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Ideal Theory of BCK/BCI-algebras Based on Double-framed Soft Sets

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Abstract: The notion of a (closed) double-framed soft ideal (briefly, a (closed) DFS-ideal) in BCK/BCI-algebras is introduced, and related properties are investigated. Several examples are provided. The relation between a DFS-algebra and a DFS-ideal is considered, and characterizations of a (closed) DFS-ideal are established. A new DFS-ideal from old one is constructed, and we show that the int-uni DFS-set of two DFS-ideals is a DFS-ideal. Conditions for a DFS-ideal to be closed are discussed.

Keywords: Inclusive (resp. Exclusive) set, Int-uni DFS-set, DFS-algebra, (Closed) DFS-ideal

1 Introduction

Molodtsov [19] introduced the concept of soft set as a new mathematical tool for dealing with uncertainties that is free from the difficulties that have troubled the usual theoretical approaches. Molodtsov pointed out several directions for the applications of soft sets. Worldwide, there has been a rapid growth in interest in soft set theory and its applications in recent years. Evidence of this can be found in the increasing number of high-quality articles on soft sets and related topics that have been published in a variety of international journals, symposia, workshops, and international conferences in recent years. Maji et al. [16] described the application of soft set theory to a decision making problem. Maji et al. [15] also studied several operations on the theory of soft sets. Jun and Park [14] studied applications of soft sets in ideal theory of BCK/BCI-algebras.

We refer the reader to the papers [1], [2], [3], [4], [5], [8], [10], [11], [12], [13], [20] and [21] for further information regarding algebraic structures/properties of soft set theory. In [9], Jun et al. introduced the notion of double-framed soft sets (briefly, DFS-sets), and applied it to BCK/BCI-algebras. They discussed double-framed soft algebras (briefly, DFS-algebras) and investigated related properties.

In this paper, we introduce the notion of a (closed) double-framed soft ideal (briefly, a (closed) DFS-ideal) in BCK/BCI-algebras. We discuss the relation between a DFS-algebra and a DFS-ideal. We establish characterizations of a (closed) DFS-ideal, and make a new DFS-ideal from old one. We show that the int-uni DFS-set of two DFS-ideals is a DFS-ideal. We provide conditions for a DFS-ideal to be closed.

2 Preliminaries

2.1 Basic results on BCK/BCI-algebras

A BCK/BCI-algebra is an important class of logical algebras introduced by K. Iséki and was extensively investigated by several researchers.

An algebra $(X;*,0)$ of type $(2,0)$ is called a *BCI-algebra* if it satisfies the following conditions:

(I) $(\forall x,y,z\in X)\ (((x*y)*(x*z))*(z*y)=0)$,
(II) $(\forall x,y\in X)\ ((x*(x*y))*y=0)$,
(III) $(\forall x\in X)\ (x*x=0)$,
(IV) $(\forall x,y\in X)\ (x*y=0, y*x=0 \Rightarrow x=y)$.

If a BCI-algebra X satisfies the following identity:

(V) $(\forall x\in X)\ (0*x=0)$,

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then X is called a *BCK-algebra*. Any BCK/BCI-algebra X satisfies the following conditions:

$$(\forall x \in X)\ (x * 0 = x), \tag{1}$$
$$(\forall x, y, z \in X)\ (x \leq y \Rightarrow x * z \leq y * z,\ z * y \leq z * x), \tag{2}$$
$$(\forall x, y, z \in X)\ ((x * y) * z = (x * z) * y), \tag{3}$$
$$(\forall x, y, z \in X)\ ((x * z) * (y * z) \leq x * y) \tag{4}$$

where $x \leq y$ if and only if $x * y = 0$. Note that $(X, \leq)$ is a partially ordered set (see [17]).

A nonempty subset S of a BCK/BCI-algebra X is called a *subalgebra* of X if $x * y \in S$ for all $x, y \in S$. A subset I of a BCK/BCI-algebra X is called an *ideal* of X if it satisfies:

$$0 \in I, \tag{5}$$
$$(\forall x \in X)\,(\forall y \in I)\,(x * y \in I\ \Rightarrow\ x \in I). \tag{6}$$

We refer the reader to the books [6] [7] and [17] for further information regarding BCK/BCI-algebras.

2.2 Basic results on soft sets

Molodtsov [19] defined the soft set in the following way: Let U be an initial universe set and E be a set of parameters. We say that the pair (U, E) is a *soft universe*. Let $\mathscr{P}(U)$ denotes the power set of U and $A, B, C, \cdots \subseteq E$.

Definition 1([19]). *A pair (α, A) is called a soft set over U, where α is a mapping given by*

$$\alpha : A \to \mathscr{P}(U).$$

In other words, a soft set over U is a parameterized family of subsets of the universe U. For $\varepsilon \in A$, $\alpha(\varepsilon)$ may be considered as the set of ε-approximate elements of the soft set (α, A). Clearly, a soft set is not a set. For illustration, Molodtsov considered several examples in [19].

In what follows, we take $E = X$, as a set of parameters, which is a BCK/BCI-algebra and $A, B, C, \cdots$ be subalgebras of E unless otherwise specified.

Definition 2([9]). *A double-framed soft pair $\langle(\alpha, \beta); A\rangle$ is called a double-framed soft set of A over U (briefly, DFS-set of A), where α and β are mappings from A to $\mathscr{P}(U)$.*

Definition 3([9]). *A DFS-set $\langle(\alpha, \beta); A\rangle$ of A is called a double-framed soft algebra of A over U (briefly, DFS-algebra of A) if it satisfies :*

$$(\forall x, y \in A)\left(\begin{array}{l}\alpha(x * y) \supseteq \alpha(x) \cap \alpha(y),\\ \beta(x * y) \subseteq \beta(x) \cup \beta(y)\end{array}\right). \tag{7}$$

