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Soft Generalized Closed Sets with Respect to an Ideal in Soft Topological Spaces

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Abstract: A soft ideal on a non empty set X is a non empty collection of soft subsets of X with heredity property which is also closed under finite unions.
The concept of soft generalized closed sets in soft topological spaces was introduced by Kannan [1]. In this paper, we introduce and study the concept of soft generalized closed sets with respect to a soft ideal, which is the extension of the concept of soft generalized closed sets.

Keywords: Soft sets, Soft topological space, Soft Ig-closed sets, soft Ig-open sets and soft mappings.

1 Introduction

In many complicated problems arising in the fields of engineering, social science, economics, medical science, etc involving uncertainties, classical methods are found to be inadequate in recent times. Molodtsov [2] pointed out that the important existing theories viz. Probability Theory, Fuzzy set Theory, Intuitionistic Fuzzy Set Theory, Rough Set Theory etc. which can be considered as a mathematical tools for dealing with uncertainties, have their own difficulties. He further pointed out that the reason for these difficulties is, possibly, the inadequacy of the parametrization tool of the theory. In 1999 he initiated the novel concept of Soft set as a new mathematical tool for dealing with uncertainties. Soft set theory, initiated by Molodtsov [2], is free of the difficulties present in these theories. Soft systems provide a very general framework with the involvement of parameters. Therefore, researches work on soft set theory and its applications in various fields are progressing rabidly. Later Maji et al [3] presented some new definitions on soft sets such as subset, the complement of a soft set and discussed in detail the application of soft set theory in decision making problems [4]. Chen et al [5,6] and Kong et al. [6] introduced a new definition of soft parametrization reduction. Xiao et al. [8] and Pei and Miao [9] discussed the relationship between soft sets and information systems. Also an attempt was made by Kostek [10] to assess sound quality based on a soft set approach. Mushrif et al. [11] presented a novel method for the classification of natural textures using the notions of soft set theory.
The topological structures of set theories dealing with uncertainties were first studied by Chang [12]. Chang introduced the notion of fuzzy topology and also studied some of its properties. Lashin et al. [13] generalized rough set theory in the framework of topological spaces. Recently, Shabir and Naz [14] introduced the notion of soft topological spaces which are defined over an initial universe with a fixed set of parameters. They also studied some of basic concepts of soft topological spaces. Later, Aygüoğlu et al. [15], Zorlutuna et al. [16] and Hussain et al. [17] continued to study the properties of soft topological spaces. They got many important results in soft topological spaces.
The study and research about near open and near closed sets have specific importance, it helps in the modifications of topological spaces via adding new concepts and facts or constructing new classes. Closed sets are fundamental objects in a topological spaces. For example, one can define the topology on a set by using either the axioms for the closed sets or the Kuratowski closure axioms. In 1970, Levine [18] introduced the notion of g-closed sets in topological spaces as a generalization of closed sets.
Indeed ideals are very important tools in general topology. It was the works of Newcomb [19], Rancin [20], Samuels [21] and Hamlet Jankovic [22,23] which

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motivated the research in applying topological ideals to generalize the most basic properties in general Topology. Jafari and Rajesh [24] introduced the concept of g-closed sets with respect to an ideal which is a extension of the concept of g-closed sets. Recently. Kannan [1] introduced the concept of g-closed sets in soft topological spaces.
In this paper, we introduce and study the concept of soft g-closed sets with respect to an ideal, which is the extension of the concept of soft g-closed set. We also study the relationship between generalized soft Ig-closed sets, soft Ig-open sets, soft g-closed sets and soft g-open sets. This research not only can form the theoretical basis for further applications of topology on soft set but also lead to the development of information systems.

2 Preliminaries

In this section, we present the basic definitions and results of soft set theory which may found in earlier studies [1,2,3,14,15,16,17]. Throughout this paper, X refers to an initial universe, E is a set of parameters, $\wp(X)$ is the power set of X, and $A \subseteq E$.

Definition 2.1 A soft set F_A over the universe X is defined by the set of ordered pairs

$$F_A = \{(e, F_A(e)) : e \in E, F_A(e) \in \wp(X)\}$$

where $F_A : E \to \wp(X)$, such that $F_A(e) \neq \phi$, if $e \in A \subseteq E$ and $F_A(e) = \phi$ if $e \notin A$.
From now on, we will use $S(X, E)$ instead of all soft sets over X.

Definition 2.2 The soft set $F_\phi \in S(X, E)$ is called null soft set, denoted by Φ, Here $F_\phi(e) = \phi, \forall\, e \in E$.

