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Estimation of the Coefficient of Variation for Lindley Distribution based on Progressive First Failure Censored Data

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Abstract: In applied statistics, the coefficient of variation (CV) of a distribution is considered as one of the useful descriptive measures for describing variability. However, inferences concerning the coefficient of variation of non-normal distributions are rarely reported. In this paper, estimation of CV using progressive first failure censored data for the Lindley distribution (LD) is developed. A point estimation as well as interval estimation of CV aure obtained using Bayesian and non-Bayesian approaches. In Bayesian approach, we obtian the Bayes estimation with both the symmetric and asymmetric loss fnctions. Results from simulation studies assessing the performance of the maximum likelihood estimation (MLE) and Bayes estimates are included.

Keywords: cofficient of variation; Lindely distribution; progressive first failure censored scheme; Bayesian approach .

1 Introduction

The CV is an important quantity to describe the variation. It provides an alternative index besides the most commonly used measurements of variation such as variance or standard deviation, which come across with difficulty in comparing the variations from different populations with different units, such as, for example, the variability of the weights of newborns (measured in grams) with the size of adults (measured in centimeters). The CV measures the variability of a series of numbers independently of the unit of measurement used for these numbers. This approach has been used by several authors to obtain the CV estimator (for details, see [1] and [2]). The CV has long widely used as a descriptive and inferential quantity in several fields such as chemistry, engineering, medical sciences, physics, and telecommunications. In chemical experiments, it is often used as a yardstick of precision of measurements, two measurement methods may be compared on the basis of their respective CV. In physiological science, the CV can be applied to assess the homogeneity of bone samples [3]. It has been used as a tool in uncertainty analysis of fault trees [4] and in assessing the strength of ceramics [5]. Many statistical procedures concerning CV are based on the normal distribution. However, several phenomena do not agree with the normality assumption due to asymmetry or to the presence of heavy and light tails in the distribution of the data.

2 Progressive First Failure

This section discussed the process of obtaining point and interval estimations of the parameter based on progressive first-failure censored data. Let $y_i = Y^R_{i;m,n,k}$ be the observed values of the lifetime y obtained from a progressive first-failure

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censoring scheme $R = (R_1, ..., R_m)$. Then the likelihood function of the observations is:

$$L(\theta) = Ak^m \prod_{i=1}^{m} f(y_i)\,[1 - F(y_i)]^{k(R_i+1)-1}, \tag{1}$$

$$L(\theta) = Ak^m \prod_{i=1}^{m} \left[\frac{\theta^2 (1+y_i)\, e^{-\theta y_i}}{1+\theta}\right] \left[\left(1 + \frac{\theta y_i}{1+\theta}\right) e^{-\theta y_i}\right]^{k(R_i+1)-1}, \tag{2}$$

the log likelihood function may have the form:

$$\begin{aligned}\ell(\theta) = \log A + m\log k - \sum_{i=1}^{m} \theta y_i + 2\sum_{i=1}^{m} \log\theta - \sum_{i=1}^{m} \log(1+\theta) \\ + \sum_{i=1}^{m} \log(1+y_i) + \sum_{i=1}^{m} (k(R_i+1)-1)\log(1+\frac{\theta y_i}{1+\theta}) \\ - \sum_{i=1}^{m} \theta y_i (k(R_i+1)-1).\end{aligned} \tag{3}$$

Differentiating equation (3) with respect to θ and equating the equation to zero.

$$\begin{aligned}\frac{\partial \ell(\theta)}{\partial\theta} = \frac{2m}{\theta} - \sum_{i=1}^{m} y_i - \sum_{i=1}^{m} y_i (k(R_i+1)-1) \\ + \sum_{i=1}^{m} (k(R_i+1))(\frac{y_i}{((1+\theta)(1+\theta+\theta y_i))}) \\ - \sum_{i=1}^{m} (\frac{y_i}{(1+\theta)(1+\theta+\theta y_i)} - \frac{m}{1+\theta} = 0.\end{aligned} \tag{4}$$

Equation (4) can't be solved analytically, but can be solved by using Newton-Raphson method.

