obtained respectively as

$$\frac{\partial^2}{\partial\alpha^2}LogL=-\frac{m}{\alpha^2}-\sum_{i=1}^{m_1}\left\{\frac{\beta k(R_i+1)-1}{\left(1-exp\left(-\frac{\alpha}{x_i}\right)\right)^2}\times\frac{exp\left(-\frac{\alpha}{x_i}\right)}{x_i^2}\right\}-\sum_{i=m_1+1}^{m}\left\{\frac{(\beta k(R_i+1)-1)\,exp\left(-\frac{\alpha}{\tilde{X}_i}\right)}{\tilde{X}_i^2\left(1-exp\left(-\frac{\alpha}{\tilde{X}_i}\right)\right)^2}\right\},$$

$$\frac{\partial^2}{\partial\beta^2}LogL=-\frac{m}{\beta^2},$$

$$\frac{\partial^2}{\partial\lambda^2}LogL=-\frac{m-m_1}{\lambda^2}+2\sum_{i=m_1+1}^{m}\left(\frac{x_i-\varepsilon}{\tilde{X}_i}\right)\left(1-\frac{\alpha}{\tilde{X}_i}\right)+\sum_{i=m_1+1}^{m}\frac{\alpha(x_i-\varepsilon)^2}{\tilde{X}_i^3}\times\left(\frac{\beta k(R_i+1)-1}{1-exp\left(-\frac{\alpha}{\tilde{X}_i}\right)}\right)$$
$$\times\left\{\frac{\alpha}{1-exp\left(-\frac{\alpha}{\tilde{X}_i}\right)}+\frac{\alpha}{\tilde{X}_i}-2\right\},$$

$$\frac{\partial^2}{\partial\alpha\,\partial\beta}LogL=\frac{\partial^2}{\partial\beta\,\partial\alpha}LogL=\sum_{i=1}^{m_1}\left\{\frac{(k(R_i+1)-1)\,exp\left(-\frac{\alpha}{x_i}\right)}{\left(1-exp\left(-\frac{\alpha}{x_i}\right)\right)x_i}\right\}+\sum_{i=m_1+1}^{m}\left\{\frac{(k(R_i+1)-1)\,exp\left(-\frac{\alpha}{\tilde{X}_i}\right)}{\left(1-exp\left(-\frac{\alpha}{\tilde{X}_i}\right)\right)\tilde{X}_i}\right\},$$

$$\frac{\partial^2}{\partial\alpha\,\partial\lambda}LogL=\frac{\partial^2}{\partial\lambda\,\partial\alpha}LogL=\sum_{i=m_1+1}^{m}\frac{x_i-\varepsilon}{\tilde{X}_i^2}\left\{1-(\beta k(R_i+1)-1)\times\frac{(\tilde{X}_i-\alpha)\,exp\left(\frac{\alpha}{\tilde{X}_i}\right)-\tilde{X}_i}{\tilde{X}_i\left(exp\left(\frac{\alpha}{\tilde{X}_i}\right)-1\right)^2}\right\}$$

and

$$\frac{\partial^2}{\partial\beta\,\partial\lambda}LogL=\frac{\partial^2}{\partial\lambda\,\partial\beta}LogL=\sum_{i=m_1+1}^{m}\left\{\frac{x_i-\varepsilon}{\tilde{X}_i}\left(\frac{\alpha}{\tilde{X}_i}\left(1+\frac{k(R_i+1)-1}{1-exp\left(\frac{\alpha}{\tilde{X}_i}\right)}\right)-2\right)\right\}.$$