Definition 2.3 Let $F_A \in S(X, E)$. If $F_A(e) = X, \forall\, e \in A$, then F_A is called A-absolute soft set, denoted by $\widetilde{A}$.
If $A = E$, then the A-absolute soft set is called absolute soft set denoted by $\widetilde{E}_X$.

Definition 2.4 Let $F_A, G_B \in S(X, E)$. F_A is a soft subset of G_B, denoted $F_A \sqsubseteq G_B$ if $F_A(e) \subseteq G_B(e), \forall\, e \in E$.

Definition 2.5 Let $F_A, G_B \in S(X, E)$. Union of F_A and G_B, is a soft set H_C defined by $H_C(e) = F_A(e) \bigcup G_B(e), \forall\, e \in E$, where $C = A \bigcup B$. That is,

$$H_C = F_A \sqcup G_B$$

Definition 2.6 Let $F_A, G_B \in S(X, E)$. Intersection of F_A and G_B, is a soft set H_C defined by $H_C(e) = F_A(e) \bigcap G_B(e), \forall\, e \in E$ where $C = A \bigcap B$. That is

$$H_C = F_A \sqcap G_B.$$

Definition 2.7 Let $F_A \in S(X, E)$. The complement of F_A, denoted by F_A^c is defined by $F_A^c(e) = X - F(e), \forall\, e \in E$.

Theorem 2.8 Let J be an index set and $F_A, G_B, H_C, (F_A)_i, (G_B)_i \in S(X, E)\ \forall\, j \in J$. Then
(1) $F_A \sqcap F_A = F_A, F_A \sqcup F_A = F_A$.
(2) $F_A \sqcap G_B = G_B \sqcap F_A, F_A \sqcup G_B = G_B \sqcup F_A$.
(3) $F_A \sqcup (G_B \sqcup H_C) = (F_A \sqcup G_B) \sqcup H_C$,
$F_A \sqcap (G_B \sqcap H_C) = (F_A \sqcap G_B) \sqcap H_C$.
(4) $F_A = F_A \sqcup (F_A \sqcap G_B), F_A = F_A \sqcap (F_A \sqcup G_B)$.
(5) $F_A \sqcap (\coprod_{i\in J} (G_B)_i) = \coprod_{i\in J} (F_A \sqcap (G_B)_i)$.
(6) $F_A \sqcup (\prod_{i\in J} (G_B)_i) = \prod_{i\in J} (F_A \sqcup (G_B)_i)$.
(7) $\Phi \sqsubseteq F_A \sqsubseteq \widetilde{A} \sqsubseteq \widetilde{E}_X$.
(7) $\Phi \sqsubseteq F_A \sqsubseteq \widetilde{A} \sqsubseteq \widetilde{E}_X$.
(8) $(F_A^c)^c = F_A$.
(9) $(\prod_{i\in J} (F_A)_i)^c = \coprod_{i\in J} (F_A)_i^c$
(10) $(\coprod_{i\in J} (F_A)_i)^c = \prod_{i\in J} (F_A)_i^c$
(11) If $F_A \sqsubseteq G_B$, then $G_B^c \sqsubseteq F_A^c$.
(12) $F_A \sqcup F_A^c = \widetilde{E}_X, F_A \sqcap F_A^c = \Phi$.

Definition 2.9 Let $F_A \in S(X, E)$ and $x \in X$. $x \tilde{\in} F_A$ read as x belongs to the soft set F_A whenever $x \in F_A(e), \forall\, e \in A$.
For any $x \in X, x \tilde{\notin} F_A$ if $x \notin F_A(e)$ for some $e \in A$.

Definition 2.10 Let $x \in X$, then x_E denotes the soft set over V for which $x_E(e) = \{x\}$ for all $e \in E$.

Definition 2.11 Let $S(X, E)$ and $S(Y, K)$ be the families of all soft sets over (X, E) and (Y, K), respectively. The soft mapping (φ, ψ) from (X, E) to (Y, K) is an ordered pair of mappings $\varphi : X \to Y$ and $\psi : E \to K$
(i) The image of a soft set F_A over (X, E) under the soft mapping (φ, ψ), denoted by $(\varphi, \psi)(F_A)$ is the soft set over (Y, K) defined by

$$((\varphi,\psi)(F_A))(k) = \begin{cases} \bigcup\limits_{e\in\psi^{-1}(k)\cap A} \varphi(F_A(e)) & \text{if } \psi^{-1}(k) \cap A \neq \emptyset, \\ \emptyset & \text{otherwise} \end{cases}$$

