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Oscillation Theorems for Second-Order Nonlinear Dynamic Equation on Time Scales

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Abstract: This paper concerns the oscillation of solutions to the second order non-linear dynamic equation

$$(r(t)x^{\Delta}(t))^{\Delta}+p(t)f(x^{\sigma}(t))g(x^{\Delta}(t))=0$$

on a time scale $\mathbb{T}$ which is unbounded above. By using a generalized Riccati transformation and integral averaging technique, we establish some new sufficient conditions which ensure that every solution of this equation oscillates.

Keywords: Oscillation, Dynamic equations, Time scales, Second-order.

1 Introduction

In this paper, we consider the second order nonlinear dynamic equation

$$(r(t)x^{\Delta}(t))^{\Delta}+p(t)f(x^{\sigma}(t))g(x^{\Delta}(t))=0, \tag{1}$$

where p, r real-valued, non-negative, right-dense continuous function on a time scale $\mathbb{T}$. It is the purpose of this paper to give oscillation criteria for equation (1).
Throughout this paper, we will assume the following hypotheses:
(A1) $p,r\in C_{rd}([t_0,\infty),\mathbb{R}^+)$ such that $\int_{t_0}^{\infty}\frac{1}{r(s)}\Delta s=\infty$,
(A2) $g:\mathbb{T}\to\mathbb{R}^+$ rd-continuous and $g(u)\geq c>0$,
(A3) $f:\mathbb{T}\to\mathbb{R}$ is continuously differentiable and satisfies $uf(u)>0$, $\frac{f(u)}{u}\geq k_1>0$, $u\neq 0$.

Much recent attention has been given to dynamic equations on time scales, or measure chains, and we refer the reader to the landmark paper of Hilger [12] for a comprehensive treatment of the subject. Since then, several authors have expounded on various aspects of this new theory; see the survey paper by Agarwal et al. [1] and the references cited therein. A book on the subject of time scales by Bohner and Peterson [3] summarizes and organizes much of the time scale calculus. In recent years there has been much research activity concerning the oscillation and nonoscillation of solutions of some different equations on time scales. We refer the reader to the papers [2, 4–6, 8, 11]. In [6], the authors consider the second order dynamic equation

$$(p(t)x^{\Delta}(t))^{\Delta}+q(t)x^{\sigma}(t)=0 \; for \; t\in[a,b]$$

and give necessary and sufficient conditions for oscillation of all solutions on unbounded time scales. Unfortunately, the oscillation criteria are restricted in usage since additional assumptions have to be imposed on the unknown solutions. In [10], the authors consider the same equation and suppose that there exists some $t_0\in T$ such that p is bounded above on $[t_0,\infty)$, $\inf\{\mu(t):t\in[t_0,\infty)\}>0$ and use the Riccati equation to prove that if

$$\int_{t_0}^{\infty}q(t)\Delta t=\infty,$$

then every solution is oscillatory on $[t_0,\infty)$.

In [5], the authors consider

$$(p(t)x^{\Delta}(t))^{\Delta}+q(t)(fox^{\sigma})=0 \; for \; t\in[a,b],$$

where p and q are positive, real-valued rd-continuous functions.

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2 Preliminary results

Lemma 2.1. Let $x(t)$ be a nonoscillatory solution of (1) and (A1)-(A3) hold. Then there exist T_0 such that

$$x(t) > 0,\ \ x^{\Delta}(t) > 0,\ \ (r(t)x^{\Delta}(t))^{\Delta} < 0\ for\ t \geq T_0.$$

Proof. Suppose that $x(t)$ is nonoscillatory solution of (1) and without loss of generality, assume $x(t) > 0$ for $t \geq T_0$. Assume that $x^{\Delta}(t) < 0$ for all large t. Then without loss of generality $x^{\Delta}(t) < 0$ for all $t \geq T_1 \geq T_0$. If $x(t) > 0$, then $f(x^{\sigma}(t)) > 0$. From (1)

$$(r(t)x^{\Delta}(t))^{\Delta} = -p(t)f(x^{\sigma}(t))g(x^{\Delta}(t)) < 0,$$

$$(r(t)x^{\Delta}(t))^{\Delta} < 0. \tag{2}$$

Define $y(t) = r(t)x^{\Delta}(t)$. So $y(t)$ is decreasing. Assume that there exist $T_1 \geq T_0$ with $y(T_1) = \kappa < 0$. Then

$$r(t)x^{\Delta}(t) = y(t) \leq y(T_1) = \kappa\ \ for\ all\ \ t \geq T_1$$

and therefore

$$x^{\Delta}(t) \leq \frac{\kappa}{r(t)}\ \ for\ all\ \ t \geq T_1$$

Then an integration for $t > T_2 \geq T_1$ gives

$$x(t) \leq x(T_2) + \kappa \int_{T_2}^{t} \frac{1}{r(s)} \Delta s \to -\infty\ \ as\ \ t \to \infty$$

which is a contradiction. Hence $x^{\Delta}(t)$ is not negative for all large t and thus $x^{\Delta}(t) > 0$ for all $t \geq T_0$.