The asymptotic variance and co-variance of the parameters under the maximum likelihood estimation are obtained from the inverse of the Fisher information matrix and it is defined as

$$I=\begin{bmatrix}-\frac{\partial^2}{\partial\alpha^2}logL & -\frac{\partial^2}{\partial\alpha\partial\beta}logL & -\frac{\partial^2}{\partial\alpha\partial\lambda}logL\\ -\frac{\partial^2}{\partial\beta\partial\alpha}logL & -\frac{\partial^2}{\partial\beta^2}logL & -\frac{\partial^2}{\partial\beta\partial\lambda}logL\\ -\frac{\partial^2}{\partial\lambda\partial\alpha}logL & -\frac{\partial^2}{\partial\lambda\partial\beta}logL & -\frac{\partial^2}{\partial\lambda^2}logL\end{bmatrix}^{-1}_{(\hat{\alpha}_{Ml},\hat{\beta}_{Ml},\hat{\lambda}_{Ml})} \quad (19)$$

A nice close mathematical expressions are further not possible to obtain, however, a numerical method has applied here again for the numerical findings. Following [4], the optimal stress change time ε is the value, which minimizes asymptotic variance of maximum likelihood estimate. The asymptotic variances of these parameters are calculated from the diagonal elements of the inverse of the Fisher information matrix given in Eq. (19), by using Wolfram Mathematica software 10.0.

Table 1: Special Cases of PFF Censoring Scheme

Case	k	m	$R_i;1,2,\ldots,m$	Different Censoring Plans
1	5	10	1 2 1 0 1 1 0 1 3 1	First-Failure Progressive Type-II Censoring (FFPC)
2	1	10	1 1 0 0 1 1 0 1 4 1	Progressive Type-II Censoring (PC)
3	5	10	0 0 0 0 0 0 0 0 0 0	First-Failure Censoring (FFC)
4	1	10	0 0 0 0 0 0 0 0 0 20	Type-II Censoring (T-II)
5	1	10	0 0 0 0 0 0 0 0 0 0	Complete Sample (CS)
1	5	15	1 1 0 0 3 2 1 0 4 1 0 0 1 4 0	First-Failure Progressive Type-II Censoring (FFPC)
2	1	15	0 2 0 1 1 0 0 2 4 1 0 1 3 1 0	Progressive Type-II Censoring (PS)
3	5	15	0 0 0 0 0 0 0 0 0 0 0 0 0 0 0	First-Failure Censoring (FFC)
4	1	15	0 0 0 0 0 0 0 0 0 0 0 0 0 0 15	Type-II Censoring (T-II)
5	1	15	0 0 0 0 0 0 0 0 0 0 0 0 0 0 0	Complete Sample (CS)

5 Numerical Illustration Based On Simulated Data

A Monte Carlo simulation technique was used for generating $10,000$ FFP censored samples for each simulation, for the analysis of the proposed methods. The samples were simulated for $n = 30, m = 10, 15$ and hyper-parametric values $a = (0.25, 0.75, 1.25, 2.00, 5.00) = b$ with different values of k given in Table (1). All the special cases of FFPC have considered in this section for the analysis.

The values of the parameters under consideration were assumed here as $(\alpha, \beta) = (2.01, 0.39), (3.50, 3.67), (5.02, 6.98)$. The selection of these values meets the criterion that the variance should be unity.

Tables (2) - (4), presents the Bayes risks for the parameters α, β and λ respectively. The assumed value for the acceleration factor and shape parameter of ILLF are $\lambda(= 0.25(0.25)5.00)$ and $c = \mp 0.25, \mp 0.50, \mp 1.00$ respectively. Based on the numerical findings, it is observed that the Bayes risk increases first as λ increases and reaches maximum at $\lambda = 1.25$ and then decrease, however the magnitude is nominal for all the parameters under consideration. Hence, the numerical findings are presented here only for $\lambda(= 0.25, 1.25, 5.00)$.

It is further noted that, the minimum Bayes risks were found for the FFP censoring scheme over the other one, whereas the complete sample case shows maximum Bayes risks. The second minimum Bayes risks were observed for the Progressive censoring scheme. It is also observed that, as the censored sample size getting wider, the Bayes risks goes to closer. An increasing trend in the Bayes risks also seen when the parameter (α, β) increases. For the assumed values of the shape parameter of ILLF, the Bayes risks were noted minimum for $c = 0.50$ and maximum for $c = -1.00$.