(ii) The inverse image of a soft set G_B over (Y, K) under the soft mapping (φ, ψ), denoted by $(\varphi, \psi)^{-1}(G_B)$, is the soft set over (X, E) defined by

$$((\varphi,\psi)^{-1}(G_B))(e) = \begin{cases} \varphi^{-1}(G_B(\psi(e))) & \text{if } e \in \psi^{-1}(B), \\ \emptyset & \text{otherwise} \end{cases}$$

Definition 2.12 A soft topology τ is a family of soft sets over X satisfying the following properties.
(1) $\Phi, \tilde{E}_X \in \tau$
(2) If $F_A, G_B \in \tau$, then $F_A \sqcap G_B \in \tau$
(3) If $(F_A)_i \in \tau \forall\, i \in I$, then $\coprod_{i\in I} (F_A)_i \in \tau$.
(X, τ, E) is called a soft topological space. Every member of τ is called soft open. A soft set G_B is called

soft closed in (X,τ) if $G_B^c \in \tau$.
Indiscrete soft topology, denoted by τ^0 contains only Φ and $\tilde{E}_X$ which the discrete soft topology, denoted by τ^1 contains all soft sets over X.

Definition 2.13 Let (X,τ,E) be a soft topological space and $F_A \in S(X,E)$. The soft interior of F_A is the soft set $int_\tau(F_A) = (F_A)^0 = \sqcup\{G_B : G_B$ is soft open set and $G_B \sqsubseteq F_A\}$.

Proposition 2.14 Let (X,τ,E) be a soft topological space and $F_A \in S(X,E).F_A$ is soft open iff $F_A = (F_A)^0$.

Definition 2.15 Let (X,τ,E) be a soft topological space and $F_A \in S(X,E)$. The soft closure of F_A is the soft set $cl_\tau(F_A) = \overline{(F_A)} = \sqcap\{G_B : G_B$ is soft closed set and $F_A \sqsubseteq G_B\}$.

Proposition 2.16 Let (X,τ,E) be a soft topological space and $F_A \in S(X,E)$. $F_A \in S(X,E)$ is soft closed iff $F_A = \overline{(F_A)}$.

Definition 2.17 Let (X,τ_1,E) and (Y,τ_2,K) be two soft topological spaces. A soft mapping $(\varphi,\psi):(X,\tau_1,E) \to (Y,\tau_2,K)$ is called
(1) Soft continuous if $(\varphi,\psi)^{-1}(G_B) \in \tau_1 \,\forall\, G_B \in \tau_2$.
(2) Soft open if $(\varphi,\psi)(F_A) \in \tau_2 \,\forall\, F_A \in \tau_1$.
(3) Soft closed if $(\varphi,\psi)(F_A)$ is soft closed in (Y,τ_2,K) whenever F_A is soft closed in (X,τ_1,E).

Definition 2.18 Let (X,τ,E) be a soft topological space and $M \subseteq X$. The set

$$\tau_M = \{\tilde{E}_M \cap F_A : F_A \in \tau\}$$

is called a soft relative topology on M and (M,τ_M,E) is called the soft sub-space of (X,τ,E).

In order to efficiently discuss, we consider only soft sets F_E over a universe X in which all the parameter set E are the same. We denote the family of these sets by $SS(X,E)$.

Definition 2.19 A soft set $F_E \in SS(X,E)$ is called soft generalized closed in a soft topological space (X,τ,E) if $\overline{F}_E \sqsubseteq G_E$ whenever $F_E \sqsubseteq G_E$ and $G_E \in \tau$.

3 Soft generalized closed sets with respect to soft ideal

Definition 3.1 A non empty collections I of soft subsets over X is called a soft ideal on X if the following holds
(1) If $F_A \in I$ and $G_B \sqsubseteq F_A$ implies $G_B \in I$ (heredity)
(2) If F_A and $G_B \in I$, then $F_A \sqcup G_B \in I$ (additivity).

Definition 3.2 A soft set $F_E \in SS(X,E)$ is called soft generalized closed with respect to in ideal I (soft Ig-closed) in a soft topological space (X,τ,E) if $\overline{F}_E \setminus G_E \in I$ whenever $F_E \sqsubseteq G_E$ and $G_E \in \tau$.

