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Simulation DNA Algorithm of Set Covering Problem

Kang Zhou and Jin Chen*

School of Math and Computer, Wuhan Polytechnic University, Wuhan 430023, P. R. China

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Abstract: Sticker model is imitated by using set, variable length vector and biochemistry operator instead of tube, memory strand and biochemistry experiment for the first time. Batch separation operator and electrophoresis operator are first put forward based on DNA algorithm of Set Covering Problem (SCP) based on sticker model. Expression way, calculation method and basic properties of variable length vector and the two biochemistry operators are analyzed in detail according to the characteristics of sticker model. Simulation DNA algorithm (SDA) of SCP, which can find out all optimization set coverings, is designed, where all feasible set coverings are extracted by using batch separation operator and all optimization set coverings are extracted by using electrophoresis operator. Minimal element and deriving element are first introduced. And minimal element contains a large amount of deriving elements, so the set of batch separation operator can be simplified. Less-than relation is established to simplify the set of electrophoresis operator. Therefore the use of 'minimal element' and 'less-than' makes SDA of SCP more effective and practical. Time complexity of SDA of SCP is proved, and it shows that SDA of SCP is an effective algorithm to solve SCP.

Keywords: sticker model, SDA, SCP, batch separation operator, electrophoresis operator, minimal element, less-than relation

1 Introduction

The application range of Set Covering Problem (SCP) is wide [1], articularly, SCP has an important application in the field of optimalization of traffic network [2,3] and the Vehicle Routing Problem (VRP) [4,5,6]. SCP is a famous NP-complete problem. Domestic and foreign scholars put forward heuristic search for solving SCP, such as genetic algorithm [7], ant colony algorithm [8], simulated annealing algorithm [9], evolutionary search algorithm [10] and so on. These algorithms can only obtain the suboptimal solution of SCP. In 2010, Zhou K designed closed circle DNA algorithm [11] of SCP to obtain all the optimization solutions of SCP by using high parallelism and huge storage capacity of DNA computing, and operating times of the DNA algorithm is linear. However, the calculation scale of DNA computing is limited because of DNA encoding problem, therefore, it is a significant subject to solve SCP by borrowing thought of DNA computing.

In 2009, Zhou K first put forward simulation DNA algorithm (SDA) [12] based on thought of DNA computing by using the properties of solutions of Eight Queens Problem(EQP) to obtain all solutions of EQP. Therefore, in order to study SCP, SDA based on sticker model [13,14] is used in the paper, and the method of solving SCP is found out to obtain all solutions of SCP. Meanwhile, in the SDA, batch separation operator and electrophoresis operator are first put forward, and simplification algorithms for the two operators are given.

2 DNA Algorithm of SCP

2.1 Introduction of Sticker Model

Computing subjects of sticker model [13,14] are composed of two types of substances, which are memory strand and separation glass.

(1) Memory strand is single strand DNA molecular. DNA encoding on position i $(i = 1,2,\cdots,n)$ of memory strand is c_i or b_i, where c_i is composed of d_i and e_i, and $|c_i| = |d_ie_i| = |d_i||e_i|,|d_i| = |b_i|$.

(2) Separation glass i stores single strand DNA d_i' which is the Watson-Crick complementary sequences of DNA encoding d_i.

Biochemistry experiments of sticker model are merge experiment, separation experiment, batch separation experiment, electrophoresis experiment and detection experiment.

(1) Merge experiment: combine memory strands of tube S_1 and tube S_2 into tube S.

* Corresponding author e-mail: zhoukang_wh@163.com

(2) Separation experiment for d_i: separate tube S into tube $+(S,d_i)$ and tube $-(S,d_i)$, where tube $+(S,d_i)$ is mixture composed of all memory strands of tube S whose substrand i is d_i, and tube $-(S,d_i)$ is mixture composed of all memory strands of tube S whose substrand i is b_i.

(3) Let substrand set F be composed of several kinds of substrand $d_{i(j)}$, where $F=\{d_{i(1)},d_{i(2)},\cdots,d_{i(p)}\}$. Batch separation experiment for Substrand set F: separate for tube S into tube $+(S,F)$ and tube $-(S,F)$, where tube $+(S,F)$ is mixture composed of all memory strands of tube S which contain at least an arbitrary substrand $d_{i(j)}$ of F, and tube $-(S,F)$ is mixture composed of all memory strands of tube S which do not contain any substrand $d_{i(j)}$ of F.

(4) Detection experiment: a method to determine memory strand sequence by electrophoresis experiment and separation experiment.