3 Main results

Theorem 3.1 Assume that (A1)-(A3) hold. Furthermore, assume that there exist a positive real rd-functions differentiable functions $z(t)$ such that

$$\limsup_{t\to\infty} \int_{T}^{t} \left[k_1 c z(s)p(s) - \frac{1}{4}\frac{(z^{\Delta}(t))^2}{z(s)}\right] \Delta s = \infty, \tag{3}$$

then every solution of (1) is oscillatory.

Proof. Suppose to the contrary that $x(t)$ is a nonoscillatory solution of (1). Without loss of generality, we may assume that $x(t) > 0$ for $t \geq T_1 > T_0$. We shall consider only this case, the proof of the case when x(t) is eventually negative is similar. From Lemma 2.1, $x^{\Delta}(t) > 0$ for $t \geq T_1 > T_0$. Define the function $w(t)$ by the Riccati substitution

$$w(t) := z(t)\frac{r(t)x^{\Delta}(t)}{x(t)},\ t \geq T_1. \tag{4}$$

Then $w(t) > 0$ satisfies

$$w^{\Delta}(t) = \left[\frac{z(t)}{x(t)}\right]^{\Delta} (r(t)x^{\Delta}(t))^{\sigma} + \frac{z(t)}{x(t)}(r(t)x^{\Delta}(t))^{\Delta}$$

$$\begin{aligned} w^{\Delta}(t) = \ & \frac{z^{\Delta}(t)x(t) - z(t)x^{\Delta}(t)}{x(t)x^{\sigma}(t)}(r(t)x^{\Delta}(t))^{\sigma} \\ & + \frac{z(t)}{x(t)}(-p(t)f(x^{\sigma}(t))g(x^{\Delta}(t))). \end{aligned}$$

From Lemma 2.1, $x(t) > 0,\ \ x^{\Delta}(t) > 0,\ \ (r(t)x^{\Delta}(t))^{\Delta} < 0$ so

$$x^{\sigma}(t) > x(t),\ (r(t)x^{\Delta}(t))^{\sigma} < r(t)x^{\Delta}(t).$$

We get

$$\begin{aligned} w^{\Delta}(t) \leq z^{\Delta}(t)\frac{w^{\sigma}(t)}{z^{\sigma}(t)} & - z(t)\frac{x^{\Delta}(t)(r(t)x^{\Delta}(t))^{\sigma}}{(x^{\sigma}(t))^2} \\ & - \frac{z(t)}{x^{\sigma}(t)}p(t)f(x^{\sigma}(t))c \end{aligned} \tag{5}$$

$$\begin{aligned} w^{\Delta}(t) \leq & - k_1 c z(t)p(t) + z^{\Delta}(t)\frac{w^{\sigma}(t)}{z^{\sigma}(t)} \\ & - z(t)\frac{x^{\Delta}(t)(w^{\sigma}(t))^2}{(z^{\sigma}(t))^2(r(t)(x^{\Delta}(t)))^{\sigma})} \end{aligned} \tag{6}$$

$$\leq -k_1 c z(t)p(t) + z^{\Delta}(t)\frac{w^{\sigma}(t)}{z^{\sigma}(t)} - z(t)\frac{(w^{\sigma}(t))^2}{(z^{\sigma}(t))^2 r^{\sigma}(t)} \tag{7}$$

$$w^{\Delta}(t) \leq -k_1 c z(t)p(t) + \frac{1}{4}\frac{r^{\sigma}(t)(z^{\Delta}(t))^2}{z(t)} - \left[\sqrt{\frac{z(t)}{r^{\sigma}(t)}}\frac{w^{\sigma}(t)}{z^{\sigma}(t)} - \frac{1}{2}\sqrt{\frac{r^{\sigma}(t)}{z(t)}}z^{\Delta}(t)\right]^2,$$

then

$$w^{\Delta}(t) \leq -k_1 c z(t)p(t) + \frac{1}{4}\frac{r^{\sigma}(t)(z^{\Delta}(t))^2}{z(t)}. \tag{8}$$