3 Double-framed soft ideals

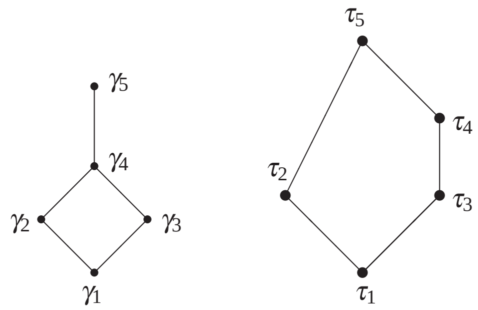
Definition 4. *A DFS-set $\langle(\alpha, \beta); A\rangle$ of A is called a double-framed soft ideal of A over U (briefly, DFS-ideal of A) if it satisfies:*

$$(\forall x \in A)\,(\alpha(0) \supseteq \alpha(x),\ \beta(0) \subseteq \beta(x)), \tag{8}$$
$$(\forall x, y \in A)\left(\begin{array}{l}\alpha(x) \supseteq \alpha(x * y) \cap \alpha(y),\\ \beta(x) \subseteq \beta(x * y) \cup \beta(y)\end{array}\right). \tag{9}$$

Example 1. Let $(U, E) = (U, X)$ where $X = \{0, 1, 2, a, b\}$ is a BCI-algebra with the following Cayley table:

$*$	0	1	2	a	b
0	0	0	0	a	a
1	1	0	1	b	a
2	2	2	0	a	a
a	a	a	a	0	0
b	b	a	b	1	0

Let $\{\gamma_1, \gamma_2, \gamma_3, \gamma_4, \gamma_5\}$ and $\{\tau_1, \tau_2, \tau_3, \tau_4, \tau_5\}$ be classes of subsets of U which are posets with the following Hasse diagrams:



Define a DFS-set $\langle(\alpha, \beta); E\rangle$ of E as follows:

$$\alpha : E \to \mathscr{P}(U),\ \ x \mapsto \begin{cases} \gamma_5 & \text{if } x = 0,\\ \gamma_2 & \text{if } x = 1,\\ \gamma_4 & \text{if } x = 2,\\ \gamma_3 & \text{if } x = a,\\ \gamma_1 & \text{if } x = b \end{cases}$$

and

$$\beta : E \to \mathscr{P}(U),\ \ x \mapsto \begin{cases} \tau_1 & \text{if } x = 0,\\ \tau_2 & \text{if } x = 1,\\ \tau_3 & \text{if } x = 2,\\ \tau_4 & \text{if } x = a,\\ \tau_5 & \text{if } x = b. \end{cases}$$

Routine calculations show that $\langle(\alpha, \beta); E\rangle$ is a DFS-ideal of E.

Proposition 1. *Every DFS-ideal $\langle(\alpha, \beta); A\rangle$ of A satisfies the following conditions:*

$$(1)(\forall x, y \in A)\,(x \leq y\ \Rightarrow\ \alpha(x) \supseteq \alpha(y),\ \beta(x) \subseteq \beta(y)),$$
$$(2)(\forall x, y, z \in A)\left(x * y \leq z \Rightarrow \begin{cases}\alpha(x) \supseteq \alpha(y) \cap \alpha(z),\\ \beta(x) \subseteq \beta(y) \cup \beta(z)\end{cases}\right).$$

Proof. (1) Let $x,y \in A$ be such that $x \leq y$. Then $x*y = 0 \in A$, and so

$$\alpha(y) = \alpha(0) \cap \alpha(y) = \alpha(x*y) \cap \alpha(y) \subseteq \alpha(x),$$
$$\beta(y) = \beta(0) \cup \beta(y) = \beta(x*y) \cup \beta(y) \supseteq \beta(x)$$

by (8) and (9).

(2) Let $x,y,z \in A$ be such that $x*y \leq z$. Then

$$\alpha(z) = \alpha(0) \cap \alpha(z) = \alpha((x*y)*z) \cap \alpha(z) \subseteq \alpha(x*y),$$
$$\beta(z) = \beta(0) \cup \beta(z) = \beta((x*y)*z) \cup \beta(z) \supseteq \beta(x*y).$$

It follows from (9) that

$$\alpha(y) \cap \alpha(z) \subseteq \alpha(y) \cap \alpha(x*y) \subseteq \alpha(x),$$
$$\beta(y) \cup \beta(z) \supseteq \beta(y) \cup \beta(x*y) \supseteq \beta(x).$$

This completes the proof.

Corollary 1. *Every DFS-ideal $\langle(\alpha,\beta);E\rangle$ of E satisfies the following conditions:*

$$(1)(\forall x,y \in E)\,(x \leq y \ \Rightarrow\ \alpha(x) \supseteq \alpha(y),\ \beta(x) \subseteq \beta(y)),$$
$$(2)(\forall x,y,z \in E)\left(x*y \leq z \ \Rightarrow\ \begin{cases} \alpha(x) \supseteq \alpha(y) \cap \alpha(z), \\ \beta(x) \subseteq \beta(y) \cup \beta(z) \end{cases}\right).$$

Proposition 2. *Every DFS-ideal $\langle(\alpha,\beta);E\rangle$ of E satisfies the following conditions:*

$$(1)(\forall x,y,z \in E)\begin{pmatrix} \alpha(x*y) \supseteq \alpha(x*z) \cap \alpha(z*y) \\ \beta(x*y) \subseteq \beta(x*z) \cup \beta(z*y) \end{pmatrix}.$$
$$(2)(\forall x,y \in E)\begin{pmatrix} \alpha(x*y) = \alpha(0) \ \Rightarrow\ \alpha(x) \supseteq \alpha(y) \\ \beta(x*y) = \beta(0) \ \Rightarrow\ \beta(x) \subseteq \beta(y) \end{pmatrix}.$$

Proof. (1) Since $(x*y)*(x*z) \leq z*y$ for all $x,y,z \in E$, it follows from Corollary 1(1) that

$$\alpha(z*y) \subseteq \alpha((x*y)*(x*z)),$$

$$\beta((x*y)*(x*z)) \subseteq \beta(z*y).$$

Using (9) we have

$$\begin{aligned} \alpha(x*y) &\supseteq \alpha((x*y)*(x*z)) \cap \alpha(x*z) \\ &\supseteq \alpha(x*z) \cap \alpha(z*y), \end{aligned}$$

$$\begin{aligned} \beta(x*y) &\subseteq \beta((x*y)*(x*z)) \cup \beta(x*z) \\ &\subseteq \beta(x*z) \cup \beta(z*y). \end{aligned}$$

(2) Let $x,y \in E$ be such that $\alpha(x*y) = \alpha(0)$ and $\beta(x*y) = \beta(0)$. Then

$$\alpha(x) \supseteq \alpha(x*y) \cap \alpha(y) = \alpha(0) \cap \alpha(y) = \alpha(y)$$
$$\beta(x) \subseteq \beta(x*y) \cup \beta(y) = \beta(0) \cup \beta(y) = \beta(y)$$

by (8) and (9).

