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Analysis of Progressively Type-I Censored Data in Competing Risks Models with Generalized Inverted Exponential Distribution

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Abstract: This paper presents estimates of the parameters involved in a competing risks model in the presence of progressive type-I censored data. We consider the case when the competing risks have generalized inverted exponential distributions. The maximum likelihood method is used to derive point and asymptotic confidence intervals for the unknown parameters. The relative risks due to each cause of failure are investigated. A real data set is used to illustrate the theoretical results and to assess the performance of relative risk and MLE estimates at different schemes of progressively type-I censored samples under causes of failure that follow the generalized inverted exponential distributions.

Keywords: Competing Risks, progressive type-I censoring scheme, generalized Inverted Exponential Distribution, maximum Likelihood Estimation.

1 Introduction

Inverted distributions have been introduced to overcome some disadvantages of the most widely used distributions in reliability and survival analysis. These disadvantages involve constant hazard (failure) rates of an exponential distribution and non-closed form of some distribution functions, such as gamma distribution. The inverted exponential distribution (IED) has been explored and used to overcome the restriction of the constant hazard rate. Lin *et al.* [1] addressed the properties of the IED, such as: reliability function, hazard rate, and estimation of the parameters using maximum likelihood method. Dey [2] explored the IED from the Bayesian viewpoint based on squared error and LINEX loss functions. The generalized inverted exponential distribution (GIED) was proposed as another useful two-parameter generalization of the inverted exponential distribution (see [3]). This lifetime distribution is capable of modeling diverse shapes of failure rates as well as aging criteria.

The cumulative distribution function (cdf) and the probability density function (pdf) of the GIED are respectively given by

$$F(x) = 1 - \left[1 - \exp\left(\frac{-\lambda}{x}\right)\right]^{\alpha}, x > 0, \lambda, \alpha > 0 \tag{1}$$

and

$$f(x) = \frac{\alpha\lambda}{x^2} \exp\left(\frac{-\lambda}{x}\right)\left[1 - \exp\left(\frac{-\lambda}{x}\right)\right]^{\alpha-1} \tag{2}$$

where α is the shape parameter and λ is the scale parameter. If $\alpha = 1$, GIED reduces to IED. Figure (1) illustrates the behavior of the GIED at $\lambda = 1$ and for some various values of α.

The survival function is given by

$$\bar{F}(x) = \left[1 - \exp\left(\frac{-\lambda}{x}\right)\right]^{\alpha}. \tag{3}$$

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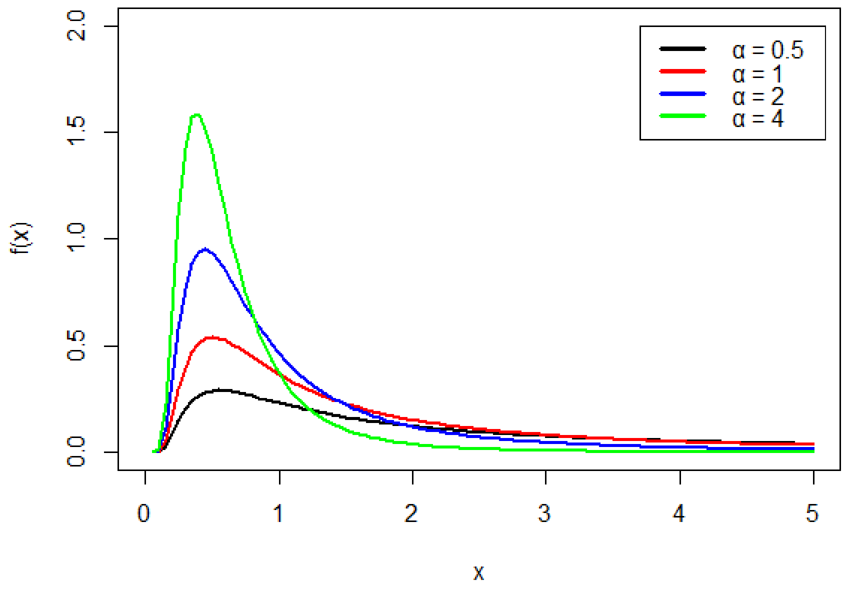

Fig. 1: Density function of GIED for some values of α

The hazard rate function of GIED can be obtained as

$$h(x) = \frac{\alpha\lambda}{x^2\left(\exp\left(\frac{\lambda}{x}\right) - 1\right)} \tag{4}$$

and its shape is illustrated in Figure (2) at $\lambda = 1$ and for some various values of α.