Example 3.3 Let $X = \{a,b,c\}$ be the set of three cars under consideration and, $E = \{e_1(costly), e_2(Luxurious)\}$. Let A_E, B_E, C_E be three soft sets representing the attractiveness of the car which Mr. X, Mr. Y and M. Z are going to buy, where
$A(e_1) = \{b\}, A(e_2) = \{a\}$
$B(e_1) = \{b,c\}, B(e_2) = \{a,b\}$
$C(e_1) = \{a,b\}, C(e_2) = \{a,c\}$.
Then A_E, B_E and C_E are soft sets over X and

$$\tau = \{\Phi, \tilde{E}_X, A_E, B_E, C_E\}$$

is the soft topology over X. Let

$$I = \{\Phi, \tilde{E}_X, F_E, G_E, D_E\}$$

be a soft ideal on X, where
$F(e_1) = \{a\} \quad F(e_2) = \phi$
$G(e_1) = \phi \quad G(e_2) = \{c\}$
$D(e_1) = \{a\} \quad D(e_2) = \{c\}$.
So

$$\tau^c = \{\Phi, \tilde{E}, A_E^c, B_E^c, C_E^c\},$$

where
$A^c(e_1) = \{a,c\} \quad A^c(e_2) = \{b,c\}$

$B^c(e_1) = \{a\} \quad B^c(e_2) = \{c\}$

$C^c(e_1) = \{c\} \quad C^c(e_2) = \{b\}$
Clearly B_E is soft Ig-closed. In fact, $B_E \sqsubseteq B_E$ and $B_E \in \tau$. But $\overline{B}_E = \tilde{E}_X$ and $\overline{B}_E \setminus B_E = \tilde{E}_X \setminus B_E = D_E \in I$.

Proposition 3.4 Every soft g-closed set is soft Ig-closed.
Proof: Let F_E be a soft g-closed set in a soft topological space (x,τ,E). We show that F_E is soft Ig-closed. Let $F_E \sqsubseteq G_E$ and $G_E \in \tau$. Since F_E is soft g-closed, then $\overline{F}_E \sqsubseteq G_E$ and hence $\overline{F}_E \setminus G_E = \Phi \in I$. Consequently F_E is soft Ig-closed.

The converse of the above proposition is not in general true. The following example supports our claim.
Example 3.5 Suppose that there are three dresses in the universe X given by $X = \{a,b,c\}$. Let $E = \{e_1(cotton), e_2(woollen)\}$ be the set of parameters showing the material of the dresses.
Let A_E, B_E, C_E be three soft sets over the common universe X, which describe the composition of the dresses, where
$A(e_1) = \{a\}, A(e_2) = X$

$B(e_1) = \{a, b\}$, $B(e_2) = X$
Then $\tau = \{\Phi, \widetilde{E}_X, A_E, B_E\}$ and $\tau^c = \{\Phi, \widetilde{E}_X, A^c_E, B^c_E\}$ where
$A^c(e_1) = \{b, c\}$, $A^c(e_2) = \phi$ and
$B^c(e_1) = \{c\}$, $B^c(e_2) = \phi$. Let
$I = \{\Phi, C_E, D_E, H_E\}$ where
$C(e_1) = \{b\}$, $C(e_2) = \phi$.
$D(e_1) = \{c\}$, $D(e_2) = \phi$.
$H(e_1) = \{b, c\}$, $H(e_2) = \phi$.
We show that A_E is soft Ig-closed but it is not soft g-closed.
For $A_E \sqsubseteq A_E \in \tau$. Then $\overline{A_E} \setminus A_E = \widetilde{E}_X \setminus A_E = H_E \in I$. Consequently A_E is soft Ig-closed. On the other hand A_E is not g-closed since

$$\overline{A_E} = \widetilde{E}_X \not\sqsubseteq A_E$$

Theorem 3.6 A soft set A_E is soft Ig-closed in a soft topological space (X, τ, E) iff $F_E \sqsubseteq \overline{A_E} \setminus A_E$ and F_E is soft closed implies $F_E \in I$

Proof:$(\Rightarrow)$ Assume that A_E is soft Ig-closed. Let $F_E \sqsubseteq \overline{A_E} \setminus A_E$. Suppose that F_E is soft closed. Then $A_E \sqsubseteq F^c_E$. By our assumption, $\overline{A_E} \setminus F^c_E \in I$. But $F_E \sqsubseteq \overline{A_E} \setminus F^c_E$, then $F_E \in I$ (from the properties of soft ideal).
$(\Leftarrow)$ Conversely, assume that $F_E \sqsubseteq \overline{A_E} \setminus A_E$ and F_E is soft closed implies $F_E \in I$. Suppose that $A_E \sqsubseteq G_E$ and $G_E \in \tau$. Then $\overline{A_E} \setminus \overline{G_E} = \overline{A_E} \sqcap \overline{G^c_E}$ is a soft closed set in (X, τ, E) and $\overline{A_E} \setminus \overline{G_E} \sqsupseteq \overline{A_E} \setminus G_E$. By assumption $\overline{A_E} \setminus G_E \in I$. This implies that A_E is soft Ig-closed.