Proposition 3. *If $\langle(\alpha,\beta);E\rangle$ is a DFS-ideal of E, then the following are equivalent:*

$$(1)\ (\forall x,y \in E)\begin{pmatrix} \alpha(x*y) \supseteq \alpha((x*y)*y) \\ \beta(x*y) \subseteq \beta((x*y)*y) \end{pmatrix}.$$

$$(2)\ (\forall x,y,z \in E)\begin{pmatrix} \alpha((x*z)*(y*z)) \supseteq \alpha((x*y)*z) \\ \beta((x*z)*(y*z)) \subseteq \beta((x*y)*z) \end{pmatrix}.$$

Proof. Assume that (1) is valid and let $x,y,z \in E$. Since

$$((x*(y*z))*z)*z = ((x*z)*(y*z))*z \leq (x*y)*z,$$

it follows from (3), (1) and Corollary 1(1) that

$$\begin{aligned} \alpha((x*z)*(y*z)) &= \alpha((x*(y*z))*z) \\ &\supseteq \alpha(((x*(y*z))*z)*z) \\ &\supseteq \alpha((x*y)*z) \end{aligned}$$

and

$$\begin{aligned} \beta((x*z)*(y*z)) &= \beta((x*(y*z))*z) \\ &\subseteq \beta(((x*(y*z))*z)*z) \\ &\subseteq \beta((x*y)*z). \end{aligned}$$

Conversely, suppose that (2) holds. If we take $y = z$ in (2), then

$$\begin{aligned} \alpha((x*z)*z) &\subseteq \alpha((x*z)*(z*z)) \\ &= \alpha((x*z)*0) \\ &= \alpha(x*z) \end{aligned}$$

and

$$\begin{aligned} \beta((x*z)*z) &\supseteq \beta((x*z)*(z*z)) \\ &= \beta((x*z)*0) \\ &= \beta(x*z) \end{aligned}$$

by (III) and (1). This proves (1).

Theorem 1. *In a BCK-algebra, every DFS-ideal is a DFS-algebra.*

Proof. Let $E = X$ be a BCK-algebra and let $\langle(\alpha,\beta);E\rangle$ be a DFS-ideal of E. For any $x,y \in E$, we have

$$\begin{aligned} \alpha(x*y) &\supseteq \alpha((x*y)*x) \cap \alpha(x) \\ &= \alpha((x*x)*y) \cap \alpha(x) \\ &= \alpha(0*y) \cap \alpha(x) \\ &= \alpha(0) \cap \alpha(x) \\ &\supseteq \alpha(x) \cap \alpha(y) \end{aligned}$$

and

$$\begin{aligned} \beta(x*y) &\subseteq \beta((x*y)*x) \cup \beta(x) \\ &= \beta((x*x)*y) \cup \beta(x) \\ &= \beta(0*y) \cup \beta(x) \\ &= \beta(0) \cup \beta(x) \\ &\subseteq \beta(x) \cup \beta(y). \end{aligned}$$

Therefore $\langle(\alpha,\beta);E\rangle$ is a DFS-algebra of E.

The converse of Theorem 1 is not true as seen in the following example.

Example 2. Let $U = \mathbb{N}$ be the initial universe set and let $E = \{0,a,b,c,d\}$ be a BCK-algebra with the following Cayley table:

$*$	0	a	b	c	d
0	0	0	0	0	0
a	a	0	0	0	0
b	b	b	0	0	0
c	c	c	c	0	0
d	d	c	c	a	0

Define a DFS-set $\langle(\alpha,\beta);E\rangle$ of E as follows:

$$\alpha : E \to \mathscr{P}(U),\ \ x \mapsto \begin{cases} \mathbb{N} & \text{if } x = 0, \\ 4\mathbb{N} & \text{if } x = a, \\ 2\mathbb{N} & \text{if } x = b, \\ 3\mathbb{N} & \text{if } x = c, \\ 8\mathbb{N} & \text{if } x = d \end{cases}$$

and

$$\beta : E \to \mathscr{P}(U),\ \ x \mapsto \begin{cases} 12\mathbb{N} & \text{if } x = 0, \\ 3\mathbb{N} & \text{if } x = a, \\ 6\mathbb{N} & \text{if } x = b, \\ 5\mathbb{N} & \text{if } x = c, \\ \mathbb{N} & \text{if } x = d. \end{cases}$$

Then $\langle(\alpha,\beta);E\rangle$ is a DFS-algebra of E, but it is not a DFS-ideal of E since

$$\alpha(d*b)\cap\alpha(b) = 3\mathbb{N}\cap 2\mathbb{N} = 6\mathbb{N} \nsubseteq \alpha(d)$$

and/or

$$\beta(d*b)\cup\beta(b) = 5\mathbb{N}\cup 6\mathbb{N} \nsupseteq \beta(d).$$

The following example shows that Theorem 1 is not true in BCI-algebras.

Example 3. Consider the BCI-algebra $(\mathbb{Z},*,0)$ as the initial universe set U, where $a*b = a-b$ for all $a,b\in\mathbb{Z}$. Let $E = X = \{0,a,b,c\}$ be a BCI-algebra with the following Cayley table:

$*$	0	a	b	c
0	0	a	b	c
a	a	0	c	b
b	b	c	0	a
c	c	b	a	0

Define a DFS-set $\langle(\alpha,\beta);E\rangle$ of E as follows:

$$\alpha : E \to \mathscr{P}(U),\ x \mapsto \begin{cases} \mathbb{Z} & \text{if } x = 0, \\ 2\mathbb{Z} & \text{if } x = a, \\ 3\mathbb{Z} & \text{if } x = b, \\ 8\mathbb{Z} & \text{if } x = c \end{cases}$$

and

$$\beta : E \to \mathscr{P}(U),\ \ x \mapsto \begin{cases} 12\mathbb{Z} & \text{if } x = 0, \\ 8\mathbb{Z} & \text{if } x = a, \\ 3\mathbb{Z} & \text{if } x = b, \\ 2\mathbb{Z} & \text{if } x = c. \end{cases}$$