2.1 Approximate Confidence Interval (CI)

In this subsection, we obtained the approximate confidence interval for LD parameter. The observed Fisher's information is given by $I(\hat{\theta}) = -\frac{\partial^2 \ell(\theta)}{\partial\theta^2}$ at $\theta = \hat{\theta}$. The sampling distribution of $\frac{\hat{\theta}-\theta}{\sqrt{var(\hat{\theta})}}$ can be approximated by a standard normal distribution. When the sample size is large, the $100(1-\gamma)\%$ confidence interval bounds for θ can be computed by :

$$(\hat{\theta}_L, \hat{\theta}_U) = \hat{\theta} \pm Z_{\frac{\gamma}{2}} \sqrt{var(\hat{\theta})}.$$

In order to find an approximate estimate of the variance of CV using the Delta method, see [8], let $G = (\frac{\partial CV}{\partial\theta})$,where $\frac{\partial CV}{\partial\theta}$ is the first derivatives of the CV.

The approximate asymptotic variance of CV is given by $Var(\hat{CV}) = [GI^{-1}G^t]_{\hat{\theta}}$. The asymptotic distribution of $\frac{\hat{CV}-CV}{\sqrt{Var(\hat{CV})}}$ has $N(0,1)$. This yields that the asymptotic $100(1-\gamma)\%$ confidence interval for CV is:

$$\hat{CV} \pm Z_{\frac{\gamma}{2}} \sqrt{var(\hat{\theta})}.$$

3 Bayes Estimators under Symmetric and Asymmetric Loss Functions

This section deals with obtaining the Bayesian estimation for the LD (for more detail about LD see [6], [7]) parameter under different loss functions. Dube et al. [8] had studied the Bayesian estimation using squared error (SE) loss function for Lindely distribution by using important sampling technique and Metropolis-Hasting algorithm . In practical works the parameters cannot be treated as a constant during the life testing time. Therefore, it would be a fact to assume the parameters used in the life time model as random variables. We have also conducted a Bayesian study by assuming the following independent gamma prior for θ:

$$g(\theta)\alpha\theta^{a-1}e^{-b\theta},\theta>0, \tag{5}$$

where a and b are hyperparameters and $a,b>0$.

3.1 Symmetric Loss Function

In this subsection, we made Bayesian estimation using SE loss function. The likelihood function has the form :

$$L(\theta)=Ak^{m}\prod_{i=1}^{m}\left[\frac{\theta^{2}\left(1+y_{i}\right)e^{-\theta y_{i}}}{1+\theta}\right]\left[\left(1+\frac{\theta y_{i}}{1+\theta}\right)e^{-\theta y_{i}}\right]^{k(R_{i}+1)-1}. \tag{6}$$

Thus, the posterior density function of θ, given the data, is given by

$$\pi\left(\theta\mid x\right)=\frac{L(\theta)g(\theta\mid a,b)}{\int_{0}^{\infty}L(\theta)g(\theta\mid a,b)d\theta}. \tag{7}$$

Therefore, the Bayes estimate of any function of θ say $h(\theta)$ under squared error loss function is

$$\hat{\theta}_{SE}=E_{(\theta|data)}\left[h(\theta)\right]=\frac{\int_{0}^{\infty}h(\theta)L(\theta)g(\theta\mid a,b)d\theta}{\int_{0}^{\infty}L(\theta)g(\theta\mid a,b)d\theta}. \tag{8}$$

The posterior density function is:

$$\pi\left(\theta\mid x\right)\propto L(\theta)g(\theta\mid a,b). \tag{9}$$

$$\pi(\theta\mid y)\propto\theta^{2m+a-1}\frac{Ak^{m}}{(1+\theta)^{m}}\prod_{i=1}^{m}(1+y_{i})(1+\frac{\theta y_{i}}{(1+\theta)})^{k(R_{i}+1)-1}\times[\exp(-\theta(b+k\sum_{i=1}^{m}y_{i}(R_{i}+1)))]. \tag{10}$$

It is not possible to compute equation (8) analytically. The posterior density function cannot be reduced analytically to well known distributions. But its plot shows that it is similar to normal distribution. So, to calculate the integral that we cannot calculate it exact, we use the Metropolis-Hasting (MH) algorithm with normal proposal distribution.