Several interesting characteristics and properties of GIED have been investigated by Abouammoh and Alshingiti [3]. The hazard rate function of GIED increases at the beginning of the aging and reduces at the end. It is inconstant based on the shape parameter. The GIED has a unimodal and right skewed density function. Furthermore, GIED provides a better fit than gamma, Weibull, generalized exponential and IED (see [3]). According to recent contributions to this distribution, e.g. [4,5,6,7,8,9,10,11,12] should be reported.
Some applications of GIED in reliability analysis are, as follows:

– Test on the endurance of deep groove ball bearings [3].
– Polished window strength for a glass airplane window [3].
– After surgery, where the hazard rate is very high.

However, models have been recently developed to assess the lifetimes of a specific risk in the presence of other competing risk factors. The data for these competing risk models consists of the failure time and an indicator variable denoting the specific cause of failure of the individual or item. The causes of failure may be assumed to be independent or dependent. In most situations, the analysis of competing risk data assumes independent causes of failure. See Crowder [13] and the monograph by David and Moeschberger [14] for an exhaustive treatment of different competing risks models.

In practical life testing experiments, censored data arise when the experiment has to be terminated before collecting complete observation. The censoring technique is common and unavoidable in practice, especially in reliability engineering, for many reasons, such as time constraint and cost reduction. Various types of censoring have been discussed in the literature, with the most common censoring schemes, i.e. type-I censoring and type-II censoring. More recently, progressive censoring scheme has gained significant attention in the literature because of its effective utilization of the available resources in comparison with traditional censoring designs. One of progressive censoring schemes is progressive type-I which is observed when a pre-fixed number of life test units are continuously removed through the

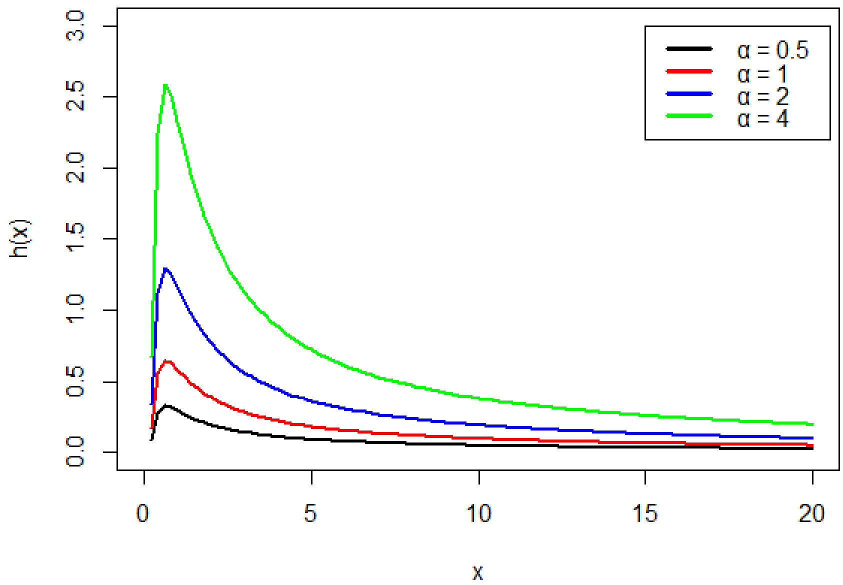

Fig. 2: Hazard rate function of GIED for some values of α

experiment at the end of each of pre-specified time intervals. It provides both the practical feature of identifying the termination time and the larger flexibility to the experimenter in the phase of design through eliminating the test units at non-terminal time points [15].