Then $\langle(\alpha,\beta);E\rangle$ is a DFS-ideal of E, but it is not a DFS-algebra of E since

$$\alpha(a)\cap\alpha(b) = 2\mathbb{Z}\cap 3\mathbb{Z} \nsubseteq 8\mathbb{Z} = \alpha(a*b)$$

and/or

$$\beta(a)\cup\beta(c) = 8\mathbb{Z}\cup 2\mathbb{Z} \nsupseteq 3\mathbb{Z} = \beta(a*c).$$

For a DFS-set $\langle(\alpha,\beta);A\rangle$ of A and two subsets γ and δ of U, the *γ-inclusive set* and the *δ-exclusive set* of $\langle(\alpha,\beta);A\rangle$, denoted by $i_A(\alpha;\gamma)$ and $e_A(\beta;\delta)$, respectively, are defined as follows:

$$i_A(\alpha;\gamma) := \{x\in A \mid \gamma\subseteq\alpha(x)\}$$

and

$$e_A(\beta;\delta) := \{x\in A \mid \delta\supseteq\beta(x)\},$$

respectively. The set

$$DF_A(\alpha,\beta)_{(\gamma,\delta)} := \{x\in A \mid \gamma\subseteq\alpha(x),\ \delta\supseteq\beta(x)\}$$

is called a *double-framed including set* ([9]) of $\langle(\alpha,\beta);A\rangle$. It is clear that

$$DF_A(\alpha,\beta)_{(\gamma,\delta)} = i_A(\alpha;\gamma)\cap e_A(\beta;\delta).$$

Theorem 2. *For a DFS-set $\langle(\alpha,\beta);A\rangle$ of A, the following are equivalent:*

(1) *$\langle(\alpha,\beta);A\rangle$ is a DFS-ideal of A.*
(2) *The nonempty γ-inclusive set and δ-exclusive set of $\langle(\alpha,\beta);A\rangle$ are ideals of A for any subsets γ and δ of U.*

Proof. Suppose that $\langle(\alpha,\beta);A\rangle$ is a DFS-ideal of A. Let γ and δ be subsets of U such that $i_A(\alpha;\gamma)\neq\emptyset\neq e_A(\beta;\delta)$. Then $\gamma\subseteq\alpha(x)$ and $\delta\supseteq\beta(a)$ for some $x,a\in A$, which imply from (8) that $\gamma\subseteq\alpha(x)\subseteq\alpha(0)$ and $\delta\supseteq\beta(a)\supseteq\beta(0)$. Hence $0\in i_A(\alpha;\gamma)\cap e_A(\beta;\delta)$. Let $x,y,a,b\in A$ be such that $x*y\in i_A(\alpha;\gamma)$, $y\in i_A(\alpha;\gamma)$, $a*b\in e_A(\beta;\delta)$ and $b\in e_A(\beta;\delta)$. Then $\gamma\subseteq\alpha(x*y)$, $\gamma\subseteq\alpha(y)$, $\delta\supseteq\beta(a*b)$ and $\delta\supseteq\beta(b)$. It follows from (9) that

$$\gamma\subseteq\alpha(x*y)\cap\alpha(y)\subseteq\alpha(x)$$

and

$$\delta\supseteq\beta(a*b)\cup\beta(b)\supseteq\beta(a)$$

so that $x\in i_A(\alpha;\gamma)$ and $a\in e_A(\beta;\delta)$. Hence $i_A(\alpha;\gamma)$ and $e_A(\beta;\delta)$ are ideals of A for any subsets γ and δ of U.

Conversely, assume that the nonempty γ-inclusive set and δ-exclusive set of $\langle(\alpha,\beta);A\rangle$ are ideals of A for any subsets γ and δ of U. Let $x,a\in A$ be such that $\alpha(x)=\gamma_x$ and $\beta(a)=\delta_a$. Then $x\in i_A(\alpha;\gamma_x)$ and $a\in e_A(\beta;\delta_a)$. Since $i_A(\alpha;\gamma_x)$ and $e_A(\beta;\delta_a)$ are ideals of A by assumption, we have $0\in i_A(\alpha;\gamma_x)\cap e_A(\beta;\delta_a)$. Hence $\alpha(x)=\gamma_x\subseteq\alpha(0)$ and $\beta(a)=\delta_a\supseteq\beta(0)$. Let $x,y,a,b\in A$ be such that $\alpha(x*y)=\gamma_1$, $\alpha(y)=\gamma_2$, $\beta(a*b)=\delta_1$ and $\beta(b)=\delta_2$. Let us take $\gamma=\gamma_1\cap\gamma_2$ and $\delta=\delta_1\cup\delta_2$. Then $x*y\in i_A(\alpha;\gamma)$, $y\in i_A(\alpha;\gamma)$, $a*b\in e_A(\beta;\delta)$ and $b\in e_A(\beta;\delta)$. It follows from (6) that $x\in i_A(\alpha;\gamma)$ and $a\in e_A(\beta;\delta)$. Thus $\alpha(x)\supseteq\gamma=\gamma_1\cap\gamma_2=\alpha(x*y)\cap\alpha(y)$ and $\beta(a)\subseteq\delta=\delta_1\cup\delta_2=\beta(a*b)\cup\beta(b)$. Therefore $\langle(\alpha,\beta);A\rangle$ is a DFS-ideal of A.

Corollary 2. *For a DFS-set $\langle(\alpha,\beta);E\rangle$ of E, the following are equivalent:*

(1) *$\langle(\alpha,\beta);E\rangle$ is a DFS-ideal of E.*
(2) *The nonempty γ-inclusive set and δ-exclusive set of $\langle(\alpha,\beta);E\rangle$ are ideals of E for any subsets γ and δ of U.*

Corollary 3. *If $\langle(\alpha,\beta);E\rangle$ is a DFS-ideal of E, then the nonempty double-framed including set of $\langle(\alpha,\beta);E\rangle$ is an ideal of E.*

For any DFS-set $\langle(\alpha,\beta);E\rangle$ of E, let $\langle(\alpha^*,\beta^*);E\rangle$ be a DFS-set of E defined by

$$\alpha^*: E\to\mathscr{P}(U),\ x\mapsto\begin{cases}\alpha(x) & \text{if } x\in i_E(\alpha;\gamma),\\ \eta & \text{otherwise,}\end{cases}$$

$$\beta^*: E\to\mathscr{P}(U),\ x\mapsto\begin{cases}\beta(x) & \text{if } x\in e_E(\beta;\delta),\\ \rho & \text{otherwise,}\end{cases}$$

where γ,δ,η and ρ are subsets of U with $\eta\subsetneq\alpha(x)$ and $\rho\supsetneq\beta(x)$.

