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Likelihood of Surviving Children using a Probability Model

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Abstract:
Child birth is a natural phenomenon which is required to develop a society. Controlled population growth is a boon for any developing country and taking care of it is the prime responsibility of the concerned government. Unfortunately, in India, child mortality had been a real curse for decades and there were no concrete models which could estimate the number of surviving children in a given time period. In the recent times, lots of initiatives have been taken by the government and other agencies for controlling the fast growing population and at the same time to reduce child mortality. Few of probability models are available to study the variation in the number of births to a female under varying sets of assumptions, but a very little work has been done to find out the distribution of number of surviving children out of the births in a given period of time. Hence, it is necessary to develop a probability model which could explain the distribution of number of surviving children apart from deriving the distribution of number of births. This paper attempts to develop a probability model for number of surviving children, using discrete probability distributions.
Keywords: Population, mortality, survival, probability model, probability distribution.

1. Introduction

Most of the developing countries have experienced a decrease in the mortality and fertility levels and a notable increase in the educational attainment of its citizens. These three major changes have put almost all the developing countries on an economic growth path which was awaited since decades. A decrease in fertility level is an indicator of financial stability, better medical assistance and lesser youth mortality, which in turn increases the investment in education [15]. The decision on the number of children is taken within the household decision making framework [17] and governed by the number of surviving children rather than the number of total children ever born. Hence, it is important to have the distribution of number of surviving children with the help of a concrete probability model. Although, a number of probability models are available to study the variation in the number of births to a female under varying sets of assumptions, but a very little work has been done to find out the distribution of number of surviving children out of the births in a given period of time. Another finite horizon stochastic model was proposed with respect to life cycle fertility within an environment where infant survival is uncertain [21]. It is thus, desirable to develop a probability model which could explain the distribution of number of surviving children apart from deriving the distribution of number of births.

Yadava and Singh (1985) attempted to develop a probability model, but they could only exclude the infant deaths out of the number of births in a given period. A probability model was proposed to study the effect of sex of their previous children on their subsequent fertility at each parity and explained the exclusion of infant deaths [5]. Another probability model was proposed to estimate current fertility including the component of infant and early childhood mortality [6]. Similarly, Yadava and Srivastava (1993) discussed the pattern of births over time in equilibrium birth process. Yadava and Tiwari (2006) derived a probability distribution of number of surviving children out of the births in a given time period in an equilibrium birth process for a defined duration.

The objective of this study is to develop a probability model for number of surviving children, utilizing the properties of Binomial and Poisson distribution. Efforts have been made to have the model assuming survival of a child as success and death as failure. It is also assumed that the probability of survival remains same for all parity. Then, the process of birth

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follows a Poisson distribution; hence, the probability of surviving children can be computed by combining the two distributions.

2. Methodology and Model

Let us assume that a female gives *'n'* number of births during her entire reproductive years and survival of a child is considered to be a success and death, a failure. Again, let, $X_1, X_2, X_3\dots$ be identically and independently distributed Bernoulli variates with

$$P(X_i = 1) = p \text{ and } P(X_i = 0) = q = (1-p)$$

Here *'p'* is considered to be the probability of survival. For a fixed *'n'*, the random variable $X = X_1 + X_2 + X_3 + \dots + X_n$ is a Binomial variate with parameters *'n'* and *'p'* and its distribution function is given as:

$$\boldsymbol{P[X = x|n, p] = \binom{n}{x} p^x (1-p)^{n-x}}; \qquad \text{where } 0 \le p \le 1, n > 0 \text{ and } x = 1, 2, 3, \dots\dots, n.$$

Let us now suppose that *'n'*, instead of being a fixed number, is also a random variable following Poisson's distribution with parameter *'λ'*, then

$$\boldsymbol{P[n = k] = \frac{e^{-\lambda} \lambda^k}{k!}}; \qquad \text{where, } k = 0, 1, 2, 3\dots \text{ and '}\lambda\text{' is the mean number of births.}$$

Now, combining the two distributions, we get:

$$P[X = x \cap n = k] = P(n = k).\, P(X = x|n = k)$$

$$= \frac{e^{-\lambda} \lambda^k}{k!} \binom{n}{x} p^x (1-p)^{n-x}$$

$$= \frac{e^{-\lambda} \lambda^k}{k!} \binom{k}{x} p^x (1-p)^{k-x} \qquad \text{(putting } n = k\text{)}$$