A common way of formulating a model of competing risks is to define $Y_1, Y_2, \ldots, Y_h$ respectively as the latent lifetimes of the unit when it is exposed to the $1^{st}, 2^{nd}, \ldots, h^{th}$ risk alone. In the simultaneous presence of all h causes, only the smallest of the y_i, $\min_j y_j$, is in fact observable, together with the actual cause of failure, say j. Correspondingly, we write the observed lifetime as x_j. Namely,

$$x_i = y_i | y_i = \min_j y_j,$$

$$i = 1, 2, \ldots, n, \qquad j = 1, 2, \ldots, h$$

the cumulative distribution function of the individual X_i is given by:

$$F_j(x) = P(X_j \le x) = 1 - P(X_j > x) = 1 - P(X_1 > x, X_2 > x, \ldots, X_j > x)$$

where $P(X_1 > x, X_2 > x, \ldots, X_j > x)$ is computed with respect to the joint distribution of $X_1, X_2, \ldots, X_j$. When the risks are independent, the cdf can be rewritten as follows:

$$\begin{aligned} F_j(x) &= 1 - P(X_1 > x)P(X_2 > x)\ldots P(X_j > x) \\ &= 1 - \bar{F}_1(x)\bar{F}_2(x)\ldots\bar{F}_j(x) \\ &= 1 - \prod_{j=1}^{h} \bar{F}_j(x) \end{aligned}$$

where $F_j(x)$ is the cdf for the j risk. Taking the derivative of $F_j(x)$ with respect to time x, the pdf of individual can be obtained as:

$$f_j(x) = \sum_{j=1}^{h} \left(f_j(x) \prod_{l \neq j}^{h} \bar{F}_l(x) \right) = \sum_{j=1}^{h} \left(\frac{f_j(x)}{\bar{F}_j(x)} \prod_{j=1}^{h} \bar{F}_j(x) \right)$$

Let the probability density function and the survivor function of X_{ij} be denoted by $f_j(x)$ and $\bar{F}_j(x)$, respectively. Thus, the pdf of X_i is given by

$$u_j(x_i) = \frac{f_j(x_i)}{\pi_j \bar{F}_j(x_i)} \prod_{l=1}^{h} \bar{F}_j(x_l) \tag{5}$$

where $\pi_j = P(X_j = \min_l X_l)$ is the probability of failure due to cause j and it is assumed to be non-zero with $\sum \pi_j = 1$ (see[16] and [17]).

Consider a life test experiment with h independent competing causes of failure where a random sample of n specimens is tested simultaneously, and where each failure is due to a single cause. Suppose that the test is conducted under progressive type-I censoring, where R_s items are removed from the survived items at predetermined time points T_s, $s = 1,2,\ldots,m$ where m is the number of stages in the test, $T_s > T_{s-1}$ and $n = r + \sum_{s=1}^{m} R_s$. The values T_s are to be predetermined by the experimenter, the choice of which depends on the prior knowledge and experience of the experimenter about the items on test. Suppose r_j specimens fail due to the jth cause and we observe

$$x_{j,i}, \qquad j = 1,2,\ldots,h; \quad i = 1,2,\ldots,r_j$$

where $x_{j,i}$ denotes the jth failure time due to the ith cause. In these situations, R_s, T_s and n are fixed and predetermined, while l_i is the number of the survivor items at time T_s and $r_j = \sum_{i=1}^{m} l_i$ are random variables. Figure (3) describe this scheme of censoring [18].

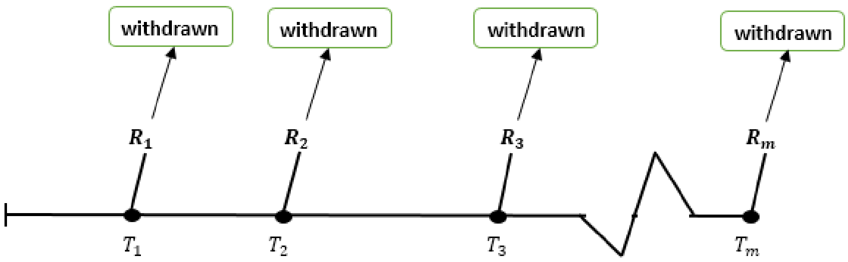

Fig. 3: Progressive Type-I Censoring Scheme

The likelihood function under progressive type-I censoring with multiple modes of failure is (see [16,17,14])

$$\ell \propto \left[\prod_{j=1}^{h} \prod_{i=1}^{r_j} \left\{ u_j(x_{ji}) \prod_{l(\neq j)=1}^{h} \bar{F}_l(x_{ji}) \right\} \right] \cdot \left[\prod_{j=1}^{h} \prod_{i'=1}^{s} \left(\bar{F}_j(T_{i'}) \right)^{R_{i'}} \right] \tag{6}$$

where i' is the stages of progressive type-I censoring scheme and $i' = 1,2,\ldots,s$.