Theorem 3. *If $\langle(\alpha,\beta);E\rangle$ is a DFS-ideal of E, then so is $\langle(\alpha^*,\beta^*);E\rangle$.*

Proof. Assume that $\langle(\alpha,\beta);E\rangle$ is a DFS-ideal of E. Then the nonempty γ-inclusive set $i_E(\alpha;\gamma)$ and the δ-exclusive set $e_E(\beta;\delta)$ of $\langle(\alpha,\beta);E\rangle$ are ideals of E for every subsets γ and δ of U. If $x\in i_E(\alpha;\gamma)\cap e_E(\beta;\delta)$, then

$$\alpha^*(0)=\alpha(0)\supseteq\alpha(x)=\alpha^*(x)$$

and

$$\beta^*(0)=\beta(0)\subseteq\beta(x)=\beta^*(x).$$

If $x\notin i_E(\alpha;\gamma)$, then $\alpha^*(x)=\eta$. Hence $\alpha^*(0)\supseteq\eta=\alpha^*(x)$. If $x\notin e_E(\beta;\delta)$, then $\beta^*(x)=\rho$. Hence $\beta^*(0)\subseteq\rho=\beta^*(x)$. Let $x,y\in E$. If $x*y,y\in i_E(\alpha;\gamma)$, then $x\in i_E(\alpha;\gamma)$. Thus

$$\alpha^*(x)=\alpha(x)\supseteq\alpha(x*y)\cap\alpha(y)=\alpha^*(x*y)\cap\alpha^*(y).$$

If $x*y\notin i_E(\alpha;\gamma)$ or $y\notin i_E(\alpha;\gamma)$, then $\alpha^*(x*y)=\eta$ or $\alpha^*(y)=\eta$. Hence

$$\alpha^*(x)\supseteq\eta=\alpha^*(x*y)\cap\alpha^*(y).$$

Now, if $x*y,y\in e_E(\beta;\delta)$, then $x\in e_E(\beta;\delta)$. Thus

$$\beta^*(x)=\beta(x)\subseteq\beta(x*y)\cup\beta(y)=\beta^*(x*y)\cup\beta^*(y).$$

If $x*y\notin e_E(\beta;\delta)$ or $y\notin e_E(\beta;\delta)$, then $\beta^*(x*y)=\rho$ or $\beta^*(y)=\rho$. Hence

$$\beta^*(x)\subseteq\rho=\beta^*(x*y)\cup\beta^*(y).$$

Therefore $\langle(\alpha^*,\beta^*);E\rangle$ is a DFS-ideal of E.

The following example shows that the converse of Theorem 3 is not true in general.

Example 4. Suppose that there are ten houses in the initial universe set U given by

$$U=\{h_1,h_2,h_3,h_4,h_5,h_6,h_7,h_8,h_9,h_{10}\}.$$

Let a set of parameters $E=\{e_0,e_1,e_2,e_3\}$ be a set of status of houses in which

e_0 stands for the parameter “beautiful”,
e_1 stands for the parameter “cheap”,
e_2 stands for the parameter “in good location”,
e_3 stands for the parameter “in green surroundings,

with the following binary operation:

$*$	e_0	e_1	e_2	e_3
e_0	e_0	e_1	e_2	e_3
e_1	e_1	e_0	e_3	e_2
e_2	e_2	e_3	e_0	e_1
e_3	e_3	e_2	e_1	e_0

Then $(E,*,e_0)$ is a BCI-algebra (see [9]). Consider a DFS-set $\langle(\alpha,\beta);E\rangle$ of E as follows:

$$\alpha: E\to\mathscr{P}(U),\ x\mapsto\begin{cases}U & \text{if } x=e_0,\\ \{h_2,h_4,h_6,h_8,h_{10}\} & \text{if } x=e_1,\\ \{h_3,h_6,h_9\} & \text{if } x=e_2,\\ \{h_8\} & \text{if } x=e_3,\end{cases}$$

$$\beta: E\to\mathscr{P}(U),\ x\mapsto\begin{cases}\{h_6\} & \text{if } x=e_0,\\ \{h_3,h_6,h_9\} & \text{if } x=e_1,\\ \{h_2,h_4,h_6,h_8,h_{10}\} & \text{if } x=e_2,\\ U & \text{if } x=e_3.\end{cases}$$

Note that $i_E(\alpha;\{h_8\})=\{e_0,e_1,e_3\}$ is not an ideal of E since $e_2*e_3=e_1\in i_E(\alpha;\{h_8\})$ and $e_2\notin i_E(\alpha;\{h_8\})$. Using Corollary 2, $\langle(\alpha,\beta);E\rangle$ is not a DFS-ideal of E. Note that $i_E(\alpha;\gamma)=\{e_0,e_1\}=e_E(\beta;\delta)$ for $\gamma=\{h_2,h_4,h_6,h_8,h_{10}\}$ and $\delta=\{h_3,h_6,h_9\}$. Let $\langle(\alpha^*,\beta^*);E\rangle$ be a DFS-set of E defined by

$$\alpha^*: E\to\mathscr{P}(U),\ x\mapsto\begin{cases}\alpha(x) & \text{if } x\in i_E(\alpha;\gamma),\\ \emptyset & \text{otherwise,}\end{cases}$$

$$\beta^*: E\to\mathscr{P}(U),\ x\mapsto\begin{cases}\beta(x) & \text{if } x\in e_E(\beta;\delta),\\ U & \text{otherwise}\end{cases}$$

that is,

$$\alpha^*: E\to\mathscr{P}(U),\ x\mapsto\begin{cases}U & \text{if } x=e_0,\\ \{h_2,h_4,h_6,h_8,h_{10}\} & \text{if } x=e_1,\\ \emptyset & \text{if } x\in\{e_2,e_3\},\end{cases}$$

$$\beta^*: E\to\mathscr{P}(U),\ x\mapsto\begin{cases}\{h_6\} & \text{if } x=e_0,\\ \{h_3,h_6,h_9\} & \text{if } x=e_1,\\ U & \text{if } x\in\{e_2,e_3\}.\end{cases}$$

It is routine to verify that $\langle(\alpha^*,\beta^*);E\rangle$ is a DFS-ideal of E.