Integrating (8) from $T \geq T_1$ to t

$$w(t) - w(T) \leq -\int_{T}^{t} \left[k_1 c z(t)p(t) - \frac{1}{4}\frac{r^{\sigma}(t)(z^{\Delta}(t))^2}{z(t)}\right] \Delta s,$$

$$\int_{T}^{t} \left[k_1 c z(t)p(t) - \frac{1}{4}\frac{r^{\sigma}(t)(z^{\Delta}(t))^2}{z(t)}\right] \Delta s \leq w(T) - w(t) < w(T).$$

Taking the lim sup of both sides of above inequality as $t \to \infty$, we obtain a contradiction to condition (3). The proof is complete. □

Corollary 3.2. Assume that (A1)-(A3) hold. If

$$\limsup_{t\to\infty} \int_{T}^{t} k_1 c p(s) \Delta s = \infty, \tag{9}$$

then every solution (1) is oscillatory.

Example 3.3. Consider the dynamic equation

$$(tx^{\Delta}(t))^{\Delta} + \tfrac{1}{\mu(t)}x^{\sigma}(t)(1 + (x^{\sigma}(t))^2)(1 + (x^{\Delta}(t))^2) = 0,\ t \in \mathbb{T}.$$

where
$r(t) = t,\ p(t) = \frac{1}{\mu(t)},\ f(x^{\sigma}(t)) = x^{\sigma}(t)(1 + (x^{\sigma}(t))^2),$
$g(x^{\Delta}(t)) = 1 + (x^{\Delta}(t))^2.$
If we take :
i)$\mathbb{T} = \mathbb{Z} \Rightarrow \sigma(t) = t \Rightarrow \mu(t) = 1 \Rightarrow p(t) = 1,$
ii) $\mathbb{T} = \{2^N : n \in \mathbb{Z}\} \cup \{0\} \Rightarrow \sigma(t) = 2t \Rightarrow \mu(t) = t \Rightarrow$

$p(t) = \frac{1}{t}$,
all conditions of Corollary 3.2 are satisfied. Hence it is oscillatory.

Corollary 3.4. Assume that (A1)-(A3) hold. If there is $\lambda \geq 1$ such that

$$\limsup_{t\to\infty} \int_T^t \left[k_1 c s^\lambda p(s) - \frac{1}{4} \frac{r^\sigma(t)((s^\lambda)^\Delta)^2}{s^\lambda} \right] \Delta s = \infty, \quad (10)$$

then every solution (1) is oscillatory.

Now, let us introduce the class of functions $\mathbb{R}$ which will be extensively used in the sequel. Let $\mathbb{D}_0 \equiv \{(t,s) \in \mathbb{T}^2 : t > s \geq t_0\}$ and $\mathbb{D} \equiv \{(t,s) \in \mathbb{T}^2 : t \geq s \geq t_0\}$. The function $H \in C_{rd}(\mathbb{D}, \mathbb{R})$ is said belong to the class $\Re$ if

(i)$H(t,t) = 0,\ t \geq t_0,\ H(t,s) > 0$, on $\mathbb{D}_0$,

(ii) H has a continuous Δ-partial derivative $H_s^\Delta(t,s)$ on $\mathbb{D}_0$ with respect to the second variable. (H is rd-continuous function if H is rd-continuous function in t and s.)

Theorem 3.6. Assume that (A1)-(A3) hold. Let z(t) be a positive differentiable function and let $H : \mathbb{D} \to \mathbb{R}$ be an rd-continuous function such that H belongs to the class $\Re$ and satisfies

$$\limsup_{t\to\infty} \frac{1}{H(t,T)} \int_T^t \left[k_1 c H(t,s) z(s) p(s) - \frac{1}{4} C(t,s) \right] \Delta s = \infty, \quad (11)$$

where

$$C(t,s) = r^\sigma(s)(z^\sigma(s)B(t,s))^2,$$

$$B(t,s) = H_s^\Delta(t,s) + H(t,s)\frac{z^\Delta(s)}{z^\sigma(s)}.$$

Then every solution of (1) is oscillatory.