Theorem 3.7 If F_E and G_E are soft Ig-closed sets in a soft topological space (X, τ, E), then there union $F_E \sqcup G_E$ is also soft Ig-closed in (X, τ, E).

Proof: Suppose that F_E and G_E are soft Ig-closed in (X, τ, E). If $F_E \sqcup F_E \sqsubseteq H_E$ and $H_E \in \tau$, then $F_E \sqsubseteq H_E$ and $G_E \sqsubseteq H_E$. By assumption $\overline{F_E} \setminus H_E \in I$ and $\overline{G_E} \setminus H_E \in I$ and hence $\overline{(F_E \sqcup G_E} \setminus H_E) = (\overline{F_E} \setminus H_E) \sqcup (\overline{G_E} \setminus H_E) \in I$. That is $F_E \sqcup G_B$ is soft Ig-closed.

Theorem 3.8 If F_E is soft Ig-closed in a soft topological space (X, τ, E) and $F_E \sqsubseteq G_E \sqsubseteq \overline{F_E}$, then G_E is soft Ig-closed in (X, τ, E).

Proof: If F_E is soft Ig-closed and $F_E \sqsubseteq G_E \sqsubseteq \overline{F_E}$ in (X, τ, E). Suppose that $G_E \sqsubseteq H_E$ and $H_E \in \tau$. Then $F_E \sqsubseteq H_E$. Since F_E is soft Ig-closed, then $\overline{F_E} \setminus H_E \in I$. Now, $G_E \sqsubseteq \overline{F_E}$ implies that $\overline{G_E} \sqsubseteq \overline{F_E}$. So $\overline{G_E} \setminus H_E \sqsubseteq \overline{F_E} \setminus H_E$ and thus $\overline{G_E} \setminus H_E \in I$. Consequently G_E is soft Ig-closed in (X, τ, E).

Remark 3.9 The intersection of two soft Ig-closed sets need not be a soft Ig-closed set as shown by the following example.

Example 3.10 Let A_E, B_E, C_E be three soft sets over the universe X, which describe the characters of the students with respect to the given parameters for finding the best student of an academic year. Let the set of the students under considerations is $X = \{a, b, c\}$, $E = \{e_1(result), e_2(formancess)\}$, $\tau = \{\Phi, \widetilde{E}_X, A_E\}$ and $I = \{\Phi\}$, $A(e_1) = \{b\}, A(e_2) = \{a\}$. So $\tau^c = \{\Phi, \widetilde{E}_X, A^c_E\}$. Let $B_E, C_E \in SS(X, E)$ s.t. $B(e_1) = \{a, b\}$, $B(e_2) = \phi$, $C(e_1) = \{b, c\}$ and $C(e_2) = \phi$. Thus B_E and C_E are Ig-closed.
In fact, $B_E \sqsubseteq \widetilde{E}_X$ and $\overline{B_E} \setminus \widetilde{E}_X = \widetilde{E}_X \setminus \widetilde{E}_X = \Phi \in I$. Also, $C_E \sqsubseteq \widetilde{E}_X$ and $\overline{C_E} \setminus \widetilde{E}_X = \widetilde{E}_X \setminus \widetilde{E}_X = \Phi \in I$. So, B_E and C_E are Ig-closed. But $B_E \sqcap C_E$ is not Ig-closed. In fact, $B_E \sqcap C_E = H_E$, where $H(e_1) = \{b\}, H(e_2) = \phi$. Also, $H_E \sqsubseteq A_E$ and $\bar{H} \setminus A_E = \bar{E} \setminus A_E = A^c_E \notin I$

Theorem 3.11 If A_E is soft Ig-closed and F_E is soft closed in a soft topological space (X, τ, E). Then $A_E \sqcap F_E$ is soft Ig-closed in (X, τ, E).