Tables (5) - (7), presents the Bayes risk under GELF for shape parametric values $d = \mp 0.50, \mp 1.00$. All the properties has seen similar as discussed above for the various selected parametric values. It is also noted that, the Bayes risk minimizes for $d = 1.00$ and maximizes for $d = -1.00$.

6 Numerical Illustration under the Real Data

To demonstrate the use of the methodology introduced in the present study, a dataset from [17] is explored. Nelson [17] reports an accelerated life test of 76 times (in minutes) to breakdowns of an insulating fluid at voltage stress. Here, the stress is defined as the natural logarithm of the ratio of voltage to insulation thickness. The validity of the model is checked by computing the Kolmogorov - Smirnov (K - S) distance test between the empirical distribution function and the fitted distribution function when the parameters are obtained by the method of maximum likelihood estimation. The resultant ML estimates of the parameters are obtained as $\hat{\alpha}_{Ml} = 2.3424$ and $\hat{\beta}_{Ml} = 1.03.1344$ with the K - S distance test value $D = 0.0674$ and $p = 0.442$. This result shows that the generalized inverted Exponential model provides a good fit to the data set.

From the considered real data set of 76 observations, 30 observations have used for the numerical illustration and presents here in Table (8) - (9) for ILLF and GELF respectively. All the properties have seen similar as discussed above, however, the magnitude of the Bayes risk getting smaller as compared to the simulated data set.

Table 2: Bayes Risk for Parameter α under ILLF

		$c = 0.50$						$c = -1.00$					
		$m_1 = 05$ $m - m_1 = 05$			$m_1 = 10$ $m - m_1 = 05$			$m_1 = 05$ $m - m_1 = 05$			$m_1 = 10$ $m - m_1 = 05$		
$(\alpha, \beta) \rightarrow$	$\lambda \downarrow$	2.01, 0.39	3.50, 3.67	5.02, 6.98	2.01, 0.39	3.50, 3.67	5.02, 6.98	2.01, 0.39	3.50, 3.67	5.02, 6.98	2.01, 0.39	3.50, 3.67	5.02, 6.98
FFPC	0.25	0.653	0.669	0.678	0.582	0.604	0.606	0.701	0.724	0.731	0.625	0.645	0.648
	1.25	0.743	0.762	0.768	0.658	0.692	0.697	0.762	0.773	0.780	0.667	0.701	0.698
	5.00	0.675	0.692	0.741	0.598	0.615	0.625	0.728	0.756	0.773	0.638	0.671	0.682
PC	0.25	0.737	0.759	0.766	0.657	0.662	0.684	0.744	0.761	0.773	0.663	0.686	0.691
	1.25	0.813	0.838	0.841	0.717	0.760	0.761	0.852	0.873	0.880	0.755	0.793	0.799
	5.00	0.745	0.769	0.821	0.661	0.683	0.710	0.772	0.791	0.847	0.685	0.703	0.714
FFC	0.25	0.817	0.835	0.848	0.728	0.753	0.757	0.827	0.873	0.876	0.737	0.771	0.776
	1.25	0.918	0.941	0.948	0.814	0.824	0.836	0.927	0.947	0.951	0.819	0.837	0.846
	5.00	0.841	0.861	0.922	0.746	0.765	0.778	0.865	0.914	0.935	0.767	0.815	0.815
T-II	0.25	0.787	0.813	0.819	0.701	0.723	0.741	0.809	0.832	0.840	0.721	0.743	0.750
	1.25	0.878	0.904	0.908	0.775	0.821	0.822	0.904	0.934	0.935	0.801	0.819	0.849
	5.00	0.812	0.838	0.893	0.719	0.744	0.752	0.818	0.846	0.899	0.725	0.752	0.758
CS	0.25	0.843	0.877	0.881	0.752	0.772	0.781	0.836	0.877	0.885	0.744	0.773	0.783
	1.25	0.932	0.951	0.953	0.792	0.849	0.855	0.969	0.983	0.992	0.828	0.872	0.884
	5.00	0.885	0.924	0.941	0.775	0.821	0.847	0.895	0.938	0.944	0.794	0.838	0.843