The present study aims to analyze progressive type-I censoring scheme under the competing risks model when lifetimes have independent GIED. We drive the maximum likelihood estimates (MLE) and we obtain the approximate two sided confidence intervals of these different parameters. We consider a real data set and see how the different models work in the practical situation.

The other sections are organized as follows: Section Two involves the model and the notation used throughout this paper. Section Three handles the maximum likelihood estimation, confidence intervals, and relative risks. A real data set is used to illustrate the theoretical results in section Four.

2 Model Description and Notation

Without loss of generality, we assume that there are two causes of failures. We assume the following notation:

– X_i: lifetime of system i
– X_{ji}: lifetime of the ith individual under causes $j, j = 1,2$

–r_j: number of failure due to causes $j, j = 1,2$
–$F_j(.)$: cumulative distribution function of X_{ji}
–$\bar{F}_j(.)$: survival function of X_{ji}.

Also, we need the following assumptions throughout this paper.

1. The random vectors $(X_{1i}, X_{2i}), i = 1,2,\ldots,n$, are n i.i.d. random vectors
2. The random variablesX_{1i} and X_{2i} are independent for all $i = 1,2,\ldots,n$, and $X_i = \min(X_{1i}, X_{2i})$
3. The random variable X_{ji} has $GIED(\alpha_j, \lambda_j), j = 1,2$, and $i = 1,2,\ldots,n$.
4. The life test is conducted under progressive type-I censoring which is discussed in the previous section.

3 Estimation Process

3.1 Maximum Likelihood Estimation

For a progressive type-I censored sample obtained from a life test experiment with two independent $GIED(\alpha_1, \lambda_1)$ and $GIED(\alpha_2, \lambda_2)$, or equivalently, from (2) with parameters α_1, λ_1 and α_2, λ_2, it follows from (5) and (6) that

$$\ell\left[\prod_{j=1}^{h}\prod_{i=1}^{r_j}\left\{\frac{\alpha_j\lambda_j}{x_{ji}^2}\right\}\left[1-\exp\left(\frac{-\lambda_j}{x_{ji}}\right)\right]^{\alpha_j-2}\prod_{l(\neq j)=1}^{h}\left[1-\exp\left(\frac{-\lambda_l}{x_{ji}}\right)\right]^{\alpha_l}\right\}\right]$$
$$\left[\prod_{j=1}^{h}\prod_{i'=1}^{s}\left(\left[1-\exp\left(\frac{-\lambda_j}{T_{i'}}\right)\right]^{\alpha_j}\right)^{R_{i'}}\right] \tag{7}$$

where $j = 1,2$. The log-likelihood function is

$$\ln\ell\sum_{j=1}^{h}\sum_{i=1}^{r_j} r_j\ln(\alpha_j\lambda_j) - 2\ln(x_{ji}) - \frac{\lambda_j}{x_{ji}} + (\alpha_j-2)\ln(\left[1-\exp\left(\frac{-\lambda_j}{x_{ji}}\right)\right]) + \sum_{l=1}^{h}\alpha_l\ln(\left[1-\exp\left(\frac{-\lambda_l}{x_{ji}}\right)\right])$$
$$+\sum_{j=1}^{h}\sum_{i'=1}^{s}R_{i'}\alpha_j\ln(\left[1-\exp\left(\frac{-\lambda_j}{T_{i'}}\right)\right]) \tag{8}$$

The first order derivations of (8) with respect to α_j and λ_j, are given, respectively, by $j = 1,2$

$$\frac{\partial\ln\ell}{\partial\alpha_j} = \sum_{j=1}^{h}\sum_{i=1}^{r_j}\frac{r_j}{\alpha_j} + \ln\left[1-\exp\left(\frac{-\lambda_j}{x_{ji}}\right)\right] + \sum_{l=1}^{h}\ln\left[1-\exp\left(\frac{-\lambda_l}{x_{ji}}\right)\right] + \sum_{j=1}^{h}\sum_{i'=1}^{s}R_{i'}\ln\left[1-\exp\left(\frac{-\lambda_j}{T_{i'}}\right)\right]$$

and

$$\frac{\partial\ln\ell}{\partial\lambda_j} = \sum_{j=1}^{h}\sum_{i=1}^{r_j}\frac{r_j}{\lambda_j} - \frac{1}{x_{ji}} + \frac{\alpha_j-2}{\exp\left(\frac{\lambda_j}{x_{ji}}\right)-1} + \sum_{l=1}^{h}\frac{\alpha_l}{\exp\left(\frac{\lambda_l}{x_{ji}}\right)-1} + \sum_{j=1}^{h}\sum_{i'=1}^{s}\frac{R_{i'}\alpha_j}{\exp\left(\frac{\lambda_j}{T_{i'}}\right)-1}$$