2.2 SCP and DNA Algorithm

To consider two sets given, where set of element is $M=\{1,2,\cdots,m\}$ and set of subsets of set M is $V=\{v_1,v_2,\cdots,v_n\}$, and $M=\bigcup_{1\le j\le n}v_j$(set V is called as a set covering of M). Weight set of set V is $C=\{c_1,c_2,\cdots,c_n\}$. SCP is to find a subset V^* of set V, which satisfies $\bigcup_{v_j\in V^*}v_j=M$(set V^* is a set covering of M), and whose sum of weight is minimum(namely, $\min\sum_{v_j\in V^*}c_j$). DNA algorithm of SCP is as follows:

Step1 Encode DNA encoding corresponding to each subset v_i, and synthesize memory strand to produce all nonempty subsets of set V.

Step2 Extract all feasible set coverings.

(1) Let $i=1$.

(2) Let $F_i=\{v_j|i\in v_j\}$.Do batch separation experiment of tube S for Substrand set F_i to obtain tube $-(S,F_i)$ and tube $+(S,F_i)$. Let $S=+(S,F_i)$.

(3) If $i=m$, then turn to step 3. Otherwise, let $i=i+1$, and turn to (2) of Step2.

Step3 Extract all optimization set coverings from all feasible set coverings by doing gel electrophoresis experiment of tube S.

Step4 Detect experiment results.□

3 Biochemical Operator of SDA

If each DNA in tube is considered as an element in a set, where the element only considers major memory strand in the tube, and does not consider residual impurity in the tube, then a tube can be considered as a set. Doing a biochemistry experiment in a tube can be considered as calculating a biochemistry operator in a set, and the calculating object and calculated results of the biochemistry operator are sets. So according to the corresponding relation, DNA algorithm can be mapped into SDA. Main biochemical experiments in DNA algorithm of SCP are batch separation experiment and electrophoresis experiment, so batch separation operator and electrophoresis operator are studied as follows.

3.1 Variable Length Vector

Definition 3.1. For set $M=\{1,2,\cdots,m\}$, set $V=\{v_1,v_2,\cdots,v_n\}$, weight set $C=\{c_1,c_2,\cdots,c_n\}$ and $V'=\{v_{k(1)},v_{k(2)},\cdots,v_{k(p)}\}\subseteq V$.

(1) Variable length vector $a'=(k(1),k(2),\cdots,k(p))$ is called as a simulation memory strand. So $a'\Leftrightarrow V'$, and $(a'\triangleq())\Leftrightarrow(V'=\emptyset)$.

(2) For variable length vector $a=(l(1),l(2),\cdots,l(q))$, $(a',a)\triangleq(k(1),k(2),\cdots,k(p),l(1),l(2),\cdots,l(q))$. Particularly, for j $(1\le j\le n)$, $(a',j)\triangleq(k(1),k(2),\cdots,k(p),j)$.

(3) Set $L=\{a'\}$ is called as a simulation tube.

(4) For $\forall M'\subseteq M$, if $M'\subseteq\bigcup_{j=1}^{p}v_{k(j)}$, then V' is a feasible set covering of M', denoted by $V'\in cover(M')$.

(5) For $\forall M'\subseteq M$, $((k(1),k(2),\cdots,k(p))\in cover(M'))\Leftrightarrow(M'\subseteq\bigcup_{j=1}^{p}v_{k(j)})$.

(6) For $a'=(k(1),k(2),\cdots,k(p))$, $\sum_{i=1}^{p}c_{k(i)}$ is denoted by $Sum(a')$. So $Sum(a')\triangleq\sum_{v_j\in V'}c_j$.□

According to Definition 1, the following conclusions are established:

For $\forall M'\subseteq M$, $a=(k(1),k(2),\cdots,k(p)))$,

(1) $((k(1),\cdots,k(i),k(i+1),\cdots,k(p))\in cover(M'))\leftrightarrow((k(1),\cdots,k(i+1),k(i),\cdots,k(p))\in cover(M'))$.

(2) If $\forall j\exists i((1\le j\le n)\wedge(1\le i\le p)\rightarrow(j=k(i)))$, then $(a\in cover(M'))\leftrightarrow((a,j)\in cover(M'))$.□

Definition 3.2. For $a=(k(1),k(2),\cdots,k(p))$.

(1) $(k(1),\cdots,k(i),k(i+1),\cdots,k(p))\triangleq(k(1),\cdots,k(i+1),k(i),\cdots,k(p))$.

(2) If $\forall j\exists i((1\le j\le n)\wedge(1\le i\le p)\rightarrow(j=k(i)))$, then $a\triangleq(a,j)$.□

For example, $(1,3,7,8)=(1,7,3,8)=(1,3,3,7,8)$.