3.2 Asymmetric Loss Function

Asymmetric loss function may be more appropriate in some fields. Recently, many authors considered asymmetric loss functions in reliability and life testing. One of the most popular asymmetric loss functions is linear-exponential (LINEX) loss function which was introduced by [9]. It used in several papers, for example, [10], [11], [12] and [13]. This function is approximately linearly on one side and rises approximately to zero on the other side. Under the assumption that the minimal loss occurs at $\hat{\theta}=\theta$, the LINEX loss function can be expressed as:

$$L_{1}(\delta)\propto\exp(c\delta)-c\delta-1, \tag{11}$$

where $\delta=\hat{\theta}-\theta,\hat{\theta}$ is the estimate of $\theta,c\neq 0$.
The magnitude of c represent the direction, and degree of symmetry. Where $c>0$ means overestimation is more serious than underestimation, and $c<0$ means the opposite. For c close to zero the LINEX loss function is approximately the (SE) loss function. The posterior expectation of the LINEX loss function of is :

$$E_{\theta}(L1(\hat{\theta}-\theta))\propto((e^{c\theta})E_{\theta}[e^{-c\theta}])-c((\hat{\theta}-E_{\theta}\theta)-1. \tag{12}$$

The Bayes estimator under the LINEX loss function is the value of

$$\hat{\theta}_{LINEX} = \frac{-1}{c}\log(E_{\theta}[\exp(-c\theta)]), \tag{13}$$

such that $E_{\theta}[\exp(-c\theta)]$ exists.

Another asymmetric loss function called a general entropy (GE) loss function was proposed by [14] which can be expressed as:

$$L_2(\hat{\theta},\theta) \propto [\frac{\hat{\theta}}{\theta}]^q - q\log\frac{\hat{\theta}}{\theta} - 1. \tag{14}$$

The weighted SE loss function results from $q = -1$. The Bayes estimate $\hat{\theta}_{GE}$ under GE loss function is

$$\hat{\theta}_{GE} = (E_{\theta}[\theta^{-q}])^{\frac{-1}{q}}, \tag{15}$$

such that $E_{\theta}[\theta^{-q}]$ exists.

Since it is not possible to compute equation (13) and (15) analytically. we used the Markov chain Mont-Carlo (MCMC) method such as Metropolis-Hastings algorithm. to draw samples from the posterior density function and then to compute the Bayes estimate.

3.3 Metropolis-Hasting Algorithm

The MH algorithm was originally introduced by [15]. Suppose that our goal was to draw samples from some distributions $f(x|\theta) = \nu g(\theta)$, where ν is the normalizing constant which may not be known or very difficult to compute. The MH algorithm provided a way of sampling from $f(x|\theta)$ without requiring us to know ν. Let $q\left(\theta^{(b)}|\theta^{(a)}\right)$ be an arbitrary transition kernel: that is the probability of moving, or jumping, from current state $\theta^{(a)}$ to $\theta^{(b)}$. This is sometimes called the proposal distribution. The following algorithm generated a sequence of values $\theta^{(1)}, \theta^{(2)}, ..., \theta^{(n)}$ which form a Markov chain with stationary distribution given by $f(x|\theta)$.

Algorithm

1.Start with $\theta^{(0)} = \theta_{MLE}$.
2.Set i=1.
3.Generate $\theta^{(*)}$ from the proposal distribution $N(\theta^{(i-1)}, var\theta^{(i-1)})$.
4.Calculate the acceptance probability $r(\theta^{(i-1)}, \theta^{(*)}) = \min\left[1, \frac{\pi(\theta^{(*)})}{\pi(\theta^{(i-1)})}\right]$.
5.Generate U from uniform on (0, 1).
6.If $U < r(\theta^{(i-1)}, \theta^{(*)})$ accept the proposal distribution and set $\theta^{(i)} = \theta^{(*)}$. Otherwise, reject the proposal distribution and set $\theta^{(i)} = \theta^{(i-1)}$.
7.Set $i = i+1$.
8.Repeat Steps $3-7$ N times.
9.Obtain the BEs of θ using MCMC under SEL function as $\hat{CV}_{SE} = \sum_{i=M+1}^{N}\frac{1}{N-M}CV^{(i)}$.
10.Obtain the BEs of θ using MCMC under LINEX function as $\hat{CV}_{LINEX} = \frac{-1}{c}\log\frac{\sum_{i=M+1}^{N}\exp(-cCV^{(i)})}{N-M}$.
11.Obtain the BEs of θ using MCMC under GE function $\hat{CV}_{GE} = [\frac{\sum_{i=M+1}^{N}(CV^{(i)})^{-q}}{N-M}]^{\frac{-1}{q}}$, where M is nburn units and N is the number of MCMC iterations.
12.Arrange the values of $\theta^{(*)}$ in ascending order.
13.Find the position of the lower bound and upper bound θ
14.Repeat the above steps N times and every time find the average value of $\theta^{(*)}_{low}$ and $\theta^{(*)}_{upp}$.