The marginal distribution of X is given by,

$$P(X = x) = \sum_{k=x}^{\infty} P[X = x \cap n = k]$$

$$= e^{-\lambda} p^x \sum_{k=x}^{\infty} \binom{k}{x} \frac{\lambda^k q^{k-x}}{k!}$$

$$= \frac{e^{-\lambda} (\lambda p)^x}{x!} \sum_{k=x}^{\infty} \frac{(\lambda q)^{k-x}}{(k-x)!}$$

$$= \frac{e^{-\lambda} (\lambda p)^x}{x!} \sum_{y=0}^{\infty} \frac{(\lambda q)^y}{y!} \qquad \text{(assuming } y = k-x\text{)}$$

$$= \frac{e^{-\lambda} (\lambda p)^x}{x!} . e^{\lambda q}$$

Now, putting $q = (1-p)$ in the above equation, we get,

$$P(X = x) = \frac{e^{-\lambda} (\lambda p)^x}{x!} . e^{\lambda(1-p)}$$

$$= \frac{e^{-\lambda} . e^{\lambda} . e^{-(\lambda p)} (\lambda p)^x}{x!}$$

After simplification, we get,

$$\boldsymbol{P(X = x) = \frac{e^{-(\lambda p)} (\lambda p)^x}{x!}}$$

The above function follows a Poisson distribution with parameter *'λp'*.

This model may be used in estimating the probability of surviving children of all parity with varying degrees of mortality.

3. Results and Discussion

The tables 1 to 9 show the probability of survival of children of all parity with varying degrees of mortality. In Table 1, it is evident that the probability of survival of children of all parity decreases as the mortality increases, except for zero births. The table also reveals that probabilities of survival, when compared in accordance with number of births, decrease as the number of births increase. This indicates that if a child of any parity dies, the probability of survival of the previous parity will increase as the number of surviving children will decrease. Fig.1 also shows that probability of survival for lower parity will be more than that for higher parity. In Table 2, it is observed that the probability of survival of children of all parity decreases as the mortality increases, except for zero and one births. From second birth onwards, the probability of survival decreases as the level of mortality increases. Fig.2 shows that the probability of survival at zero birth is lower than that for one birth and again it decreases from the second birth. This is quite obvious since in the first few births, parents are more careful and protective towards their children and as the number of children increase, the extent of care diminishes which results in lesser probability of survival than previous births. The similar trend continues till Table 9 and we observe that as the mean number of births increase, the probability of survival increases initially but it decreases afterwards with increasing mortality levels. Here, we also observe that as the mean number of births 'λ' increases, the distribution becomes more skewed in the positive direction. This clearly indicates that the probability of survival decreases as the mortality levels and number of births increase. It can also be inferred from the analysis that as the mean number of children increases (say 4.5, 5, 5.5 etc.), we get a distribution which approaches towards normality. In the initial number of births, the probability of survival increases and after achieving a desired level of births, the same decreases, which provides a more, bell shaped curve rather than a skewed curve.

Table 1: Probability of survival with mean no. of children λ=1.

No. of Births	**Probability of surviving children with varying degree of mortality and λ=1**			
	p=1	**p=0.9**	**p=0.85**	**p=0.8**
0	0.36787944	0.40656966	0.42741493	0.44932896
1	0.36787944	0.36591269	0.36330269	0.35946317
2	0.18393972	0.16466071	0.15440364	0.14378527
3	0.06131324	0.04939821	0.04374770	0.03834274
4	0.01532831	0.01111460	0.00929639	0.00766855
5	0.00306566	0.00200063	0.00158039	0.00122697
6	0.00051094	0.00030009	0.00022389	0.00016360
7	0.00007299	0.00003858	0.00002719	0.00001870
8	0.00000912	0.00000434	0.00000289	0.00000187
9	0.00000101	0.00000043	0.00000027	0.00000017

Table 2: Probability of survival with mean no. of children λ=1.5.

No. of Births	Probability of surviving children with varying degree of mortality and λ=1.5			
	p=1	p=0.9	p=0.85	p=0.8
0	0.22313016	0.25924026	0.27943097	0.30119421
1	0.33469524	0.34997435	0.35627448	0.36143305
2	0.25102143	0.23623269	0.22712498	0.21685983
3	0.12551072	0.10630471	0.09652812	0.08674393
4	0.04706652	0.03587784	0.03076834	0.02602318
5	0.01411996	0.00968702	0.00784593	0.00624556
6	0.00352999	0.00217958	0.00166726	0.00124911
7	0.00075643	0.00042035	0.00030368	0.00021413
8	0.00014183	0.00007093	0.00004840	0.00003212
9	0.00002364	0.00001064	0.00000686	0.00000428