3.2 Batch Separation Operator and Electrophoresis Operator

Definition 3.3. For $i\in M$, let set $V(i)=\{v|i\in v\wedge v\in V\}=\{v_{l(1)},v_{l(2)},\cdots,v_{l(j)}\}$, and $V(i)$ is represented as $K_i=\{l(1),l(2),\cdots,l(j)\}$. **Batch separation operator** for simulation tube L is as follows:

$$Separate(L,K_i)\triangleq\{(a,l(q))|(a\in L)\wedge(l(q)\in K_i)\}$$

□

For example, $Separate((1,3),(2,3),(4),\{3,5,8\})=\{(1,3,3),(2,3,3),(4,3),(1,3,5),(2,3,5),(4,5),(1,3,8),(2,3,8),(4,8)\}$.

Theorem 3.1. Let $L = \{a|a \in cover(M')\}$, where $M' \subseteq M$. If $L(\tilde{a}) = \{(\tilde{a}, k(l(1)), \cdots, k(l(q)))|\{k(l(1)), \cdots, k(l(q))\} \subseteq \{k(p+1), \cdots, k(n)\}\}\bigcup\{\tilde{a}\}$, where $\tilde{a} = (k(1), k(2), \cdots, k(p)) \in L$, then

(1) $L(\tilde{a}) \subseteq L$; $|L(\tilde{a})| = 2^{n-p}$.

(2) $\forall a((a \in L(\tilde{a}) - \{\tilde{a}\}) \to (Sum(\tilde{a}) < Sum(a)))$.

So for $\forall a \in L(\tilde{a}) - \{\tilde{a}\}$, vector a does not be the optimal solution of SCP.

Proof. (1) $\tilde{a} \in L \Rightarrow (k(1), k(2), \cdots, k(p)) \in cover(M')$.

For $l(i)$ $((p+1 \le l(i) \le n) \wedge (1 \le i \le q))$,

$M' \subseteq \bigcup_{j=1}^{p} v_{k(j)} \subseteq \bigcup_{j=1}^{p} v_{k(j)} \bigcup v_{k(l(i))} \Rightarrow (\tilde{a}, k(l(i))) \in cover(M')$.

So $(\tilde{a}, k(l(1)), \cdots, k(l(q))) \in cover(M') \Rightarrow (\tilde{a}, k(l(1)), \cdots, k(l(q))) \in L$.

So $L(\tilde{a}) \subseteq L$.

A combination of $\{k(p+1), \cdots, k(n)\}$ and $\tilde{a}$ merge into a vector of $L(\tilde{a})$.

The number of combinations of $\{k(p+1), \cdots, k(n)\}$ is equal to $|L(\tilde{a})|$.

So $|L(\tilde{a})| = 2^{n-p}$.

(2) The number of components of $\tilde{a}$ is the fewest in $L(\tilde{a})$,so $Sum(\tilde{a}) < Sum(a)$.□

For example, set $V = \{v_1, v_2, v_3, v_4, v_5\}$ and set $M = \{1,2,3,4\}$. If $(1,3,4) \in cover(M)$, then $(1,3,4,2), (1,3,4,5)), (1,3,4,2,5) \in cover(M)$.

Definition 3.4. Let $L(\tilde{a}) = \{(\tilde{a}, k(l(1)), \cdots, k(l(q)))| \{k(l(1)), \cdots, k(l(q))\} \subseteq \{k(p+1), \cdots, k(n)\}\}\bigcup\{\tilde{a}\}$, where $\tilde{a} = (k(1), k(2), \cdots, k(p))$.

(1) Vector $\tilde{a}$ is called as minimal element of set $L(\tilde{a})$.

(2) Vector $\tilde{a}$ is representative of set $L(\tilde{a})$, denoted by $[\tilde{a}] \triangleq L(\tilde{a})$.

(3) For $\forall a \in L(\tilde{a}) - \{\tilde{a}\}$, vector a is called as deriving element of vector $\tilde{a}$.□

According to Theorem 1 and Definition 4, the following conclusions are established:

(1) $((l(1), l(2), \cdots, l(t))$ is deriving element of $(k(1), k(2), \cdots, k(p)))$ $\leftrightarrow$ $(\{k(1), k(2), \cdots, k(p)\} \subseteq \{l(1), l(2), \cdots, l(t)\})$ $\leftrightarrow$ $([l(1), l(2), \cdots, l(t)] \subseteq [k(1), k(2), \cdots, k(p)])$.

(2) (minimal elements of set L are $a_1, a_2, \cdots, a_q$) $\to$ $(L = \bigcup_{i=1}^{q} [a_i])$.

(So set L can be represented as $\{a_1, a_2, \cdots, a_q\}$).

(3) For $M' \subseteq M$, $\forall a \exists b((a \in L) \wedge (b \in L) \to (b \in cover(M')))$.