4 Simulation Study

This section deals with obtaining some numerical results. MLE and Bayes estimates using LINEX, SE and GE loss functions with their mean square errors (MSE) , coverage probability (COV) , 95 % CIs and HPD Interval with their widths for the parameter θ when $N = 10000$, $M = 1000, \theta = 1, a = 1, b = 1, A = 2, k = 2,4,6,8,12, c = 1, q = 1$. We generate a progressively first-failure censored samples from the continuous random variable using the algorithm described in [17]. Tables (1-5) contains some results concluded from the simulation study.

Table 1: Estimators and MSE, CI and HPD

C.S	[1]	[2]	[3]
n	50	50	70
m	20	20	30
k	2	2	2
$\hat{CV}_{MIE}$	0.937	0.9363	0.9344
$\hat{CV}_{SE}$	0.9343	0.9330	0.9313
$\hat{CV}_{LINEX}$	0.9344	0.93309	0.9314
$\hat{CV}_{GE}$	0.934	0.933	0.931
MSE_{MIE}	0.00014	0.000104	0.00012
MSE_{SE}	0.000108	0.000082	0.00012
MSE_{LINEX}	0.000109	0.000089	0.00011
MSE_{GE}	0.00011	0.000090	0.00012
$95\%CI$	{0.91569, 0.95997}	{0.914, 0.959}	{0.912, 0.957}
Length	0.04428	0.045	0.045
$95\%CIforHPD$	{0.910367, 0.953985}	{0.9087, 0.9529}	{0.907, 0.9514}
Length	0.043618	0.0442	0.0444
COV_{MLE}	0.92	0.95	0.93
COV_{MCMC}	0.95	0.99	0.98

Table 2: Estimators and MSE, CI and HPD

C.S	[1]	[2]	[3]
n	80	80	80
m	40	40	40
k	2	2	2
$\hat{CV}_{MIE}$	0.9341	0.9339	0.9337
$\hat{CV}_{SE}$	0.93408	0.93391	0.9354
$\hat{CV}_{LINEX}$	0.934	0.934	0.93376
$\hat{CV}_{GE}$	0.934	0.933	0.934
MSE_{MIE}	0.000052	0.000058	0.00006
MSE_{SE}	0.00005	0.000054	0.0000564
MSE_{LINEX}	0.000048	0.000054	0.000056
MSE_{GE}	0.000048	0.000054	0.0000567
$95\%CI$	{0.9197, 0.9518}	{0.9195, 0.9517}	{0.9193, 0.9515}
Length	0.0321	0.0322	0.0322
$95\%CIforHPD$	{0.917, 0.9488}	{0.9168, 0.9487}	{0.9165, 0.9486}
Length	0.0318	0.0319	0.0321
COV_{MLE}	0.98	0.95	0.96
COV_{MCMC}	0.98	0.97	0.97

Under different combinations of n, m, we used three different censoring scheme (C.S), as:
scheme I: $R_1 = n - m, \quad R_i = 0$ for $i \neq 1$.
scheme II: $R_{\frac{m+1}{2}} = n - m, \ R_i = 0$ for $i \neq \frac{m+1}{2}$; if m odd, and
$R_{\frac{m}{2}} = n - m, \ R_i = 0$ for $i \neq \frac{m}{2}$; if m even.
scheme III: $R_m = n - m, \quad R_i = 0$ for $i \neq m$.