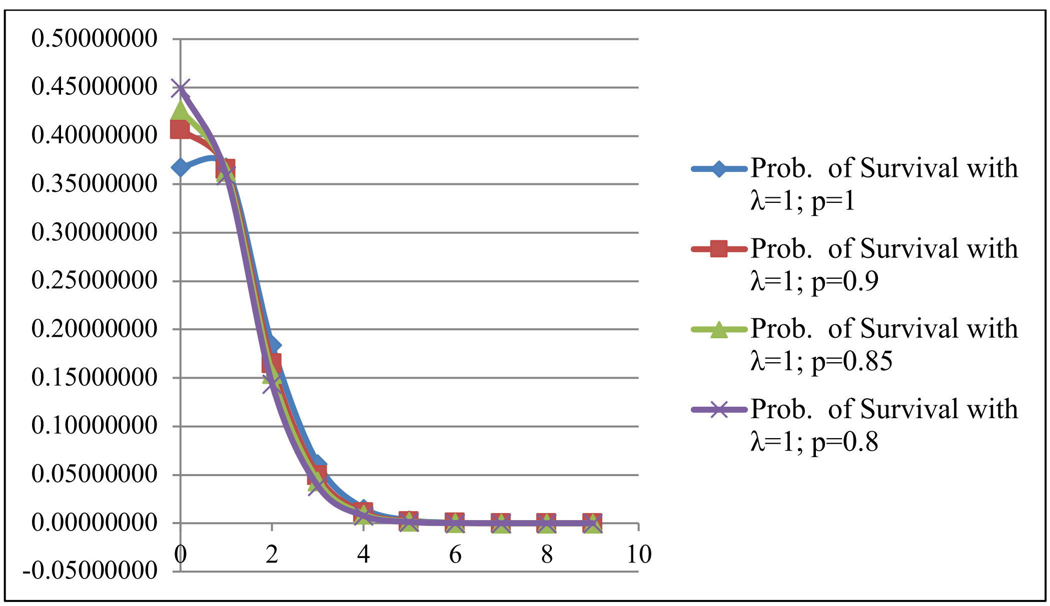

Fig.1: Probability of survival with λ=1.

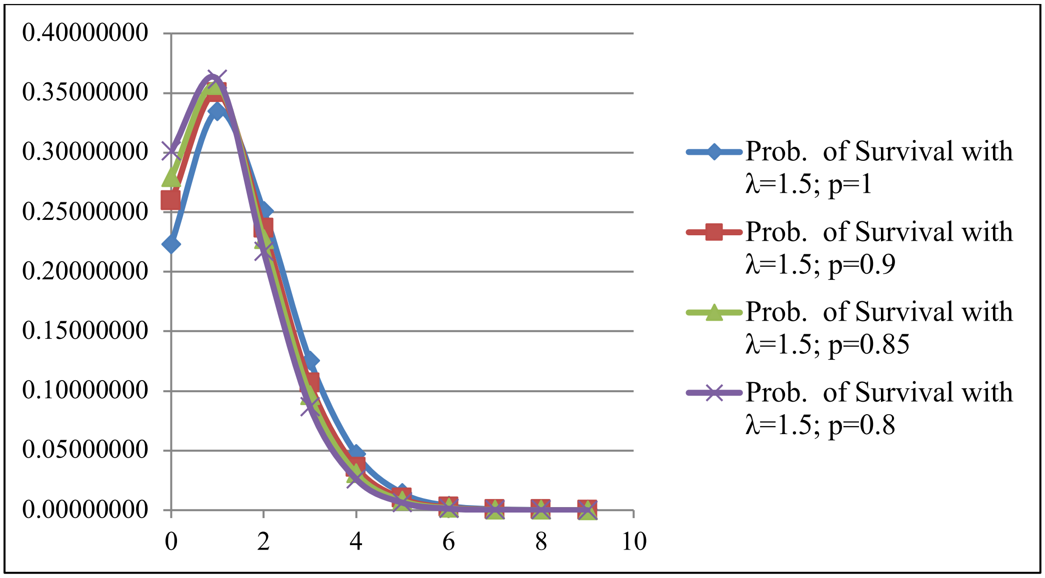

Fig.2: Probability of survival with λ=1.5.

Table 3: Probability of survival with mean no. of children λ=2.

No. of Births	Probability of surviving children with varying degree of mortality and λ=2			
	p=1	p=0.9	p=0.85	p=0.8
0	0.13533528	0.16529889	0.18268352	0.20189652
1	0.27067057	0.29753800	0.31056199	0.32303443
2	0.27067057	0.26778420	0.26397769	0.25842754
3	0.18044704	0.16067052	0.14958736	0.13782802
4	0.09022352	0.07230173	0.06357463	0.05513121
5	0.03608941	0.02602862	0.02161537	0.01764199
6	0.01202980	0.00780859	0.00612436	0.00470453
7	0.00343709	0.00200792	0.00148734	0.00107532
8	0.00085927	0.00045178	0.00031606	0.00021506
9	0.00019095	0.00009036	0.00005970	0.00003823