Table 3: Bayes Risk for Parameter β under ILLF

		$c = 0.50$						$c = -1.00$					
		$m_1 = 05$ $m - m_1 = 05$			$m_1 = 10$ $m - m_1 = 05$			$m_1 = 05$ $m - m_1 = 05$			$m_1 = 10$ $m - m_1 = 05$		
$(\alpha, \beta) \rightarrow$	$\lambda \downarrow$	2.01, 0.39	3.50, 3.67	5.02, 6.98	2.01, 0.39	3.50, 3.67	5.02, 6.98	2.01, 0.39	3.50, 3.67	5.02, 6.98	2.01, 0.39	3.50, 3.67	5.02, 6.98
FFPC	0.25	0.732	0.749	0.759	0.653	0.677	0.679	0.784	0.809	0.817	0.701	0.722	0.726
	1.25	0.794	0.815	0.821	0.701	0.738	0.743	0.815	0.827	0.834	0.710	0.736	0.745
	5.00	0.732	0.752	0.805	0.648	0.666	0.677	0.791	0.821	0.840	0.691	0.727	0.740
PC	0.25	0.824	0.848	0.855	0.736	0.742	0.766	0.831	0.851	0.863	0.742	0.768	0.771
	1.25	0.870	0.899	0.901	0.766	0.813	0.814	0.914	0.937	0.944	0.807	0.849	0.855
	5.00	0.809	0.836	0.892	0.715	0.741	0.771	0.839	0.861	0.922	0.743	0.763	0.776
FFC	0.25	0.912	0.932	0.946	0.814	0.841	0.846	0.922	0.973	0.976	0.824	0.861	0.866
	1.25	0.987	1.011	1.021	0.872	0.883	0.896	0.997	1.018	1.023	0.878	0.898	0.907
	5.00	0.914	0.937	1.005	0.811	0.831	0.846	0.942	0.995	1.018	0.833	0.887	0.887
T-II	0.25	0.879	0.908	0.913	0.784	0.809	0.827	0.902	0.928	0.937	0.806	0.830	0.838
	1.25	0.942	0.972	0.975	0.830	0.879	0.881	0.971	1.004	1.005	0.857	0.878	0.910
	5.00	0.883	0.911	0.973	0.781	0.808	0.817	0.890	0.921	0.979	0.788	0.818	0.824
CS	0.25	0.941	0.978	0.982	0.840	0.862	0.871	0.932	0.978	0.987	0.832	0.863	0.874
	1.25	1.002	1.022	1.026	0.848	0.911	0.917	1.043	1.058	1.068	0.887	0.936	0.949
	5.00	0.964	1.007	1.025	0.843	0.893	0.922	0.974	1.022	1.028	0.863	0.911	0.917

7 Conclusion

In the article we studied about some Bayes risks of the unknown parameters of generalized Inverted Exponential distribution. The optimum Step-Stress Partially Accelerated Life Test (SS-PALT) has been adopted with different censoring patterns. A comparison between Bayes risks have discussed under two different asymmetric loss functions with the help of real and simulated data set. Based on selected parametric values, it has been observed from the tables that, the minimum Bayes risks were found for the FFP censoring. Whereas the complete sample case shows maximum Bayes risks. Similar deeds also has been seen in real data set. The data set was discussed by [17] and used hereafter validity of the model by computing the Kolmogorov-Smirnov (K-S) distance test.