Let $\langle(\alpha,\beta);A\rangle$ and $\langle(\alpha,\beta);B\rangle$ be DFS-sets of A and B, respectively. The *$(\alpha_\wedge,\ \beta_\vee)$-product* (see [9]) of $\langle(\alpha,\beta);A\rangle$ and $\langle(\alpha,\beta);B\rangle$ is defined to be a DFS-set $\langle(\alpha_{A\wedge B},\beta_{A\vee B});A\times B\rangle$ of $A\times B$ in which

$$\alpha_{A\wedge B}:A\times B\to\mathscr{P}(U),\ (x,y)\mapsto\alpha(x)\cap\alpha(y),$$

$$\beta_{A\vee B}:A\times B\to\mathscr{P}(U),\ (x,y)\mapsto\beta(x)\cup\beta(y).$$

Theorem 4. *For any BCK/BCI-algebras E and F as sets of parameters, let $\langle(\alpha,\beta);E\rangle$ and $\langle(\alpha,\beta);F\rangle$ be DFS-ideals of E and F, respectively. Then the $(\alpha_\wedge,\ \beta_\vee)$-product of $\langle(\alpha,\beta);E\rangle$ and $\langle(\alpha,\beta);F\rangle$ is a DFS-ideal of $E\times F$.*

Proof. Note that $(E\times F,\ \circledast,\ (0,0))$ is a BCK/BCI-algebra. For any $(x,y)\in E\times F$, we have

$$\alpha_{E\wedge F}(0,0)=\alpha(0)\cap\alpha(0)\supseteq\alpha(x)\cap\alpha(y)=\alpha_{E\wedge F}(x,y)$$
$$\beta_{E\vee F}(0,0)=\beta(0)\cup\beta(0)\subseteq\beta(x)\cup\beta(y)=\beta_{E\vee F}(x,y).$$

Let $(x,y),(a,b)\in E\times F$. Then

$$\begin{aligned}&\alpha_{E\wedge F}((x,y)\circledast(a,b))\cap\alpha_{E\wedge F}(a,b)\\&=\alpha_{E\wedge F}(x*a,y*b)\cap\alpha_{E\wedge F}(a,b)\\&=(\alpha(x*a)\cap\alpha(y*b))\cap(\alpha(a)\cap\alpha(b))\\&=(\alpha(x*a)\cap\alpha(a))\cap(\alpha(y*b)\cap\alpha(b))\\&\subseteq\alpha(x)\cap\alpha(y)=\alpha_{E\wedge F}(x,y)\end{aligned}$$

and

$$\begin{aligned}&\beta_{E\vee F}((x,y)\circledast(a,b))\cup\beta_{E\vee F}(a,b)\\&=\beta_{E\vee F}(x*a,y*b)\cup\beta_{E\vee F}(a,b)\\&=(\beta(x*a)\cup\beta(y*b))\cup(\beta(a)\cup\beta(b))\\&=(\beta(x*a)\cup\beta(a))\cup(\beta(y*b)\cup\beta(b))\\&\supseteq\beta(x)\cup\beta(y)=\beta_{E\vee F}(x,y).\end{aligned}$$

Hence $\langle(\alpha_{E\wedge F},\beta_{E\vee F});E\times F\rangle$ is a DFS-ideal of $E\times F$.

Let $\langle(\alpha,\beta);A\rangle$ and $\langle(f,g);B\rangle$ be DFS-sets of A and B, respectively. Then $\langle(\alpha,\beta);A\rangle$ is called a *double-framed soft subset* (briefly, DFS-subset) of $\langle(f,g);B\rangle$, denoted by $\langle(\alpha,\beta);A\rangle\,\tilde{\subseteq}\,\langle(f,g);B\rangle$, (see [9]) if

(i) $A\subseteq B$,

(ii) $\left(\begin{array}{l}\alpha(e)\text{ and }f(e)\text{ are identical approximations}\\\beta(e)\text{ and }g(e)\text{ are identical approximations}\end{array}\right)$ for all $e\in A$.

Theorem 5. *Let $\langle(\alpha,\beta);A\rangle$ be a DFS-subset of $\langle(f,g);B\rangle$. If $\langle(f,g);B\rangle$ is a DFS-ideal of B, then $\langle(\alpha,\beta);A\rangle$ is a DFS-ideal of A.*

Proof. Let $x\in A$. Then $x\in B$, and so

$$\alpha(0)=f(0)\supseteq f(x)=\alpha(x),\ \beta(0)=g(0)\subseteq g(x)=\beta(x).$$

Let $x,y\in A$. Then $x,y\in B$. Hence

$$\alpha(x)=f(x)\supseteq f(x*y)\cap f(y)=\alpha(x*y)\cap\alpha(y)$$

and

$$\beta(x)=g(x)\subseteq g(x*y)\cup g(y)=\beta(x*y)\cup\beta(y).$$

Therefore $\langle(\alpha,\beta);A\rangle$ is a DFS-ideal of A.

The converse of Theorem 5 is not true as seen in the following example.

Example 5. Suppose that there are six houses in the initial universe set U given by

$$U=\{h_1,h_2,h_3,h_4,h_5,h_6\}.$$

Let a set of parameters $E=\{e_0,e_1,e_2,e_3,e_4\}$ be a set of status of houses in which

e_0 stands for the parameter "beautiful",
e_1 stands for the parameter "cheap",
e_2 stands for the parameter "in good location",
e_3 stands for the parameter "in green surroundings",
e_4 stands for the parameter "luxury",

with the following binary operation:

$*$	e_0	e_1	e_2	e_3	e_4
e_0	e_0	e_0	e_0	e_3	e_3
e_1	e_1	e_0	e_1	e_4	e_3
e_2	e_2	e_2	e_0	e_3	e_3
e_3	e_3	e_3	e_3	e_0	e_0
e_4	e_4	e_3	e_4	e_1	e_0