Equating the first derivations in the previous two equations to zero, one can obtain the MLE of the unknown parameters $\alpha_1, \alpha_2, \lambda_1$ and λ_2. The system of non-linear equations has no closed form solution in $\alpha_1, \alpha_2, \lambda_1$ and λ_2. Thus, a numerical method technique is required for computing the MLE of the parameters $\alpha_1, \alpha_2, \lambda_1$ and λ_2. Now, the asymptotic variance-covariance matrix of the MLEs of $\alpha_1, \alpha_2, \lambda_1$ and λ_2 can be obtained by inverting the observed information matrix with the negative elements of the expected values of the second order derivatives of logarithms of the likelihood functions. Cohen [19] concluded that the approximate variance covariance matrix may be obtained by replacing expected values with their MLEs. Now, the approximate sample information matrix will be

$$I(\hat{\theta}) = -\begin{bmatrix} \frac{\partial^2\ln(\ell)}{\partial\alpha_1^2} & & & \\ \frac{\partial^2\ln(\ell)}{\partial\alpha_2\partial\alpha_1} & \frac{\partial^2\ln(\ell)}{\partial\alpha_2^2} & & \\ \frac{\partial^2\ln(\ell)}{\partial\lambda_1\partial\alpha_1} & \frac{\partial^2\ln(\ell)}{\partial\lambda_1\partial\alpha_2} & \frac{\partial^2\ln(\ell)}{\partial\lambda_1^2} & \\ \frac{\partial^2\ln(\ell)}{\partial\lambda_2\partial\alpha_1} & \frac{\partial^2\ln(\ell)}{\partial\lambda_2\partial\alpha_2} & \frac{\partial^2\ln(\ell)}{\partial\lambda_2\partial\lambda_1} & \frac{\partial^2\ln(\ell)}{\partial\lambda_2^2} \end{bmatrix}_{(\hat{\alpha}_1,\hat{\alpha}_2,\hat{\lambda}_1,\hat{\lambda}_2)}$$

where the elements of the 4×4 matrix $I_{ij}(\theta)$, $i,j=1,2,3,4$ can be obtained as follows:

$$\frac{\partial^2 \ln\ell}{\partial \alpha_j^2} = -\sum_{j=1}^{h}\sum_{i=1}^{r_j}\frac{r_j}{\alpha_j^2}$$

$$\frac{\partial^2 \ln\ell}{\partial \lambda_j^2} = \sum_{j=1}^{h}\sum_{i=1}^{r_j}\frac{-r_j}{\lambda_j^2} - \frac{(\alpha_j-2)\exp\left(\frac{\lambda_j}{x_{ji}}\right)}{x_{ji}\left(\exp\left(\frac{\lambda_j}{x_{ji}}\right)-1\right)^2} - \sum_{l=1}^{h}\frac{\alpha_l\exp\left(\frac{\lambda_l}{x_{ji}}\right)}{x_{ji}\left(\exp\left(\frac{\lambda_l}{x_{ji}}\right)-1\right)^2} - \sum_{j=1}^{h}\sum_{i'=1}^{s}\frac{R_{i'}\alpha_j\exp\left(\frac{\lambda_j}{T_{i'}}\right)}{T_{i'}\left(\exp\left(\frac{\lambda_j}{T_{i'}}\right)-1\right)^2}$$

and

$$\frac{\partial^2 \ln\ell}{\partial \alpha_j\partial\lambda_j} = \sum_{j=1}^{h}\sum_{i=1}^{r_j}\frac{1}{\exp\left(\frac{\lambda_j}{x_{ji}}\right)-1} + \sum_{l=1}^{h}\frac{1}{\exp\left(\frac{\lambda_l}{x_{ji}}\right)-1} + \sum_{j=1}^{h}\sum_{i'=1}^{s}\frac{1}{\exp\left(\frac{\lambda_j}{T_{i'}}\right)-1}$$