Table 4: Bayes Risk for Parameter λ under ILLF

		$c=0.50$						$c=-1.00$					
		$m_1=05$ $m-m_1=05$			$m_1=10$ $m-m_1=05$			$m_1=05$ $m-m_1=05$			$m_1=10$ $m-m_1=05$		
$(\alpha,\beta)\rightarrow$	$\lambda\downarrow$	2.01, 0.39	3.50, 3.67	5.02, 6.98	2.01, 0.39	3.50, 3.67	5.02, 6.98	2.01, 0.39	3.50, 3.67	5.02, 6.98	2.01, 0.39	3.50, 3.67	5.02, 6.98
FFPC	0.25	0.898	0.919	0.931	0.803	0.831	0.834	0.960	0.992	1.001	0.860	0.886	0.890
	1.25	0.937	0.962	0.971	0.823	0.869	0.875	0.963	0.976	0.985	0.835	0.879	0.877
	5.00	0.875	0.899	0.963	0.773	0.796	0.808	0.946	0.983	1.006	0.826	0.869	0.885
PC	0.25	1.009	1.039	1.047	0.903	0.911	0.939	1.018	1.041	1.056	0.910	0.941	0.946
	1.25	1.029	1.064	1.067	0.902	0.959	0.961	1.082	1.109	1.119	0.952	1.003	1.011
	5.00	0.969	1.001	1.068	0.855	0.886	0.921	1.004	1.030	1.105	0.888	0.913	0.928
FFC	0.25	1.116	1.141	1.157	0.997	1.031	1.036	1.129	1.190	1.194	1.009	1.054	1.061
	1.25	1.171	1.199	1.211	1.031	1.045	1.061	1.182	1.208	1.214	1.038	1.062	1.074
	5.00	1.096	1.123	1.205	0.971	0.995	1.013	1.129	1.193	1.221	0.998	1.062	1.062
T-II	0.25	1.076	1.111	1.118	0.961	0.991	1.013	1.104	1.135	1.146	0.988	1.017	1.027
	1.25	1.116	1.152	1.156	0.980	1.041	1.042	1.151	1.191	1.192	1.013	1.038	1.077
	5.00	1.058	1.092	1.166	0.935	0.967	0.978	1.066	1.104	1.174	0.943	0.979	0.986
CS	0.25	1.151	1.195	1.201	1.028	1.056	1.066	1.141	1.196	1.206	1.019	1.057	1.070
	1.25	1.188	1.213	1.217	1.002	1.078	1.086	1.238	1.256	1.269	1.050	1.108	1.125
	5.00	1.155	1.208	1.229	1.009	1.069	1.104	1.168	1.226	1.234	1.034	1.092	1.099

Table 5: Bayes Risk for Parameter α under GELF

		$d=1.00$						$d=-1.00$					
		$m_1=05$ $m-m_1=05$			$m_1=10$ $m-m_1=05$			$m_1=05$ $m-m_1=05$			$m_1=10$ $m-m_1=05$		
$(\alpha,\beta)\rightarrow$	$\lambda\downarrow$	2.01, 0.39	3.50, 3.67	5.02, 6.98	2.01, 0.39	3.50, 3.67	5.02, 6.98	2.01, 0.39	3.50, 3.67	5.02, 6.98	2.01, 0.39	3.50, 3.67	5.02, 6.98
FFPC	0.25	0.457	0.468	0.474	0.409	0.423	0.425	0.489	0.505	0.510	0.438	0.451	0.453
	1.25	0.482	0.495	0.499	0.424	0.447	0.451	0.495	0.502	0.507	0.431	0.452	0.451
	5.00	0.449	0.461	0.494	0.397	0.408	0.414	0.485	0.504	0.516	0.423	0.446	0.454
PC	0.25	0.514	0.529	0.534	0.460	0.463	0.478	0.519	0.531	0.538	0.464	0.481	0.482
	1.25	0.529	0.547	0.548	0.464	0.494	0.494	0.556	0.571	0.575	0.491	0.516	0.521
	5.00	0.496	0.513	0.547	0.438	0.454	0.472	0.515	0.528	0.566	0.455	0.468	0.476
FFC	0.25	0.569	0.581	0.59	0.508	0.525	0.528	0.575	0.607	0.609	0.514	0.537	0.541
	1.25	0.601	0.616	0.622	0.531	0.537	0.545	0.607	0.621	0.623	0.534	0.546	0.552
	5.00	0.561	0.575	0.617	0.497	0.511	0.519	0.578	0.611	0.626	0.511	0.544	0.544
T-II	0.25	0.548	0.566	0.57	0.489	0.505	0.516	0.563	0.579	0.584	0.503	0.518	0.523
	1.25	0.574	0.592	0.594	0.504	0.535	0.536	0.591	0.612	0.612	0.521	0.534	0.554
	5.00	0.542	0.560	0.597	0.479	0.496	0.501	0.546	0.565	0.601	0.483	0.502	0.505
CS	0.25	0.586	0.609	0.612	0.524	0.538	0.543	0.581	0.609	0.615	0.519	0.538	0.545
	1.25	0.610	0.623	0.625	0.515	0.554	0.558	0.636	0.645	0.652	0.540	0.569	0.578
	5.00	0.592	0.619	0.63	0.517	0.548	0.566	0.598	0.628	0.632	0.530	0.560	0.563