Proof. Suppose to the contrary that $x(t)$ is a nonoscillatory solution of (1). Without loss of generality, we may assume that $x(t) > 0$ for $t \geq T_1 > T_0$. We shall consider only this case, the proof of the case when $x(t)$ is eventually negative is similar. From Lemma 2.1 and (7), $x^\Delta(t) > 0$ for $t \geq T_1 > T_0$, it follows that

$$w^\Delta(t) \leq -k_1 c z(t) p(t) + z^\Delta(t)\frac{w^\sigma(t)}{z^\sigma(t)} - z(t)\frac{(w^\sigma(t))^2}{(z^\sigma(t))^2 r^\sigma(t)}. \quad (12)$$

We multiply both sides by $H(t,s)$ to get

$$H(t,s)w^\Delta(t) \leq -k_1 c H(t,s) z(t) p(t) + H(t,s) z^\Delta(t) \frac{w^\sigma(t)}{z^\sigma(t)} - H(t,s) z(t) \frac{(w^\sigma(t))^2}{(z^\sigma(t))^2 r^\sigma(t)}. \quad (13)$$

Using the integration by parts formula, we have

$$\int_T^t k_1 c H(t,s) z(s) p(s) \Delta s \leq -H(t,t)w(t) + H(t,T)w(T) + \int_T^t H_s^\Delta(t,s) w^\sigma(s) \Delta s + \int_T^t H(t,s) z^\Delta(s) \frac{w^\sigma(s)}{z^\sigma(s)} \Delta s - \int_T^t H(t,s) z(s) \frac{((w^\sigma(s))^2}{(z^\sigma(s))^2 r^\sigma(s)} \Delta s$$

from $H(t,t) = 0$, we obtain

$$\int_T^t k_1 c H(t,s) z(s) p(s) \Delta s \leq H(t,T)w(T) + \int_T^t \left[H_s^\Delta(t,s) + H(t,s)\frac{z^\Delta(s)}{z^\sigma(s)} \right] w^\sigma(s) \Delta s - \int_T^t H(t,s) z(s) \frac{((w^\sigma(s))^2}{(z^\sigma(s))^2 r^\sigma(s)} \Delta s,$$

$$\int_T^t k_1 c H(t,s) z(s) p(s) \Delta s \leq H(t,T)w(T) + \int_T^t B(t,s) w^\sigma(s) \Delta s - \int_T^t H(t,s) z(s) \frac{((w^\sigma(s))^2}{(z^\sigma(s))^2 r^\sigma(s)} \Delta s.$$

Therefore, by completing the square as in Theorem 3.1, we obtain

$$\int_T^t k_1 c H(t,s) z(s) p(s) \Delta s \leq H(t,T)w(T) + \int_T^t \frac{1}{4} r^\sigma(s)(z^\sigma(s))^2 B^2(t,s) \Delta s$$

$$- \int_T^t \left[\sqrt{\frac{H(t,s)z(s)}{r^\sigma(s)}} \frac{w^\sigma(s)}{z^\sigma(s)} - \frac{1}{2}\sqrt{\frac{r^\sigma(s)}{H(t,s)z(s)}} B(t,s) z^\sigma(s) \right]^2 \Delta s.$$

Hence, we obtain

$$\int_T^t k_1 c H(t,s) z(s) p(s) \Delta s \leq H(t,T)w(T) + \int_T^t \frac{1}{4} r^\sigma(s)(z^\sigma(s)B(t,s))^2 \Delta s = H(t,T)w(T) + \int_T^t \frac{1}{4} C(t,s) \Delta s$$

where

$$C(t,s) = r^\sigma(s)(z^\sigma(s)B(t,s))^2.$$

Then for all $t \geq T$, we have

$$\int_T^t \left[k_1 c H(t,s) z(s) p(s) - \frac{1}{4} C(t,s) \right] \Delta s \leq H(t,T)w(T)$$

and this implies that

$$\limsup_{t\to\infty} \frac{1}{H(t,T)} \int_T^t \left[k_1 c H(t,s) z(s) p(s) - \frac{1}{4} C(t,s) \right] \Delta s \leq w(T)$$

which contradicts (11). The proof is complete.□
As consequences of Theorem 3.6 we get the following.

Corollary 3.7. Suppose that the assumptions of Theorem 3.6 hold. If

$$\limsup_{t\to\infty} \frac{1}{H(t,T)} \int_T^t \left[k_1 c H(t,s) p(s) - \frac{1}{4} r^\sigma(s)(H_s^\Delta(t,s))^2 \right] \Delta s = \infty,$$

then every solution of (1) is oscillatory.