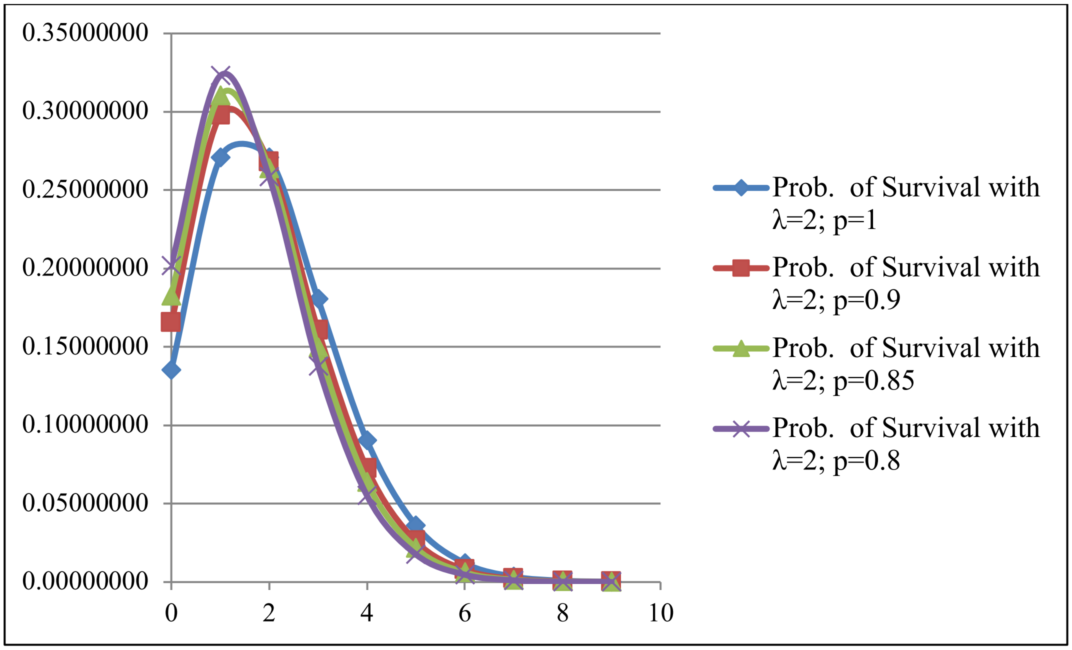

Fig.3: Probability of survival with λ=2.

Table 4: Probability of survival with mean no. of children λ=2.5.

No. of Births	Probability of surviving children with varying degree of mortality and λ=2.5			
	p=1	p=0.9	p=0.85	p=0.8
0	0.08208500	0.10539922	0.11943297	0.13533528
1	0.20521250	0.23714826	0.25379506	0.27067057
2	0.25651562	0.26679179	0.26965725	0.27067057
3	0.21376302	0.20009384	0.19100722	0.18044704
4	0.13360189	0.11255279	0.10147258	0.09022352
5	0.06680094	0.05064875	0.04312585	0.03608941
6	0.02783373	0.01899328	0.01527374	0.01202980
7	0.00994062	0.00610498	0.00463667	0.00343709
8	0.00310644	0.00171703	0.00123162	0.00085927
9	0.00086290	0.00042926	0.00029080	0.00019095