Table 6: Bayes Risk for Parameter β under GELF

		$d = 1.00$						$d = -1.00$					
		$m_1 = 05$ $m - m_1 = 05$			$m_1 = 10$ $m - m_1 = 05$			$m_1 = 05$ $m - m_1 = 05$			$m_1 = 10$ $m - m_1 = 05$		
$(\alpha, \beta) \rightarrow$	$\lambda \downarrow$	2.01, 0.39	3.50, 3.67	5.02, 6.98	2.01, 0.39	3.50, 3.67	5.02, 6.98	2.01, 0.39	3.50, 3.67	5.02, 6.98	2.01, 0.39	3.50, 3.67	5.02, 6.98
FFPC	0.25	0.574	0.588	0.595	0.514	0.532	0.534	0.604	0.613	0.641	0.522	0.542	0.548
	1.25	0.586	0.602	0.607	0.524	0.543	0.547	0.612	0.621	0.637	0.551	0.567	0.571
	5.00	0.552	0.567	0.597	0.487	0.501	0.509	0.597	0.621	0.635	0.521	0.548	0.558
PC	0.25	0.645	0.664	0.669	0.578	0.582	0.601	0.651	0.665	0.675	0.582	0.602	0.605
	1.25	0.655	0.666	0.678	0.586	0.601	0.601	0.678	0.695	0.701	0.596	0.628	0.633
	5.00	0.611	0.632	0.674	0.539	0.558	0.581	0.634	0.651	0.697	0.561	0.575	0.585
FFC	0.25	0.713	0.728	0.739	0.637	0.658	0.662	0.721	0.761	0.762	0.645	0.673	0.678
	1.25	0.734	0.752	0.759	0.646	0.665	0.665	0.742	0.768	0.772	0.65	0.666	0.673
	5.00	0.692	0.709	0.751	0.612	0.628	0.639	0.713	0.754	0.771	0.629	0.671	0.675
T-II	0.25	0.687	0.709	0.714	0.614	0.633	0.647	0.705	0.725	0.732	0.631	0.651	0.656
	1.25	0.701	0.722	0.735	0.613	0.651	0.653	0.722	0.747	0.748	0.634	0.658	0.675
	5.00	0.668	0.689	0.726	0.589	0.611	0.617	0.673	0.697	0.741	0.594	0.617	0.622
CS	0.25	0.734	0.761	0.764	0.627	0.675	0.681	0.729	0.763	0.771	0.651	0.675	0.684
	1.25	0.746	0.763	0.766	0.657	0.676	0.681	0.777	0.789	0.797	0.657	0.695	0.705
	5.00	0.729	0.743	0.756	0.637	0.675	0.697	0.737	0.774	0.779	0.652	0.689	0.694