Proof: Assume that $A_E \sqcap F_E \sqsubseteq G_E$ and $G_E \in \tau$. Then $A_E \sqsubseteq G_E \sqcup F^c_E$. Since A_E is soft Ig-closed, we have $\overline{A_E} \setminus (G_E \sqcup F^c_E) \in I$. Now, $\overline{(A_E \sqcap F_E)} \sqsubseteq \overline{A_E} \sqcap \overline{F_E} = \overline{A_E} \sqcap F_E = \overline{A_E} \sqcap F_E \setminus F^c_E$. Therefore $\overline{(A_E \sqcap F_E)} \setminus G_E \sqsubseteq (\overline{A_E} \sqcap F_E) \setminus G_E \sqcap F^c_E \sqsubseteq \overline{A_E} \setminus (G_E \sqcup F^c_E) \in I$. Hence $A_E \sqcap F_E$ is soft Ig-closed.

Theorem 3.12 Let $M \subseteq X$ and $F_E \sqsubseteq \widetilde{E}_M \sqsubseteq \widetilde{E}_X$. Suppose that F_E is soft Ig-closed in (X, τ, E). Then F_E is soft Ig-closed relative to the soft topological Subspace τ_M of X and with respect to the soft ideal $I_M = \{H_E \sqsubseteq \widetilde{E}_M : H_E \in I\}$.

Proof: Suppose that $F_E \sqsubseteq B_E \sqcap \widetilde{E}_M$ and $B_E \in \tau$. So $B_E \sqcap \widetilde{E}_M \in \tau_M$ and $F_E \sqsubseteq B_E$. Since F_E is soft Ig-closed in (X, τ, E), then $\overline{F_E} \setminus B_E \in I$. Now, $(\overline{F_E} \sqcap \widetilde{E}_M) \setminus (B_E \sqcap \widetilde{E}_M) = (\overline{F_E} \setminus B_E) \sqcap \widetilde{E}_M \in I_M$ whenever F_E is soft Ig-closed relative to the subspace (M, τ_M, E).

4 Soft generalized open sets with respect to soft ideal

Definition 4.1 A soft set $F_E \in SS(X, E)$ is called soft open set with respect to a soft ideal I (soft Ig-open)in a soft topological space (X, τ, E) iff the relative complement F^c_E is soft Ig-closed in (X, τ, E).

Theorem 4.2 A soft set A_E is soft Ig-open in a soft topological space (X, τ, E) iff $F_E \setminus B_E \sqsubseteq A^0_E$ for some $B_E \in I$, whenever $F_E \sqsubseteq A_E$ and F_E is soft closed in (X, τ, E).

Proof: $(\Rightarrow)$ Suppose that A_E is soft Ig-open. Suppose $F_E \sqsubseteq A_E$ and F_E is soft closed. We have $A^c_E \sqsubseteq F^c_E, A^c_E$ is soft Ig-closed and $F^c_E \in \tau$. By assumption,

$\overline{A_E^c} \setminus F_E^c \in I$. Hence $\overline{A_E^c} \sqsubseteq F_E^c \sqcup B_E$ for some $B_E \in I$. So $(F_E^c \sqcup B_E)^c \sqsubseteq (\overline{A_E^c})^c = A_E^0$ and therefore

$$F_E \setminus B_E = F_E \sqcap B_E^c \sqsubseteq A_E^0.$$

$(\Leftarrow)$ Conversely, assume that $F_E \sqsubseteq A_E$ and F_E is soft closed. These imply that $F_E \setminus B_E \sqsubseteq A_E^0$ for some $B_E \in I$. Consider $G_E \in \tau$ such that $A_E^c \sqsubseteq G_E$. Then $G_E^c \sqsubseteq A_E$. By assumption $G_E^c \setminus B_E \sqsubseteq A_E^0 - (\overline{A_E^c})^c$ for some $B_E \in I$. This gives that $(G_E \sqcup B_E)^c \sqsubseteq (\overline{A_E^c})^c$. Then $\overline{A_E^c} \sqsubseteq G_E \sqcup B_E$ for some $B_E \in I$. This shows that $\overline{A_E^c} \setminus G_E \in I$. Hence A_E^c is soft Ig- closed and therefore A_E is soft Ig-open.

Definition 4.3 Two soft sets A_E and B_E are said to be soft separated in a soft topological space (X,τ,E) if $A_E^c \sqcap B_E = \Phi$ and $A_E \sqcap \overline{B_E} = \Phi$.