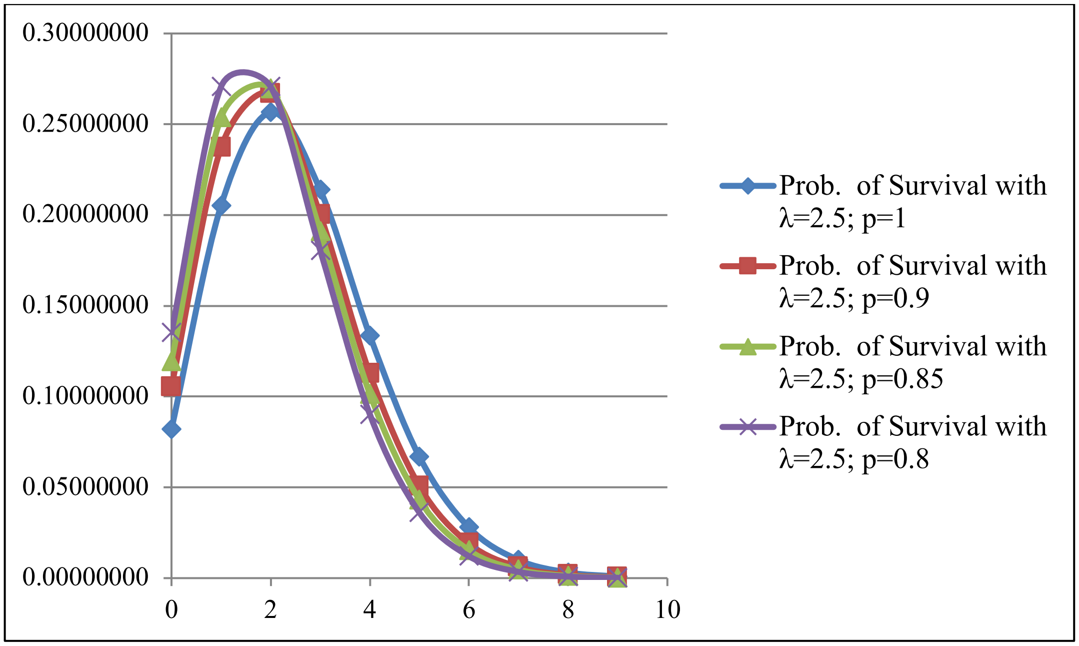

Fig.4: Probability of survival with λ=2.5.

Table 5: Probability of survival with mean no. of children λ=3.

No. of Births	**Probability of surviving children with varying degree of mortality and λ=3**			
	p=1	**p=0.9**	**p=0.85**	**p=0.8**
0	0.04978707	0.06720551	0.07808167	0.09071795
1	0.14936121	0.18145488	0.19910825	0.21772309
2	0.22404181	0.24496409	0.25386302	0.26126771
3	0.22404181	0.22046768	0.21578356	0.20901416
4	0.16803136	0.14881569	0.13756202	0.12540850
5	0.10081881	0.08036047	0.07015663	0.06019608
6	0.05040941	0.03616221	0.02981657	0.02407843
7	0.02160403	0.01394828	0.01086175	0.00825546
8	0.00810151	0.00470755	0.00346218	0.00247664
9	0.00270050	0.00141226	0.00098095	0.00066044

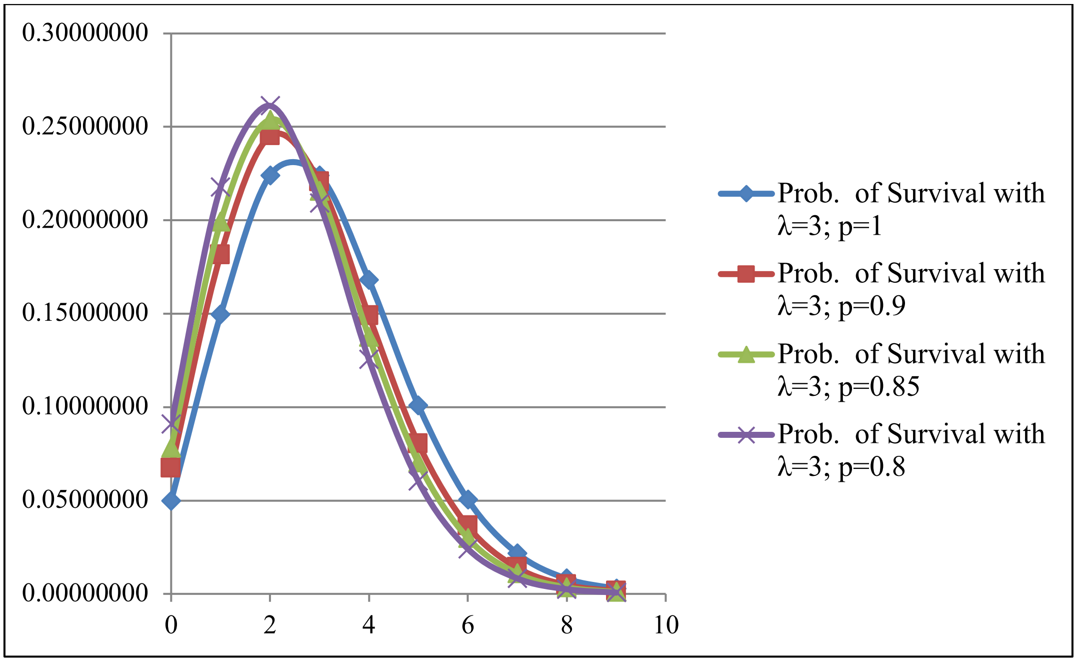

Fig.5: Probability of survival with λ=3.

Table 6: Probability of survival with mean no. of children λ=3.5.