Table 7: Bayes Risk for Parameter λ under GELF

		$d = 1.00$						$d = -1.00$					
		$m_1 = 05$ $m - m_1 = 05$			$m_1 = 10$ $m - m_1 = 05$			$m_1 = 05$ $m - m_1 = 05$			$m_1 = 10$ $m - m_1 = 05$		
$(\alpha, \beta) \rightarrow$	$\lambda \downarrow$	2.01, 0.39	3.50, 3.67	5.02, 6.98	2.01, 0.39	3.50, 3.67	5.02, 6.98	2.01, 0.39	3.50, 3.67	5.02, 6.98	2.01, 0.39	3.50, 3.67	5.02, 6.98
FFPC	0.25	0.641	0.659	0.665	0.561	0.593	0.597	0.661	0.669	0.676	0.569	0.601	0.599
	1.25	0.651	0.665	0.673	0.582	0.602	0.604	0.694	0.717	0.723	0.623	0.641	0.645
	5.00	0.611	0.627	0.673	0.539	0.554	0.563	0.661	0.687	0.704	0.576	0.607	0.618
PC	0.25	0.707	0.731	0.733	0.617	0.657	0.658	0.735	0.752	0.762	0.649	0.671	0.684
	1.25	0.729	0.751	0.756	0.653	0.658	0.679	0.744	0.764	0.771	0.652	0.688	0.694
	5.00	0.677	0.731	0.748	0.596	0.618	0.644	0.702	0.721	0.764	0.621	0.637	0.648
FFC	0.25	0.805	0.822	0.834	0.702	0.744	0.748	0.814	0.834	0.838	0.713	0.731	0.739
	1.25	0.807	0.828	0.835	0.708	0.752	0.759	0.815	0.857	0.861	0.729	0.761	0.765
	5.00	0.767	0.787	0.825	0.678	0.696	0.708	0.791	0.837	0.857	0.698	0.743	0.744
T-II	0.25	0.769	0.794	0.797	0.672	0.714	0.716	0.786	0.819	0.813	0.704	0.724	0.731
	1.25	0.776	0.801	0.826	0.694	0.716	0.732	0.793	0.822	0.822	0.726	0.743	0.751
	5.00	0.741	0.765	0.818	0.653	0.676	0.684	0.746	0.773	0.821	0.659	0.684	0.689
CS	0.25	0.821	0.837	0.841	0.727	0.745	0.749	0.822	0.861	0.869	0.736	0.763	0.772
	1.25	0.829	0.861	0.865	0.743	0.762	0.769	0.855	0.868	0.877	0.721	0.763	0.775
	5.00	0.811	0.847	0.862	0.706	0.749	0.774	0.819	0.861	0.865	0.724	0.765	0.771