Table 3: Estimators and MSE, CI and HPD

C.S	[1]	[2]	[3]
n	130	200	250
m	60	120	140
k	2	2	2
$\hat{C}V_{MIE}$	0.9363	0.9351	0.9234
$\hat{C}V_{SE}$	0.93408	0.991	0.954
$\hat{C}V_{LINEX}$	0.934	0.921	0.95676
$\hat{C}V_{GE}$	0.934	0.9345	0.9674
MSE_{MIE}	0.000042	0.000022	0.000023
MSE_{SE}	0.0000386	0.000021848	0.00002241
MSE_{LINEX}	0.000039	0.000021822	.00002239
MSE_{GE}	0.00003873	0.0000219	0.00002246
$95\%CI$	{0.9234, 0.9492}	{0.9258, 0.944}	{0.9266, 0.9437}
Length	0.0258	0.0185	0.0171
$95\%CI forHPD$	{0.9215, 0.9472}	{0.9249, 0.9433}	{0.9258, 0.9429}
Length	0.0257	0.0184	0.0171
COV_{MLE}	0.935	0.9667	0.94
COV_{MCMC}	0.965	0.96	0.94

Table 4: Estimators and MSE, CI and HPD

C.S	[1]	[2]	[3]
n	50	50	50
m	20	20	20
k	4	4	4
$\hat{C}V_{MIE}$	0.9367	0.9241	0.9354
$\hat{C}V_{SE}$	0.938	0.991	0.922
$\hat{C}V_{LINEX}$	0.9356	0.931	0.95346
$\hat{C}V_{GE}$	0.934	0.935	0.9334
MSE_{MIE}	0.000143	0.00011	0.000145
MSE_{SE}	0.00011881	0.0001008	0.0001248
MSE_{LINEX}	0.0001184	0.0001003	0.0001243
MSE_{GE}	0.000119	0.0001020	0.0001259
$95\%CI$	{0.9145, 0.9588}	{0.913, 0.9573}	{0.9139, 0.9579}
Length	0.0443	0.0443	0.044
$95\%CI forHPD$	{0.9094, 0.953}	{0.9082, 0.9517}	{0.9088, 0.9523}
Length	0.0436	0.0435	0.0435
COV_{MLE}	0.8933	0.9667	0.92
COV_{MCMC}	0.9533	0.9733	0.9667

Table 5: Estimators and MSE, CI and HPD

C.S	[1]	[2]	[3]
n	80	80	50
m	40	40	20
k	6	8	12
$\hat{CV}_{MlE}$	0.9457	0.9367	0.9233
$\hat{CV}_{SE}$	0.9345	0.9351	0.922
$\hat{CV}_{LINEX}$	0.9344	0.9351	0.94346
$\hat{CV}_{GE}$	0.956	0.935	0.9334
MSE_{MlE}	0.000055	0.0000612	0.0000640
MSE_{SE}	0.0000509	0.0000532	0.0000543
MSE_{LINEX}	0.0000508	0.0000531	0.0000542
MSE_{GE}	0.0000511	0.0000533	0.0000544
$95\%CI$	$\{0.9212, 0.9522\}$	$\{0.913, 0.9573\}$	$\{0.9216, 0.9524\}$
Length	0.0313	0.031	0.0308
$95\%CI forHPD$	$\{0.9177, 0.9487\}$	$\{0.9185, 0.9494\}$	$\{0.9191, 0.9496\}$
Length	0.031	0.0309	0.0305
COV_{MLE}	0.9467	0.9467	0.925
COV_{MCMC}	0.98	0.9733	0.955

5 Conclusion

Point and interval estimation using symmetric and asymmetric Bayesian estimation by two methods for LD parameter based on progressive first failure samples are derived and computed. Asymmetric Bayesian estimation is always better than symmetric Bayesian estimation, the MSE of $\hat{CV}_{LINEX}$ is always smaller than MSE of $\hat{CV}_{SE}$, the HPD interval length and the approximate CI length of the parameter decreases as n,m increase, olso as the diffrence between n,m deacreses the MSE error of the paramter deacreses, all the results concludede are:

1. For all censoring schemes and all values of k as n,m increase the MSE of $\hat{CV}_{MLE}$, $\hat{CV}_{SE}$, $\hat{CV}_{LINEX}$, $\hat{CV}_{GE}$ decrease.
2. For all censoring schemes and all values of k as n,m increase the HPD interval length and the CI length of the parameter decreases.
3. The MSE of $\hat{CV}_{SE}$, $\hat{CV}_{LINEX}$, $\hat{CV}_{GE}$ (Bayesian estimators) is always smaller than MSE of $\hat{CV}_{MLE}$.
4. The MSE of $\hat{CV}_{SE}$, $\hat{CV}_{LINEX}$ is always smaller than MSE of $\hat{CV}_{GE}$.
5. The MSE of $\hat{CV}_{LINEX}$ is always smaller than MSE of $\hat{CV}_{SE}$.

References

[1] W. K. Pang, W. T. Y. Bosco, M. D. Troutt and H. H. Shui, *A simulation-based approach to the study of coefficient of variation of dividend yields,* European Journal of Operational Research, 189, (2008), 559-569.
[2] W. K. Pang, P. K. Leung, W. K. Huang and L. Wei, *On interval estimation of the coefficient of variation for the three-parameter Weibull, lognormal and gamma distribution: A simulation based approach,* European Journal of Operational Research, 164, (2005), 367-377.
[3] A. J. Hamer, J. R. Strachan, M. M. Black, C. Ibbotson and R. A. Elson, *A new method of comparative bone strength measurement,* Journal of Medical Engineering and Technology, 19 (1), (1995), 1-5.
[4] K. Ahn, *Use of coefficient of variation for uncertainty analysis in fault tree analysis,* Reliability Engineering and System Safety, 47(3), (1995), 229–230.
[5] J. Gong and Y. Li, *Relationship between the estimated Weibull modulus and the coefficient of variation of the measured strength for ceramics,* Journal of the American Ceramics Society, 82 (2), (1999), 449-452.
[6] D. V. Lindley, *Fudicial distributions and Bayes's theorem*, Journal of the Royal Statistical Society B, 20, (1965), 102-107.
[7] B. Atieh, M. E. Ghitany and S. Nadarajah, *Lindley distribution and its application, Mathematics and Computers in Simulation*, 78(4), (2008), 493-506.
[8] M. Dube, R. Garg , H. Krishna, *On progressively first failure censored Lindley distribution*, Comput Stat, 31, (2016), 139-163.
[9] H. R. Varian, *A Bayesian Approach to Real Estate Assessment*, North Holland, Ams terdam, (1975), 195-208.
[10] U. Balaso oriya and N. Balakrishnan, *Reliability sampling plans for log-normal distribution based on progressively censored samples*, IEEE Transactions on Reliability, 49, (2000), 199-203.
[11] A. A. Soliman, *Estimation of parameters of life from progressively censored data using Burr-XII model*, IEEE Transactions on Reliability, 54 (1) (2005), 34-42.

[12] A. A. Soliman, *Estimation for Pareto model using general progressive censored data and asymmetric loss*, Communications in Statistics-Theory and Methods, 37, (2008), 1353-1370.
[13] G. Prakash and D. C. Singh, *Shrinkage estimation in exponential Type-II censored data under LINEX loss*, Journal of the Korean Statistical Society, 37, (2008), 53-61.
[14] R. Calabria and G. Pulcini, *Point estimation under asymmetric loss functions for left truncated exponential samples*, Communications in Statistics Theory and Methods, 25, (1996), 585-600.
[15] N. Metropolis, A. W. Rosenbluth , M. N. Rosenbluth, A. H. Teller and E. Teller, *Equations of state calculations by fast computing machines*, Journal Chemical Physics, 21, (1953), 1087-1091.
[16] M. H. Chen, Q.M. Shao, *Monte Carlo Estimation of Bayesian credible and HPD intervals*, J Comput Graph Stat, 8, (1999), 69-92.
[17] N. Balakrishnan and R. A. Sandhu, *A simple Simulation Algorithm for Generating Progressively Type-II Censored Samples*, The American Statistician, 49, (1995), 229-230.