(For example, b is deriving element of a).□

The calculating process of $Simplify(L)$ (**algorithm 1**) is as follows:

Step1 For $\forall a = (k(1), k(2), \cdots, k(p)) \in L$, sort a according to the ascending order of components of a. Namely, $\forall j((1 \le j \le p-1) \to (k(j) \le k(j+1)))$.

Step2 For $\forall a = (k(1), k(2), \cdots, k(i-1), k(i), k(i+1), \cdots, k(p)) \in L$, eliminate component of the same value of a:

for $i = p, to, 2$ *do*

if $k(i) = k(i-1)$ *then* $a = (k(1), k(2), \cdots, k(i-1), k(i+1), \cdots, k(p))$

end

Step3 Eliminate all deriving elements of L:

(1) Sort L according to length $|a|$ of vector $a \in L$,

(2) For $\forall a \forall b((a \in L) \wedge (b \in L) \wedge (|a| = |b|))$, if $a = b$, then $L = L - \{b\}$.

(3)For $\forall a \forall b((a \in L) \wedge (b \in L) \wedge (|a| < |b|))$, if $[b] \subseteq [a]$, then $L = L - \{b\}$.

(4) Output $Simplify(L) = L$ (set of all minimal elements of L).□

For example, $Separate(\{(1,3), (2,3), (4)\}, \{3,5,8\}) = \{(1,3,3), (2,3,3), (4,3), (1,3,5), (2,3,5), (4,5), (1,3,8), (2,3,8), (4,8)\} = \{(1,3), (2,3), (4,3), (4,5), (4,8), (1,3,5), (2,3,5), (1,3,8), (2,3,8)\} = \{(1,3), (2,3), (4,3), (4,5), (4,8)\}$.

Theorem 3.2.

$Separate(L, K_i) = \{a|(a \in cover(i)) \wedge (a \in L)\} \subseteq L$.

Proof. Let $L_1 = \{(a, l(q))|(a \in L) \wedge (l(q) \in K_i)\}$ and $L_2 = \{a|(a \in cover(i)) \wedge (a \in L)\}$.

For $\forall (a, l(q)) \in L_1 \Rightarrow a \in L$. And $(a, l(q))$ is a deriving element of a.

So $(a, l(q)) \in L$ and $Separate(L, K_i) = L_1 \subseteq L$.

$l(q) \in K_i \Leftrightarrow (l(q)) \in cover(i) \Rightarrow (a, l(q)) \in cover(i)$. So $L_1 \subseteq L_2$.

For $\forall a = (k(1), k(2), \cdots, k(p)) \in L_2 \Rightarrow (a \in cover(i)) \wedge (a \in L)$.

So $\exists j((1 \le j \le q) \to (k(j)) \in cover(i))$, namely, $k(j) \in K_i$ and $(a, k(j)) = a \in L_1$. So $L_2 \subseteq L_1$.□

Definition 3.5. Let weight set $C = \{c_1, c_2, \cdots, c_n\}$, **electrophoresis operator** for set L is as follows:

$$Electrophoresis(L, C) \triangleq \{a'|Sum(a') = \min_{a \in L} Sum(a)\}$$

□

For example, $Electrophoresis(\{(1,3), (2,3), (4,3), (4,5), (4,8)\}, (2,1,3,2,4,6,3,1)) = \{(i,j)|\min\{5(1,3), 4(2,3), 5(4,3), 6(4,5), 3(4,8)\}\} = \{(4,8)\}$.

Reducing vector of set L can reduce the calculation amount to calculate electrophoresis operator. So sorting weight set C, set V and set L according to weight c_i of C, then less-than relation is established in set L.

We can sort $X = (x_1, x_2, \cdots, x_n)^\mathsf{T}$, $Y = (y_1, y_2, \cdots, y_n)^\mathsf{T}$ and $L = (a_{ij})_{n \times m}$ according to x_i of X. The calculating process of $Sort(X,Y,L)$ (**algorithm 2**) is as follows:

Step1 Merge X, Y, L to obtain matrix $B = (X,Y,L)$.

Step2 Sort B according to x_i by elementary row transformation of matrix B to obtain matrix $B' = (X',Y',L')$, which satisfies $x'_i \le x'_{i+1}$ $(1 \le i \le n-1)$.

Step3 Output $Sort(X,Y,L) = (X',Y',L')$.□

For example, for $C = (c_1, c_2, \cdots, c_n)^\mathsf{T}$ and $V = (1, 2, \cdots, n)^\mathsf{T}$, let $X = C$ and $Y = V$ to call algorithm 2 to obtain $Sort(C,V,L) = (C',V',L')$, where $C' = (c'_1, c'_2, \cdots, c'_n)^\mathsf{T}$ satisfies $c'_i \le c'_{i+1}$ $(1 \le i \le n-1)$. Then let $X = V'$, $Y = C'$ and $L = L'$ to call algorithm 2 to obtain $Sort(V',C',L') = (V,C,L)$. So the conclusion is: $Sort(V',C',L')$ is the inverse operation of $Sort(C,V,L)$.