No. of Births	Probability of surviving children with varying degree of mortality and λ=3.5			
	p=1	p=0.9	p=0.85	p=0.9
0	0.03019738	0.04285213	0.05104743	0.06081006
1	0.10569084	0.13498420	0.15186612	0.17026818
2	0.18495897	0.21260011	0.22590085	0.23837545
3	0.21578547	0.22323012	0.22401834	0.22248375
4	0.18881229	0.17579372	0.16661364	0.15573862
5	0.13216860	0.11075004	0.09913512	0.08721363
6	0.07709835	0.05814377	0.04915450	0.04069969
7	0.03854917	0.02616470	0.02089066	0.01627988
8	0.01686526	0.01030235	0.00776871	0.00569796
9	0.00655871	0.00360582	0.00256799	0.00177270

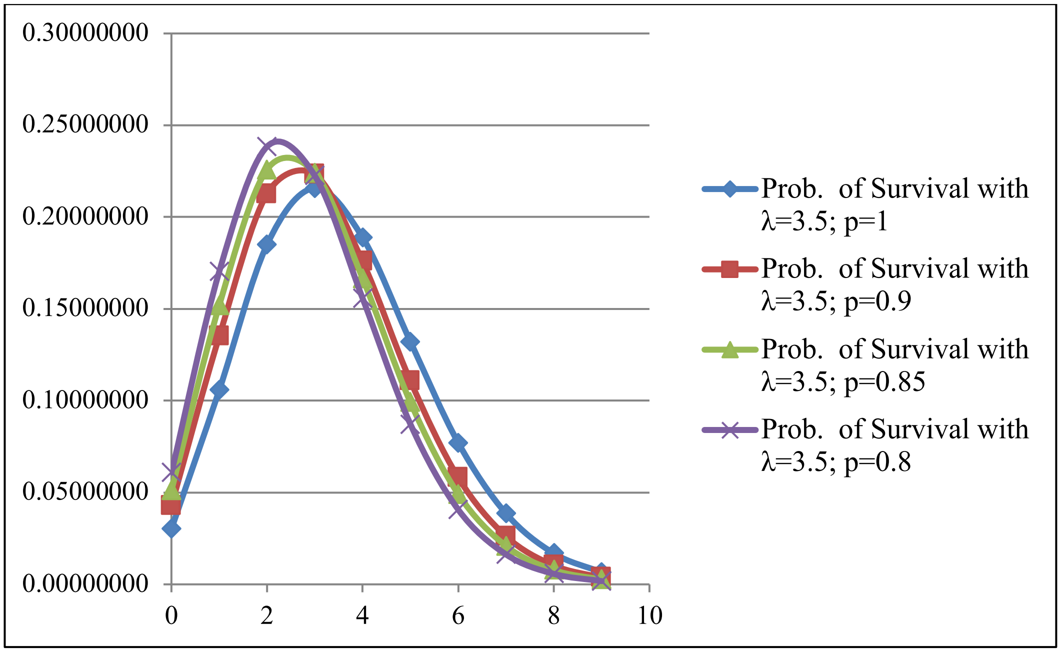

Fig.6: Probability of survival with λ=3.5.

Table 7: Probability of survival with mean no. of children λ=4.

No. of Births	Probability of surviving children with varying degree of mortality and λ=4			
	p=1	p=0.9	p=0.85	p=0.8
0	0.01831564	0.02732372	0.03337327	0.04076220
1	0.07326256	0.09836540	0.11346912	0.13043905
2	0.14652511	0.17705772	0.19289750	0.20870248
3	0.19536681	0.21246927	0.21861717	0.22261598
4	0.19536681	0.19122234	0.18582459	0.17809279
5	0.15629345	0.13768008	0.12636072	0.11397938
6	0.10419563	0.08260805	0.07160441	0.06078900
7	0.05954036	0.04248414	0.03477928	0.02778926
8	0.02977018	0.01911786	0.01478120	0.01111570
9	0.01323119	0.00764715	0.00558401	0.00395225