Corollary 3.8. Let the assumption (11) in Theorem 3.6 be replaced by

$$\limsup_{t\to\infty} \frac{1}{H(t,T)} \int_T^t k_1 c H(t,s) z(s) p(s) = \infty,$$

$$\limsup_{t\to\infty} \frac{1}{H(t,T)} \int_T^t \left[\frac{r^\sigma(s)(z^\sigma(s))^2}{4} \left(\frac{H(t,s)z^\Delta(s)}{z^\sigma(s)} + H_s^\Delta(t,s) \right)^2 \right] \Delta s < \infty.$$

Then every solution of (1) is oscillatory.

Lemma 3.9. Let $H(t,s) = (t-s)^n$, $(t,s) \in \mathbb{D}$ with $n > 1$, we see that H belongs to the class $\mathfrak{R}$. Hence

$((t-s)^n)^\Delta \le -n(t-\sigma(s))^{n-1}$.

Proof. We consider the following two cases:
Case 1. If $\mu(t) = 0$ then

$$((t-s)^m)^\Delta = -m(t-s)^{m-1}.$$

Case 2. If $\mu(t) \neq 0$, then we have

$$\begin{aligned} ((t-s)^m)^\Delta &= \frac{1}{\mu(s)}[(t-\sigma(s))^m - (t-s)^m] \\ &= -\frac{1}{\sigma(s)-s}[(t-s)^m - (t-\sigma(s))^m]. \end{aligned} \tag{14}$$

Using the Hardy, Littlewood and Polya inequality
$x^m - y^m \ge \gamma y^{m-1}(x-y)$ for all $x \ge y > 0$ and $m \ge 1$,
we have

$$[(t-s)^m - (t-\sigma(s))^m] \ge m(t-\sigma(s))^{m-1}(\sigma(s)-s). \tag{15}$$

Then, from (14) and (15), we have

$$((t-s)^m)^\Delta \le -m(t-\sigma(s))^{m-1}.$$

Corollary 3.10. Assume that (A1)-(A3) hold. Let $z(t) = 1$, $H : \mathbb{D} \to \mathbb{R}$ be rd-continuous function such that H belongs to the class $\mathfrak{R}$. If

$$\limsup_{t\to\infty} \frac{1}{t^n} \int_T^t \left[k_1 c (t-s)^n p(s) - \frac{r^\sigma(s)}{4} (n(t-\sigma(s))^{n-1})^2 \right] \Delta s = \infty, \ for\ n > 1,$$

then equation (1) is oscillatory on $[t_0,\infty)$.

4 *Assume that* f *is differentiable .*

In this section, we assume that $f'(u) \ge k_2$ for $u \neq 0$ and some $k_2 > 0$.

Theorem 4.1. Assume that (A1)-(A3) hold. Furthermore, assume that there exists a positive real rd-continuous function $z(t)$ such that

$$\limsup_{t\to\infty} \int_T^t \left[cz(s)p(s) - \frac{1}{4}\frac{r(s)(z^\Delta(t))^2}{k_2 z(s)} \right] \Delta s = \infty, \tag{16}$$

then every solution of (1) is oscillatory.

Proof. Suppose to the contrary that $x(t)$ is a nonoscillatory solution of (1). Without loss of generality, we may assume that $x(t) > 0$ for $t \ge T_1 > T_0$. We shall consider only this case, the proof of the case when x(t) is eventually negative is similar. From Lemma 2.1, $x^\Delta(t) > 0$ for $t \ge T_1 > T_0$. Define the function $w(t)$ by

$$w(t) := z(t)\frac{r(t)x^\Delta(t)}{f(x(t))},\ t \ge T_1. \tag{17}$$

Then $w(t)$ satisfies

$$w^\Delta(t) = (r(t)(x^\Delta(t)))^\sigma \left[\frac{z(t)}{f(x(t))}\right]^\Delta + \frac{z(t)}{f(x(t))}(r(t)x^\Delta(t))^\Delta. \tag{18}$$

In view of (1), $f(x^\sigma) \ge f(x)$ and Lemma 2.1 , we have

$$\begin{aligned} w^\Delta(t) \le z^\Delta(t)\frac{(r(t)x^\Delta(t))^\sigma}{f(x^\sigma(t))} &- z(t)\frac{f^\Delta(x(t))(r(t)x^\Delta(t))^\sigma}{f^2(x^\sigma(t))} \\ &- cz(t)p(t)\frac{f(x^\sigma(t))}{f(x^\sigma(t))} \end{aligned}$$

$$w^\Delta(t) \le -cz(t)p(t) + z^\Delta(t)\frac{w^\sigma(t)}{z^\sigma(t)} - \frac{z(t)}{(z^\sigma(t))^2}\frac{f^\Delta(x(t))(w^\sigma(t))^2}{(r(t)x^\Delta(t))^\sigma}.$$

Using chain rule [4]

$f^\Delta(x(t)) = f'(x(\tau))x^\Delta(t) \ge k_2 x^\Delta(t),\ \tau \in [t,\sigma(t)]$.