Table 8: Bayes Risk Under ILLF

$n=30$		$c=0.50$						$c=-1.00$					
		$m_1=05$ $m-m_1=05$			$m_1=10$ $m-m_1=05$			$m_1=05$ $m-m_1=05$			$m_1=10$ $m-m_1=05$		
$\leftarrow(\alpha,\beta)\rightarrow$		2.01, 0.39	3.50, 3.67	5.02, 6.98	2.01, 0.39	3.50, 3.67	5.02, 6.98	2.01, 0.39	3.50, 3.67	5.02, 6.98	2.01, 0.39	3.50, 3.67	5.02, 6.98
α	FFPC	0.625	0.641	0.646	0.555	0.573	0.587	0.659	0.668	0.674	0.579	0.587	0.605
	PC	0.683	0.704	0.706	0.604	0.639	0.642	0.734	0.752	0.757	0.653	0.685	0.691
	FFC	0.771	0.788	0.795	0.659	0.692	0.702	0.797	0.814	0.817	0.706	0.721	0.739
	T-II	0.737	0.758	0.762	0.635	0.689	0.691	0.777	0.801	0.803	0.691	0.706	0.731
	CS	0.782	0.797	0.799	0.666	0.713	0.718	0.832	0.844	0.851	0.714	0.751	0.761
β	FFPC	0.667	0.685	0.692	0.593	0.621	0.625	0.693	0.703	0.709	0.605	0.627	0.634
	PC	0.731	0.754	0.756	0.644	0.683	0.684	0.776	0.795	0.801	0.686	0.721	0.726
	FFC	0.827	0.847	0.854	0.732	0.741	0.752	0.845	0.863	0.867	0.746	0.762	0.771
	T-II	0.791	0.815	0.817	0.697	0.738	0.739	0.824	0.851	0.852	0.728	0.746	0.747
	CS	0.842	0.856	0.859	0.752	0.764	0.769	0.884	0.896	0.905	0.753	0.794	0.805
λ	FFPC	0.786	0.806	0.813	0.691	0.729	0.734	0.817	0.828	0.835	0.712	0.747	0.745
	PC	0.862	0.891	0.893	0.757	0.804	0.806	0.917	0.939	0.948	0.808	0.854	0.857
	FFC	0.979	1.003	1.012	0.864	0.875	0.888	1.011	1.022	1.027	0.882	0.965	0.913
	T-II	0.934	0.964	0.967	0.821	0.871	0.873	0.974	1.008	1.009	0.859	0.885	0.912
	CS	0.994	1.014	1.018	0.841	0.902	0.909	1.047	1.062	1.073	0.893	0.938	0.953

Table 9: Bayes Risk Under GELF

$n=30$		$d=1.00$						$d=-1.00$					
		$m_1=05$ $m-m_1=05$			$m_1=10$ $m-m_1=05$			$m_1=05$ $m-m_1=05$			$m_1=10$ $m-m_1=05$		
$\leftarrow(\alpha,\beta)\rightarrow$		2.01, 0.39	3.50, 3.67	5.02, 6.98	2.01, 0.39	3.50, 3.67	5.02, 6.98	2.01, 0.39	3.50, 3.67	5.02, 6.98	2.01, 0.39	3.50, 3.67	5.02, 6.98
α	FFPC	0.401	0.412	0.413	0.353	0.371	0.374	0.433	0.436	0.442	0.375	0.393	0.393
	PC	0.438	0.452	0.453	0.385	0.409	0.409	0.482	0.493	0.498	0.426	0.448	0.451
	FFC	0.496	0.508	0.513	0.438	0.444	0.451	0.525	0.536	0.538	0.463	0.473	0.478
	T-II	0.474	0.488	0.491	0.417	0.442	0.443	0.511	0.529	0.529	0.452	0.463	0.481
	CS	0.503	0.513	0.515	0.426	0.458	0.461	0.549	0.557	0.563	0.468	0.493	0.532
β	FFPC	0.495	0.508	0.513	0.444	0.462	0.463	0.523	0.532	0.544	0.472	0.485	0.489
	PC	0.552	0.561	0.571	0.495	0.508	0.508	0.578	0.592	0.597	0.509	0.536	0.542
	FFC	0.618	0.633	0.638	0.545	0.561	0.561	0.632	0.654	0.657	0.555	0.568	0.574
	T-II	0.592	0.608	0.619	0.518	0.549	0.551	0.615	0.636	0.637	0.541	0.561	0.576
	CS	0.628	0.642	0.644	0.554	0.573	0.574	0.661	0.671	0.678	0.561	0.592	0.601
λ	FFPC	0.549	0.561	0.567	0.492	0.508	0.512	0.592	0.611	0.616	0.532	0.547	0.551
	PC	0.614	0.632	0.636	0.551	0.555	0.572	0.633	0.652	0.656	0.556	0.587	0.592
	FFC	0.678	0.696	0.701	0.596	0.633	0.638	0.693	0.728	0.731	0.621	0.648	0.651
	T-II	0.653	0.673	0.694	0.585	0.603	0.616	0.675	0.699	0.699	0.618	0.633	0.639
	CS	0.696	0.723	0.726	0.625	0.641	0.647	0.726	0.737	0.745	0.614	0.649	0.659

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