3.2 Asymptotic Confidence Intervals

In this subsection, we derive the confidence intervals of the vector of the unknown parameters $\theta=(\alpha_1,\alpha_2,\lambda_1,\lambda_2)$. Based on the asymptotic distribution of the MLE of the parameters, it is known that

$$(\hat{\theta}-\theta)\rightarrow N_4(0, I^{-1}(\hat{\theta}_j))$$

where $I(\theta)$ is the Fisher information matrix. Under particular regularity conditions, the two-sided $100(1-\gamma)\%, 0<\gamma<1$, asymptotic confidence intervals for the vector of unknown parameters $\theta=(\alpha_1,\alpha_2,\lambda_1,\lambda_2)$ can be obtained as follows:

$$\hat{\theta}_j \pm Z_{\frac{\gamma}{2}}\sqrt{Var(\hat{\theta}_j)}$$

where $Var(\hat{\theta}_j)$ is the element of the main diagonal of $I^{-1}(\hat{\theta}_j)$ and $Z_{\frac{\gamma}{2}}$ is the upper $\frac{\gamma}{2}$th percentile of the standard normal distribution.

3.3 Relative Risk

Using the independence of the latent failure times $X_{1i}, X_{2i}, i=1,2,\ldots,n$, we can obtain the relative risk rate due to a particular cause (say, cause 1) as follows:

$$\begin{aligned}\pi_1 = P(X_{1i}\le X_{2i}) &= \int_0^\infty f_1(x)\cdot\bar{F}_2(x)dx \\ &= \alpha_1\lambda_1\int_0^\infty \exp\left(\frac{-\lambda_1}{x}\right)\left[1-\exp\left(\frac{-\lambda_1}{x}\right)\right]^{\alpha_1-1}\cdot\left[1-\exp\left(\frac{-\lambda_2}{x}\right)\right]^{\alpha_2}dx \qquad (9)\end{aligned}$$

and

$$\begin{aligned}\pi_2 = P(X_{2i}\le X_{1i}) &= \int_0^\infty f_2(x)\cdot\bar{F}_1(x)dx \\ &= \alpha_2\lambda_2\int_0^\infty \exp\left(\frac{-\lambda_2}{x}\right)\left[1-\exp\left(\frac{-\lambda_2}{x}\right)\right]^{\alpha_2-1}\cdot\left[1-\exp\left(\frac{-\lambda_1}{x}\right)\right]^{\alpha_1}dx \qquad (10)\end{aligned}$$

Once π_1 is computed, we define π_2 using the relation $\pi_2=1-\pi_1$. The relative risk rates of π_1 and π_2 can be obtained by replacing the MLE of α_1, α_2, λ_1 and λ_2 in (9) and (10). As the integral in the right side of (9) and (10) have no analytical solution, we have to use a numerical technique to solve these integrals.

4 Real Data Application

We analyze a real data set for an illustrative purpose as well as to assess the statistical performances of the MLE estimators under different progressive type-I censoring scheme. We used R-statistical programming language for calculation utilizing *bbmle* package to compute MLEs and relative risks.

The following original data set was first analyzed by Hoel [20] and later by Ashour and Nassar [21]. The data was obtained from a laboratory experiment in which male mice received a radiation dose of 300 roentgens at 35 days to 42 days (5-6 weeks) of age. The cause of death for each mouse was defined by reticulum cell sarcoma as cause 1 and other causes of death as cause 2.The analysis listed below exhibits $n = 77$ observations

Time of failure due to cause 1: 317, 318, 399, 495, 525, 536, 549, 552, 554, 557, 558, 571, 586, 594, 596, 605, 612, 621, 628, 631, 636, 643, 647, 648, 649, 661, 663, 666, 670, 695, 697, 700, 705, 712, 713, 738, 748, 753.

Time of failure due to cause 2: 40, 42, 51, 62, 163, 179, 206, 222, 228, 252, 249, 282, 324, 333, 341, 366, 385, 407, 420, 431, 441, 461, 462, 482, 517, 517, 524, 564, 567, 586, 619, 620, 621, 622, 647, 651, 686, 761, 763

From the original data, we generate three progressively type-I censored samples with different m stages and removed items R_i at the time censoring T_i, where $i = 1, 2, \ldots, m$. These different schemes are presented in Table (1).