Then $(E,*,e_0)$ is a BCI-algebra (see [9]). Consider a DFS-set $\langle(\alpha,\beta);A\rangle$ of $A=\{e_0,e_1,e_2\}$ as follows:

$$\alpha:A\to\mathscr{P}(U),\ x\mapsto\begin{cases}U&\text{if }x=e_0,\\\{h_2,h_4,h_6\}&\text{if }x=e_1,\\\{h_1,h_3,h_5\}&\text{if }x=e_2,\end{cases}$$

$$\beta:A\to\mathscr{P}(U),\ x\mapsto\begin{cases}\{h_6\}&\text{if }x=e_0,\\\{h_4,h_6\}&\text{if }x=e_1,\\\{h_2,h_4,h_6\}&\text{if }x=e_2.\end{cases}$$

It is routine to verify that $\langle(\alpha,\beta);A\rangle$ is a DFS-ideal of A. Define a DFS-set $\langle(f,g);E\rangle$ of E by

$$f:E\to\mathscr{P}(U),\ x\mapsto\begin{cases}U&\text{if }x=e_0,\\\{h_2,h_4,h_6\}&\text{if }x=e_1,\\\{h_1,h_3,h_5\}&\text{if }x=e_2,\\\{h_2,h_4\}&\text{if }x=e_3,\\\{h_1,h_5\}&\text{if }x=e_4,\end{cases}$$

$$g:E\to\mathscr{P}(U),\ x\mapsto\begin{cases}\{h_6\}&\text{if }x=e_0,\\\{h_4,h_6\}&\text{if }x=e_1,\\\{h_2,h_4,h_6\}&\text{if }x=e_2,\\\{h_1,h_3\}&\text{if }x=e_3,\\\{h_2,h_5,h_6\}&\text{if }x=e_4.\end{cases}$$

Then $i_E(f;\{h_1,h_5\})=\{e_0,e_2,e_4\}$ is not an ideal of E. Also, $e_E(g;\{h_1,h_3\})=\{e_3\}$ is not an ideal of E. Using Corollary 2, we know that $\langle(f,g);E\rangle$ is not a DFS-ideal of E.

For two DFS-sets $\langle(\alpha,\beta);A\rangle$ and $\langle(f,g);A\rangle$ of A, the *int-uni double-framed soft set* (briefly, int-uni DFS-set) of

$\langle(\alpha,\beta);A\rangle$ and $\langle(f,g);A\rangle$ is defined to be the DFS-set $\langle(\alpha\tilde{\cap} f,\beta\tilde{\cup} g);A\rangle$ of A where

$$\alpha\tilde{\cap} f : A \to \mathscr{P}(U),\ x \mapsto \alpha(x)\cap f(x),$$
$$\beta\tilde{\cup} g : A \to \mathscr{P}(U),\ x \mapsto \beta(x)\cup g(x).$$

It is denoted by

$$\langle(\alpha,\beta);A\rangle \sqcap \langle(f,g);A\rangle = \langle(\alpha\tilde{\cap} f,\beta\tilde{\cup} g);A\rangle$$

(see [9]).

Theorem 6. *The int-uni DFS-set $\langle(\alpha,\beta);A\rangle \sqcap \langle(f,g);A\rangle$ of two DFS-ideals $\langle(\alpha,\beta);A\rangle$ and $\langle(f,g);A\rangle$ of A is a DFS-ideal of A.*

*Proof.*For any $x\in A$, we have

$$(\alpha\tilde{\cap} f)(0) = \alpha(0)\cap f(0) \supseteq \alpha(x)\cap f(x) = (\alpha\tilde{\cap} f)(x),$$
$$(\beta\tilde{\cup} g)(0) = \beta(0)\cup g(0) \subseteq \beta(x)\cup g(x) = (\beta\tilde{\cup} g)(x).$$

For any $x,y\in A$, we have

$$\begin{aligned}(\alpha\tilde{\cap} f)(x) &= \alpha(x)\cap f(x)\\ &\supseteq (\alpha(x*y)\cap\alpha(y))\cap(f(x*y)\cap f(y))\\ &= (\alpha(x*y)\cap f(x*y))\cap(\alpha(y)\cap f(y))\\ &= (\alpha\tilde{\cap} f)(x*y)\cap(\alpha\tilde{\cap} f)(y),\end{aligned}$$

$$\begin{aligned}(\beta\tilde{\cup} g)(x) &= \beta(x)\cup g(x)\\ &\subseteq (\beta(x*y)\cup\beta(y))\cup(g(x*y)\cup g(y))\\ &= (\beta(x*y)\cup g(x*y))\cup(\beta(y)\cup g(y))\\ &= (\beta\tilde{\cup} g)(x*y)\cup(\beta\tilde{\cup} g)(y).\end{aligned}$$

Therefore $\langle(\alpha,\beta);A\rangle \sqcap \langle(f,g);A\rangle$ is a DFS-ideal of A.

Corollary 4. *The int-uni DFS-set $\langle(\alpha,\beta);E\rangle \sqcap \langle(f,g);E\rangle$ of two DFS-ideals $\langle(\alpha,\beta);E\rangle$ and $\langle(f,g);E\rangle$ of E is a DFS-ideal of E.*

Definition 5. *Let $(U,E) = (U,X)$ where X is a BCI-algebra. A DFS-ideal $\langle(\alpha,\beta);E\rangle$ of E is said to be closed if it satisfies:*

$$(\forall x\in E)\,(\alpha(0*x)\supseteq\alpha(x),\ \beta(0*x)\subseteq\beta(x)). \tag{10}$$

Example 6. Let (U,E) be a soft universe which is given in Example 5. Consider a DFS-set $\langle(\alpha,\beta);E\rangle$ of E as follows:

$$\alpha : E\to\mathscr{P}(U),\ x\mapsto \begin{cases} U & \text{if } x=e_0,\\ \{h_1,h_2\} & \text{if } x=e_1,\\ \{h_1,h_3,h_4\} & \text{if } x=e_2,\\ \{h_1,h_3\} & \text{if } x=e_3,\\ \{h_1\} & \text{if } x=e_4,\end{cases}$$

$$\beta : E\to\mathscr{P}(U),\ x\mapsto \begin{cases} \{h_5\} & \text{if } x=e_0,\\ \{h_4,h_5\} & \text{if } x=e_1,\\ \{h_3,h_5\} & \text{if } x=e_2,\\ \{h_3,h_4,h_5\} & \text{if } x\in\{e_3,e_4\}.\end{cases}$$

Then $\langle(\alpha,\beta);E\rangle$ is a closed DFS-ideal of E.