Definition 3.6. Let set $C = \{c_1, c_2, \cdots, c_n\}$ satisfy $c_i \le c_{i+1}$ $(1 \le i \le n-1)$. Let $a_1 = (k(1), k(2), \cdots, k(p))$

and $a_2 = (l(1), l(2), \cdots, l(q)) \in L$ satisfy $k(i) < k(i+1)$ $(1 \le i \le p-1)$ and $l(i) < l(i+1)$ $(1 \le i \le q-1)$.

(1) If $(p = q) \wedge (\forall i (1 \le i \le p) \to (k(i) \le l(i))) \wedge (\exists j (1 \le j \le p) \to ((k(j) < l(j)) \wedge (c_{k(j)} < c_{l(j)})))$, then called $a_1 \prec a_2$.

(2) If $(p < q) \wedge (\forall i ((1 \le i \le p) \to (k(i) \le l(i+q-p))))$, then called $a_1 \prec a_2$. □

According to Definition 5 and Definition 6, the following conclusion is established: if $a_1 \prec a_2$, then $Sum(a_1) < Sum(a_2)$.

Calculation of electrophoresis operator can be simplified. For $C = \{c_1, c_2, \cdots, c_n\}$ satisfy $c_i \le c_{i+1}$ $(1 \le i \le n-1)$, the calculating process of $Electrophoresis(L,C)$ (**algorithm 3**) is as follows:

Step1 For $\forall i$ $(1 \le i \le n)$, for $\forall a_1, a_2 \in L$ and $|a_1| = |a_2| = i$, if $a_1 \prec a_2$, then $L = L - \{a_2\}$.

Step2 For $\forall i, j$ $(1 \le i < j \le n)$, if $a_i \prec a_j$, then $L = L - \{a_j\}$.

Step3 Compute $Sum(a^*) = \min_{a \in L} Sum(a)$. For $\forall a \in L$, if $Sum(a^*) < Sum(a)$, then $L = L - \{a\}$.

Step4 Output $Electrophoresis(L,C) = L$.□

For example, $C = (1,1,2,2,4,5,6,6)$, $Electrophoresis(\{(4),(1,3),(2,3),(3,4),(4,5),(4,8),(1,2,3),(1,3,5)\},C)$ $= Electrophoresis(\{(4),(1,3),(2,3),(1,2,3)\},C)$ $= Electrophoresis(\{(4),(1,3),(2,3)\},C) = \{(4)\}$.

4 SDA of SCP

Referring to DNA algorithm of SCP, the calculation process of SDA is as follows: first batch separation operator is used to obtain all feasible set coverings from all possible set coverings, then electrophoresis operator is used to obtain all optimization set coverings from all feasible set coverings.

4.1 SDA

To consider SCP, where set $M = \{1,2,\cdots,m\}$, set $V = \{v_1, v_2, \cdots, v_n\}$ and weight set $C = \{c_1, c_2, \cdots, c_n\}$. For $i = 1,2,\cdots,m$, $K_i = \{i(1), i(2), \cdots, i(r_i)\}$ satisfies $v_{i(j)} \in cover(i)$ $(1 \le j \le r_i)$. **SDA of SCP** is as follows:

Step1 Generate matrix $A = (a_{ij})_{n \times m}$ by set K_i.

(1) For $\forall i \forall j$ $(1 \le i \le n \wedge 1 \le j \le m)$, let $a_{ij} = 0$.

(2) For $\forall j$ $(1 \le j \le m)$ to computer $r = r_j$ from set $K_j = \{i(1), i(2), \cdots, i(r_j)\}$, for $\forall i$ $(1 \le i \le r)$, let $a_{j(i)j} = 1$.

(3) Let $A_{n \times m} = (a_{ij})$.

Step2 According to weight c_i of C to sort weight set C, set V and matrix A.

(1) Let $C = (c_1, c_2, \cdots, c_n)^\mathsf{T}$ and $V = (1,2,\cdots,n)^\mathsf{T}$.

(2) Let $X = C, Y = V, L = A$ to call algorithm 2 to obtain $Sort(C,V,L) = (C', V', A')$, where $A' = (K'_1, K'_2, \cdots, K'_m)$, $C' = (c'_1, c'_2, \cdots, c'_n)^\mathsf{T}$ satisfies $c'_i \le c'_{i+1}$ $(1 \le i \le n-1)$.