Theorem 4.4 If A_E and B_E are soft separated and soft Ig-open sets in a soft topological space (X,τ,E) then $A_E \sqcup B_E$ is soft Ig-open in (X,τ,E).
Proof: Suppose that A_E and B_E are soft separated Ig-open sets in (X,τ,E) and F_E is soft closed subset of $A_E \sqcup B_E$. Then $F_E \sqcap \overline{A_E} \sqsubseteq A_E$ and$F_E \sqcap \overline{B_E} \sqsubseteq B_E$. By Theorem 4.2, $(F_E \sqcap \overline{A_E}) \setminus D_E \sqsubseteq A_E^0$ and $(F_E \sqcap \overline{B_E}) \setminus C_E \sqsubseteq B_E^0$ for some $D_E, C_E \in I$. This means that $(F_E \sqcap \overline{A_E}) \setminus A_E^0 \in I$ and $(F_E \sqcap \overline{B_E}) \setminus A_E^0 \in I$. Then $((F_E \sqcap \overline{A_E}) \setminus A_E^0) \sqcup ((F_E \sqcap \overline{B_E}) \setminus B_E^0) \in I$. Hence $(F_E \sqcap (\overline{A_E} \sqcup \overline{B_E})) \setminus (A_E^0 \sqcup B_E^0) \in I$. But $F_E = F_E \sqcap (A_E \sqcup B_E) \sqsubseteq F_E \sqcap (\overline{A_E \sqcup B_E})$, and we have
$F_E \setminus (A_E \sqcup B_E)^0 \sqsubseteq (F_E \sqcap (\overline{A_E \sqcup B_E})) \setminus (A_E \sqcup B_E)^0 \sqsubseteq (F_E \sqcap (\overline{A_E \sqcup B_E})) \setminus (A_E^0 \sqcup B_E^0) \in I$. Hence $F_E \setminus H_E \sqsubseteq (A_E \sqcup B_E)^0$ for some $H_E \in I$. This proves that $A_E \sqcup B_E$ is soft Ig-open.

Corollary 4.5 Let A_E and B_E be soft Ig-closed sets and suppose that A_E^c and B_E^c are soft separated in a soft topological space (X,τ,E). Then $A_E \sqcap B_E$ is soft Ig-closed in (X,τ,E).

Corollary 4.6 If A_E and B_E are soft Ig-open sets in a soft topological space (X,τ,E), then $A_E \sqcap B_E$ is soft Ig-open in (X,τ,E).
Proof: If A_E and B_E are soft Ig-open, then A_E^c and B_E^c are soft Ig-closed. By Theorem 3.7, $(A_E \sqcap B_E)^c = A_E^c \sqcup B_E^c$ is soft Ig-closed, which implies $A_E \sqcup B_E$ is soft Ig-open.

Theorem 4.7 Let $M \subseteq X$ and $A_E \sqsubseteq \widetilde{E}_M \sqsubseteq \widetilde{E}_X$, A_E is soft Ig-open in (M,τ_M,E) and $\widetilde{E}_M$ is soft Ig-open in (X,τ,E). Then A_E is soft Ig-open in (X,τ,E).
Proof: Suppose that $A_E \sqsubseteq \widetilde{E}_M \sqsubseteq \widetilde{E}_X$, A_E is soft Ig-open in (M,τ_M,E) and $\widetilde{E}_M$ is soft Ig-open in (X,τ,E). We show that A_E is soft Ig-open in (X,τ,E). Suppose that $F_E \sqsubseteq A_E$ and F_E is soft τ-closed. Since A_E is soft Ig-open relative to $\widetilde{E}_M$, by Theorem 4.2, $F_E \setminus D_E \sqsubseteq int_{\tau_M}(A_E)$ for some $D_E \in I_M$. This implies that there exists a soft τ-open sets G_E such that $F_E \setminus D_E \sqsubseteq G_E \sqcap \widetilde{E}_M \sqsubseteq A_E$ for some $D_E \in I$. Then $F_E \sqsubseteq \widetilde{E}_M$ and F_E is soft τ-closed. Since $\widetilde{E}_M$ is soft Ig-open, then $F_E \setminus H_E \sqsubseteq int_\tau(\widetilde{E}_M)$ for some $H_E \in I$. This implies that there exists a soft τ-open set K_E such that $F_E \setminus H_E \sqsubseteq K_E \sqsubseteq \widetilde{E}_M$ for some $H_E \in I$. Now, $F_E \setminus (D_E \sqcup H_E) = (F_E \setminus D_E) \sqcap (F_E \setminus H_E) \sqsubseteq G_E \sqcap K_E \sqsubseteq G_E \sqcap \widetilde{E}_M \sqsubseteq A_E$. This implies that $F_E \setminus (D_E \sqcup H_E) \sqsubseteq int_\tau(A_E)$ for some $D_E \sqcup H_E \in I$ and hence A_E is soft Ig-open is (X,τ,E).