Example 7. Let $(U,E)=(U,X)$ where $X=\{2^n \mid n\in\mathbb{Z}\}$ is a *BCI*-algebra with a binary operation "$\div$" (usual division). Let $\langle(\alpha,\beta);E\rangle$ be a DFS-set of E defined as follows:

$$\alpha : E\to\mathscr{P}(U),\ x\mapsto \begin{cases}\gamma_1 & \text{if } n\geq 0,\\ \gamma_2 & \text{if } n<0,\end{cases}$$

$$\beta : E\to\mathscr{P}(U),\ x\mapsto \begin{cases}\delta_1 & \text{if } n\geq 0,\\ \delta_2 & \text{if } n<0,\end{cases}$$

where γ_1, γ_2, δ_1 and δ_2 are subsets of U with $\gamma_1\supsetneq\gamma_2$ and $\delta_1\subsetneq\delta_2$. Then $\langle(\alpha,\beta);E\rangle$ is a DFS-ideal of E which is not closed since

$$\alpha\left(1\div 2^3\right) = \alpha\left(2^{-3}\right) = \gamma_2 \not\supseteq \gamma_1 = \alpha\left(2^3\right)$$

and/or

$$\beta\left(1\div 2^3\right) = \beta\left(2^{-3}\right) = \delta_2 \not\subseteq \delta_1 = \beta\left(2^3\right).$$

Theorem 7. *Let $(U,E)=(U,X)$ where X is a BCI-algebra. Then a DFS-ideal of E is closed if and only if it is a DFS-algebra of E.*

*Proof.*If $\langle(\alpha,\beta);E\rangle$ is a closed DFS-ideal of E, then $\alpha(0*x)\supseteq\alpha(x)$ and $\beta(0*x)\subseteq\beta(x)$ for all $x\in E$. It follows from (3), (III) and (9) that

$$\begin{aligned}\alpha(x*y) &\supseteq \alpha((x*y)*x)\cap\alpha(x)\\ &= \alpha(0*y)\cap\alpha(x)\\ &\supseteq \alpha(x)\cap\alpha(y)\end{aligned}$$

and

$$\begin{aligned}\beta(x*y) &\subseteq \beta((x*y)*x)\cup\beta(x)\\ &= \beta(0*y)\cup\beta(x)\\ &\subseteq \beta(x)\cup\beta(y)\end{aligned}$$

for all $x,y\in E$. Hence $\langle(\alpha,\beta);E\rangle$ is a DFS-algebra of E.

Conversely, let $\langle(\alpha,\beta);E\rangle$ be a DFS-ideal of E which is also a DFS-algebra of E. Then

$$\alpha(0*x)\supseteq\alpha(0)\cap\alpha(x)=\alpha(x),$$

$$\beta(0*x)\subseteq\beta(0)\cup\beta(x)=\beta(x)$$

for all $x\in E$. Therefore $\langle(\alpha,\beta);E\rangle$ is closed.

Let X be a *BCI*-algebra and $B(X) := \{x\in X \mid 0\leq x\}$. For any $x\in X$ and $n\in\mathbb{N}$, we define x^n by

$$x^1 = x,\ x^{n+1} = x*(0*x^n).$$

If there is an $n\in\mathbb{N}$ such that $x^n\in B(X)$, then we say that x is of *finite periodic* (see [18]), and we denote its period $|x|$ by

$$|x| = \min\{n\in\mathbb{N} \mid x^n\in B(X)\}.$$

Otherwise, x is of infinite period and denoted by $|x|=\infty$.

We provide conditions for a DFS-ideal to be closed.

Theorem 8. *Let $(U,E)=(U,X)$ where X is a BCI-algebra in which every element is of finite period. Then every DFS-ideal of E is closed.*

Proof. Let $\langle(\alpha,\beta);E\rangle$ be a DFS-ideal of E. For any $x\in E$, assume that $|x|=n$. Then $x^n\in B(X)$. Note that

$$\begin{aligned}\left(0*x^{n-1}\right)*x &= \left(0*\left(0*\left(0*x^{n-1}\right)\right)\right)*x\\ &= (0*x)*\left(0*\left(0*x^{n-1}\right)\right)\\ &= 0*\left(x*\left(0*x^{n-1}\right)\right)\\ &= 0*x^n=0,\end{aligned}$$

and so

$$\alpha\left(\left(0*x^{n-1}\right)*x\right)=\alpha(0)\supseteq\alpha(x)$$

and

$$\beta\left(\left(0*x^{n-1}\right)*x\right)=\beta(0)\subseteq\beta(x)$$

by (8). It follows from (9) that

$$\begin{aligned}\alpha\left(0*x^{n-1}\right)&\supseteq\alpha\left(\left(0*x^{n-1}\right)*x\right)\cap\alpha(x)\supseteq\alpha(x),\\ \beta\left(0*x^{n-1}\right)&\subseteq\beta\left(\left(0*x^{n-1}\right)*x\right)\cup\beta(x)\subseteq\beta(x).\end{aligned}\tag{11}$$

Also, note that

$$\begin{aligned}\left(0*x^{n-2}\right)*x &= \left(0*\left(0*\left(0*x^{n-2}\right)\right)\right)*x\\ &= (0*x)*\left(0*\left(0*x^{n-2}\right)\right)\\ &= 0*\left(x*\left(0*x^{n-2}\right)\right)\\ &= 0*x^{n-1},\end{aligned}$$

which implies from (11) that

$$\begin{aligned}\alpha\left(\left(0*x^{n-2}\right)*x\right)&=\alpha\left(0*x^{n-1}\right)\supseteq\alpha(x),\\ \beta\left(\left(0*x^{n-2}\right)*x\right)&=\beta\left(0*x^{n-1}\right)\subseteq\beta(x).\end{aligned}$$

Using (9), we have

$$\begin{aligned}\alpha\left(0*x^{n-2}\right)&\supseteq\alpha\left(\left(0*x^{n-2}\right)*x\right)\cap\alpha(x)\supseteq\alpha(x),\\ \beta\left(0*x^{n-2}\right)&\subseteq\beta\left(\left(0*x^{n-2}\right)*x\right)\cup\beta(x)\subseteq\beta(x).\end{aligned}$$

Continuing this process, we have $\alpha(0*x)\supseteq\alpha(x)$ and $\beta(0*x)\subseteq\beta(x)$ for all $x\in E$. Therefore $\langle(\alpha,\beta);E\rangle$ is closed.

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