Step3 Check set covering beginning with set $\{()\}$ for each element of set M, which is set of all possible solutions of SCP, till to extract all feasible set coverings of M.

(1) Let $i = 1, L_i = \{()\}$.

(2) Compute batch separation operator $Separate(L_i, K'_i) = \tilde{L}$.

(3) Let $L = \tilde{L}$ to call algorithm 1 to obtain $Simplify(L) = L$. Let $L_{i+1} = L$.

(4) If $i = m$, then set L_{m+1} is set of all feasible set coverings of M, return to step 4; otherwise, let $i = i+1$, return to (2) of step 3.

Step4 Compute electrophoresis operator for L_{m+1} to extract all optimization set coverings of M.

(1) Let $L = L_{m+1}, C = C'$ to call algorithm 3 to obtain $Electrophoresis(L,C) = L$.

(2) Let $L^* = L = \{a_1, a_2, \cdots, a_t\}$ (set L^* is set of all optimization set coverings of M).

Step5 Restore the original order of L^* to obtain all optimization set coverings of M.

(1) For $\forall i \forall j$ $(1 \le i \le n \wedge 1 \le j \le t)$, let $a_{ij} = 0$.

(2) For $\forall j$ $(1 \le j \le t)$, $a_j = (j(1), j(2), \cdots, j(s_j)) \in L^*$. For $\forall i$ $(1 \le i \le s_j)$, let $a_{j(i)j} = 1$.

(3) Let $L_{n \times t} = (a_{ij})$, $X = V'$ and $Y = C'$.

(4) Call algorithm 2 to obtain $Sort(V', C', L) = (V, C, L')$, where $V = (1,2,\cdots,n)^\mathsf{T}$, $C = (c_1, c_2, \cdots, c_n)^\mathsf{T}$ and $L' = (a'_{ij})_{n \times t}$.

(5) For $\forall j$ $(1 \le j \le t)$, let $a_j = ()$, and to add again component of a_j.

$for\ \ j = 1, to, t\ \ do$
$\quad for\ \ i = 1, to, n\ \ do$
$\quad\quad if\ \ a'_{ij} = 1\ \ \text{then let}\ \ a_j = (a_j, i)$
$\quad end$
end

(6) Let $L^* = \{a_1, a_2, \cdots, a_t\}$ (all optimal solutions of SCP), and to output L^*.□

4.2 Analysis on Complexity of SDA

To consider SCP, where set $M = \{1,2,\cdots,m\}$, set $V = \{v_1, v_2, \cdots, v_n\}$ and weight set $C = \{c_1, c_2, \cdots, c_n\}$. For $i = 1,2,\cdots,m$, $K_i = \{i(1), i(2), \cdots, i(r_i)\}$ satisfies $v_{i(j)} \in cover(i)$ $(1 \le j \le r_i)$.

Theorem 4.1. Time complexity of SDA of SCP is $O(|L|^2 nm + |L| n^2 m)$.

Proof. In algorithm 1, time complexity of Step1 is $O(|L| n^2)$; time complexity of Step2 is $O(|L| n)$; time complexity of Step3 is $O(|L|^2 n)$, so time complexity of algorithm 1 is $O(|L|^2 n + |L| n^2)$. Time complexity of algorithm 2 is $O(n^2)$. In algorithm 3, time complexity of Step1 and Step2 is $O(|L|^2 n)$; time complexity of Step3 is $O(|L| n^2)$, so time complexity of algorithm 3 is $O(|L|^2 n + |L| n^2)$.

In SDA of SCP, time complexity of Step2 is $O(n^2)$; time complexity of Step3 is $O(|L|^2 nm + |L| n^2 m)$; time complexity of Step4 is $O(|L|^2 n + |L| n^2)$; time complexity of Step5 is $O(n^2)$; so time complexity of SDA of SCP is $O(|L|^2nm+|L|n^2m)$.□

Time complexity of SDA of SCP is decided by the size of set L of subsets of set M. The proposed algorithm 1 can simplify maximum and effectively set L to make the size of set L minimum, which can reduce effectively computation of SDA. Therefore, SDA of SCP is an effective algorithm to solve SCP.

4.3 Analysis on Validity and Feasibility of SDA

We do an example of SCP to verify validity and feasibility of SDA of SCP. Set $M=\{1,2,\cdots,9\}$, set $V=\{v_1,v_2,\cdots,v_7\}$. The data of SCP sees Table 1.

Table 1: the data of SCP

subset	v_1	v_2	v_3	v_4	v_5	v_6	v_7
weight	4	3	2	5	2	3	1
1	1	0	0	1	0	1	0
2	0	1	0	1	1	0	1
3	1	0	1	0	0	1	1
4	1	1	1	0	0	0	1
5	0	1	1	1	0	1	0
6	1	0	1	1	1	0	0
7	1	0	0	1	1	0	1
8	0	1	0	1	0	1	0
9	1	1	1	0	1	0	1

Step1 Sort C, V and A. Calculated results see Table 2.