Table 1: Different schemes for progressively type-I censored samples

Scheme	m	Censoring time T_i	Removed items R_i
I	3	(225, 440, 610)	(5, 5, R_m)
II	4	(225, 335, 525, 610)	(5, 5, 5, R_m)
II	5	(225, 335, 440, 525, 610)	(5, 5, 5, 5, R_m)

In Table (2), we calculate the MLEs of the parameters $\alpha_1, \alpha_2, \lambda_1$, and λ_2 and their associated 95 % asymptotic confidence interval estimates at different schemes for progressively type-I censored samples.

Table 2: Maximum likelihood and associated interval estimates for real data set

Scheme	Estimate	Parameters			
		λ_1	α_1	λ_2	α_2
I	MLE	0.112	2.330	0.109	2.278
	Asy CI	(0.021,0.179)	(1.124,3.347)	(0.062,0.205)	(1.231,4.033)
II	MLE	0.134	2.330	0.109	1.961
	Asy CI	(0.069,0.212)	(1.284,4.044)	(0.051,0.181)	(1.081,3.383)
III	MLE	0.104	1.785	0.113	2.412
	Asy CI	(0.049, 0.173)	(1.020, 2.986)	(0.047, 0.194)	(1.212, 4.518)

Note: Asy CI-asymptotic confidence interval.

In Table (2) all the estimated values of maximum likelihood and associated interval estimates for the real data sets are presented under three different progressively type-I censored samples. In Table (3), we compute the values of the negative log-likelihood criterion (NLC), Akaike's information criterion (AIC) and Bayesian information criterion (BIC) at different censoring schemes. The lower the values of these criteria, the better the fit. The relative risk at each censoring scheme will be computed.

Table (3) shows goodness of fit test for different three progressively type-I censored samples schemes. We indicate that the more number of stages for progressive type-I censoring schemes, the better fit under the assumption of the data that follows GIED in presences of competing risks.

Example: we apply a progressive type-I censoring in scheme II as follows:

$$n = 77, m = 4, R_1 = 5, R_2 = 5, R_3 = 5 \quad \text{and} \quad T_1 = 225, T_2 = 335, T_3 = 525, T_4 = 610$$

Thus, the progressively type-I censored sample from the original data by applying the above-mentioned scheme is given by:

Table 3: A goodness of fit test for different progressively type-I censoring schemes

Scheme	Removed items	Failure items	Goodness of fit			Relative risk	
			AIC	BIC	NLC	π_1	π_2
I	(5, 5,22)	$r=45$	296.902	300.885	144.45	0.6236	0.3764
II	(5, 5, 5,24)	$r=38$	296.532	300.514	144.27	0.6220	0.3780
III	(5, 5, 5, 5, 16)	$r=41$	286.002	289.985	139.001	0.5454	0.4546

(2,40), (2,42), (2,51), (2,62), (2,163), (2,179), (2,206), (2,222), (1,317), (1,318), (2,228), (2,252), (2,249), (2,282), (2,324), (2,333), (1,495), (2,341), (2,366), (2,385), (2,407), (2,420), (2,431), (2,462), (2,482), (2,517), (2,524), (1,525), (1,536), (1.549), (1,552), (1,554), (1,557), (1,571), (2,586), (1,594), (1,596), (2,567)

The first component denotes the life time and the second component indicates the cause of failure. There were $R_1 = 5, R_2 = 5, R_3 = 5, R_4 = 24$ and $r = 38$. From the above-mentioned data, we obtain the following: where the asymptotic

Estimate	Parameters			
	λ_1	α_1	λ_2	α_2
MLE	0.134	2.330	0.109	1.961
Asy CI	(0.069,0.212)	(1.284,4.044)	(0.051,0.181)	(1.081,3.383)

confidence interval is reported within brackets. Also, the relative risk due to cause 1 is 0.622, and due to cause 2 is 0.378. Finally, the value of AIC, BIC, and NLC is given by 296.532, 300.514, and 144.27, respectively.

5 Perspective

The present paper explored the problem of competing risks model for generalized inverted exponential distribution under progressive type-I censoring. We derived maximum likelihood estimates and associated asymptotic confidence interval estimates for the unknown parameters of a GIED under the assumption of two causes of failure and different progressive type-I censoring schemes. Furthermore, we calculated AIC, BIC, NLC, and relative risk for each assumed censoring scheme. We conclude that: the more number of stages for progressive type-I censoring schemes, the better fit. The present work can be extended to investigate Bayesian estimation and optimal progressive censoring sampling plan under competing risks model.

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