We have

$$\begin{aligned} w^\Delta(t) &\le -cz(t)p(t) + z^\Delta(t)\frac{w^\sigma(t)}{z^\sigma(t)} - \frac{z(t)}{(z^\sigma(t))^2}\frac{k_2 x^\Delta(t)(w^\sigma(t))^2}{r(t)x^\Delta(t)} \\ &\le -cz(t)p(t) + z^\Delta(t)\frac{w^\sigma(t)}{z^\sigma(t)} - \frac{z(t)}{(z^\sigma(t))^2}\frac{k_2(w^\sigma(t))^2}{r(t)} \end{aligned} \tag{19}$$

$$\le -cz(t)p(t) + \frac{1}{4}\frac{r(t)(z^\Delta(t))^2}{k_2 z(t)} - \left[\sqrt{\frac{k_2 z(t)}{r(t)}}\frac{w^\sigma(t)}{z^\sigma(t)} - \frac{1}{2}\sqrt{\frac{r(t)}{k_2 z(t)}}z^\Delta(t)\right]^2,$$

then

$$w^\Delta(t) \le -cz(t)p(t) + \frac{1}{4}\frac{r(t)(z^\Delta(t))^2}{k_2 z(t)}.$$

We proceed as in the proof of Theorem 3.1 and we obtain a contradiction.

Corollary 4.2. Assume that (A1)-(A3) hold. If

$$\limsup_{t\to\infty} \int_T^t \left[cs^\lambda p(s) - \frac{r(t)((s^\lambda)^\Delta)^2}{4k_2 r(s)} \right] \Delta s = \infty,\ \ \lambda \ge 1,$$

then every solution of (1) is oscillatory.
Different choices of z(t) lead to different corollaries of the above theorem.

Theorem 4.3. Assume that (A1)-(A3) hold. Let z(t) be a positive differentiable function and let $H : \mathbb{D} \to \mathbb{R}$ be an rd-continuous function such that H belongs to the class $\mathfrak{R}$ and

$$\limsup_{t\to\infty} \frac{1}{H(t,T)} \int_T^t \left[cH(t,s)z(s)p(s) - \frac{r(s)}{4k_2 z(s)H(t,s)} E^2(t,s) \right] \Delta s = \infty,$$

where

$E(t,s) = H(t,s)z^\Delta(t) + z^\sigma(t)H_s^\Delta(t,s)$.

Then every solution of (1) is oscillatory.

Proof. The proof is similar to that of Theorem 3.6 and hence is omitted. □

As an immediate consequence of Theorem 4.3 using $z(t) = 1$, $H(t,s) = (t-s)^m$ and $m = n-1$, we get the following two results.

Corollary 4.4. Assume that (A1)-(A3) hold. The condition in Theorem 4.3 is replaced by

$$\limsup_{t\to\infty} \frac{1}{H(t,T)} \int_T^t cH(t,s)z(s)p(s)\Delta s = \infty$$

and

$$\limsup_{t\to\infty} \frac{1}{H(t,T)} \int_T^t \frac{r(s)(H_s^{\Delta}(t,s)z^{\sigma}(s)+H(t,s)z^{\Delta}(s))^2}{k_2 H(t,s)z(s)} \Delta s < \infty,$$

then every solution of (1) is oscillatory on $[t_0,\infty)$.

Corollary 4.5. Assume that (A1)-(A3) hold. If for $n > 2$

$$\limsup_{t\to\infty} \frac{1}{t^{n-1}} \int_T^t c(t-s)^{n-1} p(s)\Delta s = \infty$$

and

$$\limsup_{t\to\infty} \frac{1}{t^{n-1}} \int_T^t \frac{r(s)((n-1)(t-\sigma(s))^{n-2})^2}{4k_2(t-s)^{n-1}} \Delta s < \infty, \; t \geq s \geq T,$$

then every solution of (1) is oscillatory on $[T,\infty)$.

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