Table 2: results of sorting C, V and A

subset	v_7	v_3	v_5	v_2	v_6	v_1	v_4
weight	1	2	2	3	3	4	5
1	0	0	0	0	1	1	1
2	1	0	1	1	0	0	1
3	1	1	0	0	1	1	0
4	1	1	0	1	0	1	0
5	0	1	0	1	1	0	1
6	0	1	1	0	0	1	1
7	1	0	1	0	0	1	1
8	0	0	0	1	1	0	1
9	1	1	1	1	0	1	0

So calculated results are:
$C'=(1,2,2,3,3,4,5)$; $V'=(7,3,5,2,6,1,4)$; $K_1'=\{5,6,7\}$; $K_2'=\{1,3,4,7\}$; $K_3'=\{1,2,5,6\}$; $K_4'=\{1,2,4,6\}$; $K_5'=\{2,4,5,7\}$; $K_6'=\{2,3,6,7\}$; $K_7'=\{1,3,6,7\}$; $K_8'=\{4,5,7\}$; $K_9'=\{1,2,3,4,6\}$.

Step2 Compute batch separation operator.

(1) $Separate(\{()\},\{5,6,7\})=\{(5),(6),(7)\}$;

(2) $Separate(\{(5),(6),(7)\},\{1,3,4,7\})=\{(5,1),(6,1),(7,1),(5,3),(6,3),(7,3),(5,4),(6,4),(7,4),(5,7),(6,7),(7)\}=\{(7),(1,5),(1,6),(3,5),(3,6),(4,5),(4,6)\}$;

(3) $Separate(\{(7),(1,5),(1,6),(3,5),(3,6),(4,5),(4,6)\},\{1,2,5,6\})=\{(1,7),(1,5),(1,6),(1,3,5),(1,3,6),(1,4,5),(1,4,6),(2,7),(1,2,5),(1,2,6),(2,3,5),(2,3,6),(2,4,5),(2,4,6),(5,7),(1,5),(1,5,6),(3,5),(3,5,6),(4,5),(4,5,6),(6,7),(1,5,6),(1,6),(3,5,6),(3,6),(4,5,6),(4,6)\}=\{(1,5),(1,6),(1,7),(2,7),(3,5),(3,6),(4,5),(4,6),(5,7),(6,7)\}$;

(4) $Separate(\{(1,5),(1,6),(1,7),(2,7),(3,5),(3,6),(4,5),(4,6),(5,7),(6,7)\},\{1,2,4,6\})=\{(1,5),(1,6),(1,7),(2,7),(2,3,5),(2,3,6),(2,4,5),(2,4,6),(3,4,5),(3,4,6),(4,5),(4,6),(4,5,7),(4,6,7),(3,5,6),(3,6),(4,6),(5,6,7),(6,7)\}=\{(1,5),(1,6),(1,7),(2,7),(3,6),(4,5),(4,6),(6,7),(2,3,5)\}$;

(5) $Separate(\{(1,5),(1,6),(1,7),(2,7),(3,6),(4,5),(4,6),(6,7),(2,3,5)\},\{2,4,5,7\})=\{(1,2,5),(1,2,6),(2,7),(2,3,6),(2,4,5),(2,4,6),(2,3,5),(1,4,7),(4,5),(4,6),(1,5),(3,5,6),(5,6,7),(1,7),(3,6,7),(6,7)\}=\{(1,5),(1,7),(2,7),(4,5),(4,6),(6,7),(1,2,6),(2,3,6),(2,3,5),(3,5,6)\}$;

(6) $Separate(\{(1,5),(1,7),(2,7),(4,5),(4,6),(6,7),(1,2,6),(2,3,6),(2,3,5),(3,5,6)\},\{2,3,6,7\})=\{(1,2,5),(2,7),(2,4,5),(2,4,6),(1,2,6),(2,3,6),(2,3,5),(1,3,5),(1,3,7),(3,4,5),(3,4,6),(3,6,7),(3,5,6),(1,5,6),(4,6),(6,7),(1,7),(4,5,7)\}=\{(1,7),(2,7),(4,6),(6,7),(1,2,5),(1,2,6),(1,3,5),(1,5,6),(2,3,5),(2,3,6),(2,4,5),(3,4,5),(3,5,6),(4,5,7)\}$;