Theorem 4.8 If $A_E^0 \sqsubseteq B_E \sqsubseteq A_E$ and A_E is soft Ig-open in a soft topological space (X,τ,E), then B_E is soft Ig-open in (X,τ,E).
Proof: Suppose that $A_E^0 \sqsubseteq B_E \sqsubseteq A_E$ and A_E is soft Ig-open. Then $A_E^c \sqsubseteq B_E^c \sqsubseteq \overline{A_E^c}$ and A_E^c is soft Ig-closed. By Theorem 3.8, B_E^c is soft Ig-closed and hence B_E is soft Ig-open.

Theorem 4.9 A soft set A_E is soft Ig-closed in a soft topological space (X,τ,E) iff $\overline{A_E} \setminus A_E$ is soft Ig-open.
Proof:$(\Rightarrow)$ Suppose that $F_E \sqsubseteq \overline{A_E} \setminus A_E$ and F_E is soft closed. Then $F_E \in I$ and this implies that $F_E \setminus D_E = \Phi$ for some $D_E \in I$. Clearly, $F_E \setminus D_E \sqsubseteq (\overline{A_E} \setminus A_E)^0$. By Theorem 4.2 $\overline{A_E} \setminus A_E$ is soft Ig-open.
$(\Leftarrow)$ Suppose that $A_E \sqsubseteq G_E$ and G_E is soft open in (X,τ,E). Then, $\overline{A_E} \sqcap G_E^c \sqsubseteq \overline{A_E} \sqcap A_E^c = \overline{A_E} \setminus A_E$. By hypothesis, $\overline{A_E} \sqcap G_E^c \setminus D_E \sqsubseteq (\overline{A_E} \setminus A_E)^0 = \Phi$, for some $D_E \in I$. This implies that $\overline{A_E} \sqcap G_E^c \sqsubseteq D_E \in I$ and therefore $\overline{A_E} \setminus G_E \in I$. Thus, A_E is soft Ig-closed.

Theorem 4.10 Let $(\varphi,\psi) : (X,\tau,E) \to (Y,\sigma,K)$ be soft continuous and soft closed mapping. If $A_E \in SS(X,E)$ is soft Ig-closed in (X,τ,E), then $(\varphi,\psi)(A_E)$ is soft $(\varphi,\psi)(I)g$-closed in (Y,σ,K), where $(\varphi,\psi)(I) = \{(\varphi,\psi)(D_E) : D_E \in I\}$.
Proof: Suppose that $A_E \in SS(X,E)$ is soft Ig-closed in (X,τ,E). Suppose that $(\varphi,\psi)(A_E) \sqsubseteq G_E$ and G_E is soft open in (Y,σ,k). Then $A_E \sqsubseteq (\varphi,\psi)^{-1}(G_E)$. By definition, $\overline{A_E} \setminus (\varphi,\psi)^{-1}(G_E) \in I$ and hence $(\varphi,\psi)(\overline{A_E}) \setminus G_E \in (\varphi,\psi)(I)$. Since (φ,ψ) is soft closed, then $\overline{(\varphi,\psi)(A_E)} \sqsubseteq \overline{(\varphi,\psi)(\overline{A_E})} = (\varphi,\psi)(\overline{A_E})$. Then $\overline{(\varphi,\psi)(A_E)} \setminus G_E \sqsubseteq (\varphi,\psi)(\overline{A_E}) \setminus G_E \in (\varphi,\psi)(I)$ and hence $(\varphi,\psi)(A_E)$ is soft $(\varphi,\psi)(I)g$-closed in (Y,σ,K).

5 Conclusion

Topology is an important and major area of mathematics and it can give many relationships between other scientific areas and mathematical models. Recently, many

scientists have studied the soft set theory, which is initiated by Molodtsov and easily applied to many problems having uncertainties from social life. In the present work, we have continued to study the properties of soft topological spaces. We introduce the notions of soft Ig-closed and soft Ig-open sets and have established several interesting properties. Because there exists compact connections between soft sets and information systems [8,9], we can use the results deducted from the studies on soft topological space to improve these kinds of connections. We hope the findings in this paper will help researcher enhance and promote the further study on soft topology to carry out a general framework for their applications in practical life.
Granular computing is a recent approach in the field of computer science that uses topological structure as granulation models. The suggested approach for soft Ig-closed sets give new methods for generating the classes of subsets whose lower and upper approximations are contained in elementary sets which in turn help in the process of decision making under both quantities and qualitative information.

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