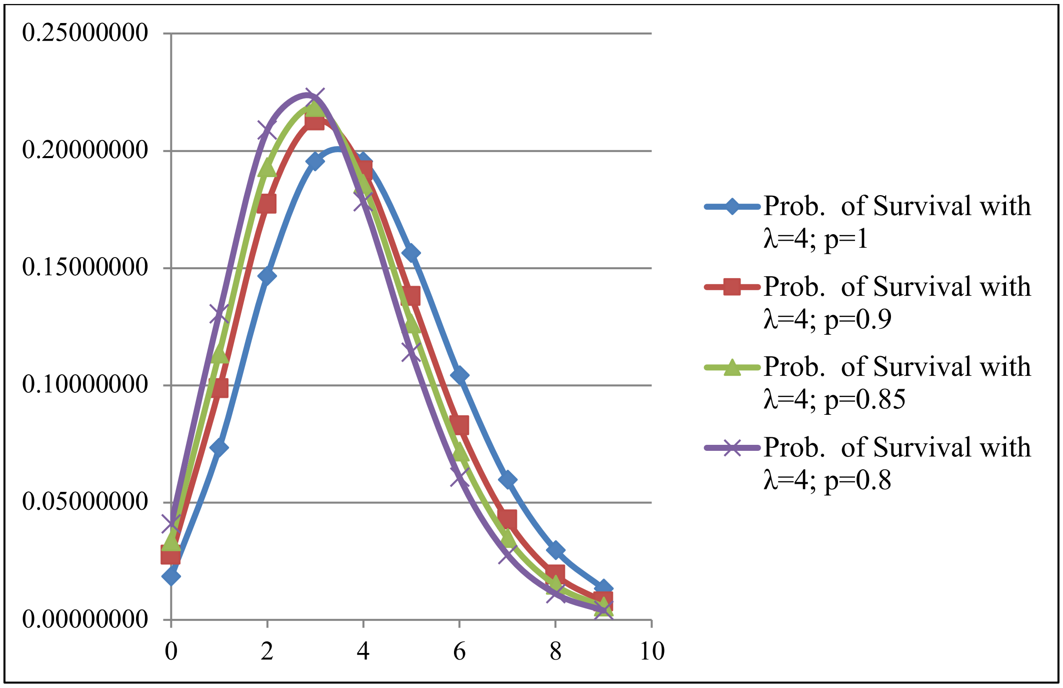

Fig.7: Probability of survival with λ=4.

Table 8: Probability of survival with mean no. of children λ=4.5.

No. of Births	**Probability of surviving children with varying degree of mortality and λ=4.5**			
	p=1	**p=0.9**	**p=0.85**	**p=0.8**
0	0.01110900	0.01742237	0.02181844	0.02732372
1	0.04999048	0.07056062	0.08345552	0.09836540
2	0.11247859	0.14288525	0.15960867	0.17705772
3	0.16871788	0.19289509	0.20350106	0.21246927
4	0.18980762	0.19530628	0.19459789	0.19122234
5	0.17082686	0.15819808	0.14886738	0.13768008
6	0.12812014	0.10678371	0.09490296	0.08260805
7	0.08236295	0.06178200	0.05185769	0.04248414
8	0.04632916	0.03127714	0.02479446	0.01911786
9	0.02316458	0.01407471	0.01053764	0.00764715

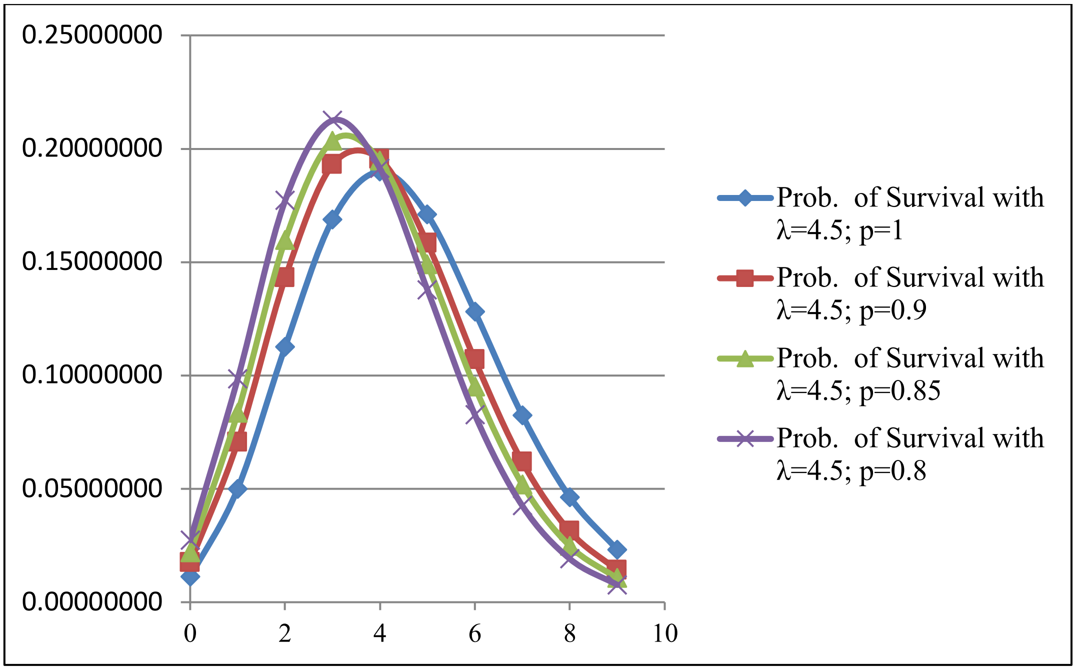

Fig.8: Probability of survival with λ=4.5.

Table 9: Probability of survival with mean no. of children λ=5.