(7) $Separate(\{(1,7),(2,7),(4,6),(6,7),(1,2,5),(1,2,6),(1,3,5),(1,5,6),(2,3,5),(2,3,6),(2,4,5),(3,4,5),(3,5,6),(4,5,7)\},\{1,3,6,7\})=\{(1,7),(1,4,6),(1,2,5),(1,2,6),(1,3,5),(1,5,6),(2,3,7),(3,4,6),(3,6,7),(2,3,5),(2,3,6),(3,4,5),(3,5,6),(4,6),(6,7),(2,7),(4,5,7)\}=\{(1,7),(2,7),(4,6),(6,7),(1,2,5),(1,2,6),(1,3,5),(1,5,6),(2,3,5),(2,3,6),(3,4,5),(3,5,6),(4,5,7)\}$;

(8) $Separate(\{(1,7),(2,7),(4,6),(6,7),(1,2,5),(1,2,6),(1,3,5),(1,5,6),(2,3,5),(2,3,6),(3,4,5),(3,5,6),(4,5,7)\},\{4,5,7\})=\{(1,4,7),(2,4,7),(4,6),(1,2,4,5),(3,4,5),(4,5,7),(1,5,7),(2,5,7),(5,6,7),(1,2,5),(1,3,5),(1,5,6),(2,3,5),(2,3,5,6),(3,5,6),(1,7),(2,7),(6,7)\}=\{(1,7),(2,7),(4,6),(6,7),(1,2,5),(1,3,5),(1,5,6),(2,3,5),(3,4,5),(3,5,6),(4,5,7)\}$;

(9) $Separate(\{(1,7),(2,7),(4,6),(6,7),(1,2,5),(1,3,5),(1,5,6),(2,3,5),(3,4,5),(3,5,6),(4,5,7)\},\{1,2,3,4,6\})=\{(1,7),(1,4,6),(1,2,5),(1,3,5),(1,5,6),(2,7),(2,4,6),(2,3,5),(3,4,6),(3,6,7),(3,4,5),(3,5,6),(4,6),(4,5,7),(6,7)\}=\{(1,7),(2,7),(4,6),(6,7),(1,2,5),(1,3,5),(1,5,6),(2,3,5),(3,4,5),(3,5,6),(4,5,7)\}$.

Step3 Compute electrophoresis operator.

$Electrophoresis(\{(1,7),(2,7),(4,6),(6,7),(1,2,5),(1,3,5),(1,5,6),(2,3,5),(3,4,5),(3,5,6),(4,5,7)\},(1,2,2,3,3,4,5))=Electrophoresis(\{(1,7),(4,6),(1,2,5),(1,3,5)\},(1,2,2,3,3,4,5))=\{6(1,7),7(4,6),6(1,2,5),6(1,3,5)\}=\{(1,7),(1,2,5),(1,3,5)\}$.

Step4 Obtain all optimization set coverings of M.

$$(V',C',L)=\left(\begin{array}{cc|ccc} 7&1&1&1&1\\ 3&2&0&1&0\\ 5&2&0&0&1\\ 2&3&0&0&0\\ 6&3&0&1&1\\ 1&4&0&0&0\\ 4&5&1&0&0 \end{array}\right)\sim\left(\begin{array}{cc|ccc} 1&4&0&0&0\\ 2&3&0&0&0\\ 3&2&0&1&0\\ 4&5&1&0&0\\ 5&2&0&0&1\\ 6&3&0&1&1\\ 7&1&1&1&1 \end{array}\right)$$

So all optimal solutions of SCP $L^*=\{(4,7),(3,6,7),(5,6,7)\}$, the optimal solution value is 6.

And all optimization set coverings of M are: $\{v_4,v_7\},\{v_3,v_6,v_7\},\{v_5,v_6,v_7\}$.

5 Conclusion

This paper solves SCP first by using SDA, where batch separation operator and electrophoresis operator are for the first time put forward. SDA of SCP has the following characteristics:

(1) Algorithm can obtain all exact optimal solutions of SCP.

(2) Minimal element and deriving element are introduced in batch separation operator to simplify calculation and to reduce elements of set.

(3) The binary relation of less-than on set L is established in electrophoresis operator to reduce calculation scale of the algorithm and to improve calculation efficiency of the algorithm.

Therefore, SDA of SCP is an effective and practical algorithm methods for SCP.

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Kang Zhou received the MS degree in Operational Research and Cybernetics from Department of Mathematics, Central China Normal University in 1994, and the PhD degree in Control Science and Engineer from Department of Control Science and Engineer, Huazhong University of Science and Technology in 2009. He is currently a professor in Wuhan Polytechnic University. His research interests are in the areas of DNA computing, the modern optimization algorithm, and operations research.

Jin Chen is received the B.S degree at Hubei Polytechnic University in 2010. Now, he is a Master degree candidate in Wuhan Polytechnic University. His research area is Applied modern intelligence optimization algorithm.