No. of Births	Probability of surviving children with varying degree of mortality and λ=5			
	p=1	**p=0.9**	**p=0.85**	**p=0.8**
0	0.00673795	0.01110900	0.01426423	0.01831564
1	0.03368973	0.04999048	0.06062299	0.07326256
2	0.08422434	0.11247859	0.12882386	0.14652511
3	0.14037390	0.16871788	0.18250047	0.19536681
4	0.17546737	0.18980762	0.19390675	0.19536681
5	0.17546737	0.17082686	0.16482074	0.15629345
6	0.14622281	0.12812014	0.11674802	0.10419563
7	0.10444486	0.08236295	0.07088273	0.05954036
8	0.06527804	0.04632916	0.03765645	0.02977018
9	0.03626558	0.02316458	0.01778221	0.01323119

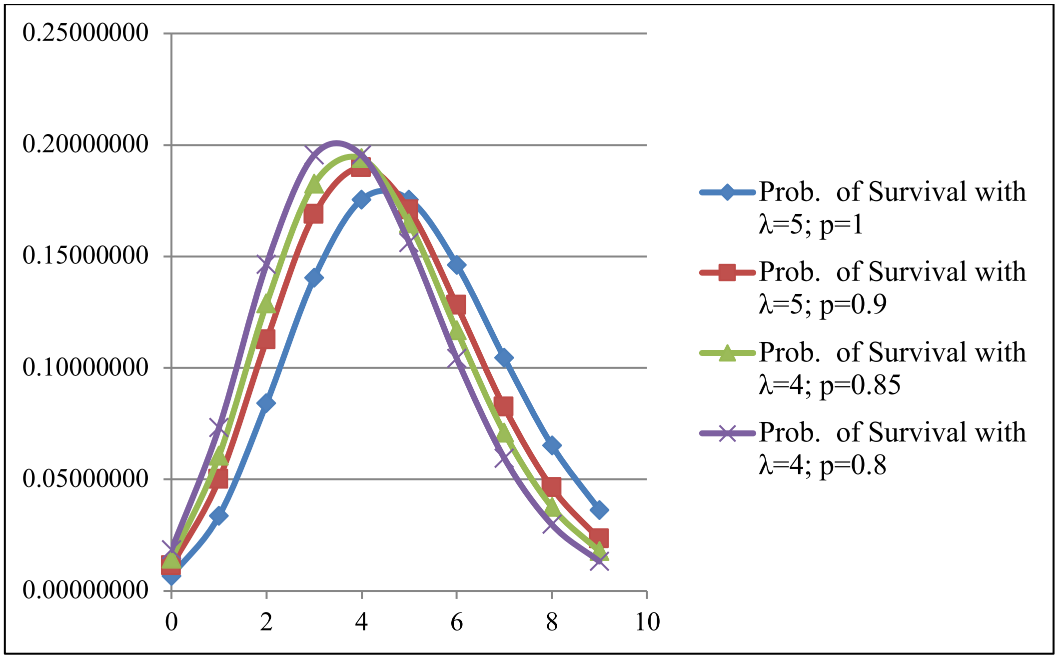

Fig.9: Probability of survival with λ=5.

4. Summary and Conclusion

In particular, the analyses presented here show a strong relationship between mean number of child births and probability of their survival. From the above discussion, it is evident that the probability of survival is more when couples have lesser children, but it diminishes with increasing child births. There may be many causes of increased mortality along with increased child births but the primary reason lies in the behavioral change among parents. If the number of children remains within the desired family size, parents will take good care of them which certainly will increase the probability of survival. Naturally, parents will become ignorant towards their children when the number is more than the desired family size. When the family size is small, parents take care of even the negligible health issues of their children but with increased size of the family, they sometimes find it difficult (though reasons are not specified) to address major health issues. Thus, the probability of child survival decreases with the increased number of children in a family. The probability of survival can also increase by increasing the birth intervals as short birth intervals increases child mortality substantially. Decreased probability with increased parity shows that child death decreases the number of children and couples will become more protective. This phenomenon is evident from all the tables and figures of the study.

Conflict of interest

The author declares that he has no known competing financial interests or personal relationships that could have appeared to influence the work reported in this paper.

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Shubhagata Roy has a rich experience of over 18 years in academia and industry. He holds a Ph.D. in Statistics from Banaras Hindu University apart from a Masters degree in Financial Management and professional certificates in life and non-life insurance. His research areas include demography, healthcare analytics, insurance analytics and spirituality. He is currently working as Assistant Professor in IBS Hyderabad, IFHE University. He is also a visiting faculty in the College of Business Administration, American University in the Emirates, Dubai. Apart from teaching, he has a rich experience of working in the insurance industry as sales and training manager. He has authored several research papers in renowned international and national journals which are indexed in high quality databases including Scopus