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Autoregressive Distributed Lag Model of COVID-19 Infected Cases and Deaths

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Abstract: The primary objectives of the present study are to investigate the short- and long-term cointegration relationships between the cumulative number of new cases of COVID-19 infections (X) and the cumulative numbers of deaths due to COVID-19 (Y), to investigate the long-run equilibrium relationship between these variables using an autoregressive distributed lag model and bounds cointegration tests, and to study the stability of the estimated model. The cumulative sum of recursive residuals test and the cumulative sum of recursive residuals squares tests are used to assess the consistency of the model parameters.

Keywords: Autoregressive distributed lag model, error correction model, unit root tests, residual diagnostics, bounds cointegration test, stability tests

1 INTRODUCTION

1.1 Background of the study

The first case of a COVID-19 infection in Tamil Nadu, India, was identified on 7^{th} March 2020. Tamil Nadu ranks fifth in terms of states with the highest number of confirmed cases in India, after Maharashtra, Karnataka, Andhra Pradesh and Kerala. All 37 districts in Tamil Nadu have been affected by the COVID-19 epidemic, with the capital district of Chennai being the most heavily affected region.

The initial increase in cases in Tamil Nadu was considered to be due to the cluster of cases linked to the Tablighi Jamaat religious congregation that took place in Delhi in early April 2020. Koyambedu in Chennai was identified as another heavily affected place that witnessed a surge in May 2020.

To understand the disease dynamics and to make appropriate decisions to control the disease, knowledge of the number of new COVID-19 cases and the number of deaths due to COVID-19 infection as well as an estimation of the long-run equilibrium relationship between cases and deaths are essential for decision makers.

Jiang et al. [1] established a time-series-based kinetic model for infectious diseases and obtained trends and short-term predictions for the transmission of COVID-19. Al-Rousan and Al-Najjar [2] analysed the effect of various factors, such as sex, region,infection mode and birth year, on recovered and deceased cases in the South Korean region. Gondauri et al. [3] considered a chain-binomial type of Bailey's model for studying and analysing the correlation between the total numbers of COVID-19 cases and recoveries in different countries. Most of the studies investigated COVID-19 cases based on various regression and time-series models because these models are frequently applied to examine the growth or trend of diseases. This paper has the following objectives.

1.2 Objectives of the present study

The main objectives of the present study are to investigate the short- and long-run cointegration relations between the cumulative number of new COVID-19 cases and the cumulative number of deaths due to COVID-19, to estimate the

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long-run equilibrium relationship between these using an Autoregressive Distributed Lag model (ARDL) and bounds cointegration tests and to study the stability of the model.

1.3 Literature review

Granger [4] demonstrated that causal relations among variables could be examined within the framework of an error correction model (ECM) with cointegrated variables. While the short-run dynamics are captured by the individual coefficients on the lagged terms, the error correction term (ECT) contains information on long-run causality. The significance of the lagged explanatory variable identifies short-run causality, while a negative and statistically significant ECT signifies long-run causality.

Pesaran and Pesaran [5] suggested applying the cumulative sum of recursive residuals (CUSUM) and the CUSUM squares (CUSUMSQ) tests once the ECM is estimated to assess parameter consistency.

Alimi [6] investigated the relationship between expected inflation and nominal interest rates in Nigeria and the extent to which the Fisher effect held for the period 1970-2021. He applied ARDL bounds testing and vector error correction (VECM), and the stability of the function was also tested by the CUSUM and CUSUMSQ tests. Finally, the CUSUM test confirmed the long-run relationship between the variables and showed the stability of the coefficients.

Moawad [7] examined the relationship between the development of the financial sector and economic growth in France and Malaysia to verify the relations' existence and to determine its direction using the ARDL. The main finding of this study was the existence of a long-term relationship between the development of the financial sector and economic growth in the two countries studied.

Agiwal et al. [8] stated that most economic time series, such as GDP, real exchange rate and banking series, were irregular by nature, as they might be affected by a variety of discrepancies, including political changes, policy reforms, and import-export market instability, and such changes entailed serious consequences for time-series modelling. Various researchers managed this problem by applying a structural break. Having this idea as the main aim of this paper was to develop a generalised structural break time-series model. The paper discussed a panel autoregressive model with multiple breaks present in all parameters, i.e., in the autoregressive coefficient and mean and error variance, which was a generalisation of various submodels. The Bayesian approach was applied to estimate the model parameters and obtained the highest posterior density interval. Strong evidence was observed to support the Bayes estimator, and then it was compared with the maximum likelihood estimator. A simulation experiment was conducted, and an empirical application on the SARRC association's GDP per capita time series was used to illustrate the performance of the proposed model. This model was also extended to a temporary shift model.

Liu et al. [9] stated in their paper that COVID-19 was declared a pandemic by the World Health Organization (WHO) on 11 March 2020 and studied the dynamics of this epidemic using a generalised logistic function model and extended compartmental models with and without delays. For a chosen population, it was shown how forecasting was done on the spreading of the infection by using a generalised logistic function model, which could be interpreted as a basic compartmental model. In an extended compartmental model, which was a modified form of the SEIQR model, the population was divided into susceptible, exposed, infectious, quarantined, and removed (recovered or dead) compartments, and a set of delay integral equations was used to describe the system dynamics. Time-varying infection rates were allowed in the model to capture the responses to control measures taken, and distributed delay distributions were used to capture variability in individual responses to an infection. The constructed extended compartmental model was a nonlinear dynamical system with distributed delays and time-varying parameters. The critical role of the data was elucidated, and it was discussed how the compartmental model could be used to capture responses to various measures, including quarantining. Data for different parts of the world were considered, and comparisons were made in terms of the reproductive number. The obtained results could be useful for furthering the understanding of disease dynamics as well as for planning purposes.

Adiga et al. [10] stated that the COVID-19 pandemic represents an unprecedented global health crisis, unlike any the world has experienced in the last 100 years. The economic, social and health impact of the pandemic continues to grow, and it is likely to end up as one of the worst global disasters since the 1918 pandemic and the two world wars. Mathematical models have played an important role in the ongoing crisis; they have been used to inform public policies and have been instrumental in many of the social distancing measures that were instituted worldwide. In this article, they reviewed some of the important mathematical models used to support ongoing planning and response efforts. These models differ in their use, mathematical form and scope.

Atangana and Araz [11], by using the existing collected data from European and African countries, presented a statistical analysis of forecasting the future number of daily deaths and infections up to 10 September 2020. Additionally, they presented numerous statistical analyses of collected data from both continents using numerous existing statistical theories. The predictions showed the possibility of the second wave of infection in Europe in the worst scenario and exponential growth in the number of infections in Africa. The projection of statistical analysis leads us to introduce an extended version of the well-balanced function to further capture the spread with fractal properties. A mathematical model depicting the spread with nine subclasses was considered, first converted to a stochastic system, where the existence and uniqueness were presented. Then, the model was extended to the concept of nonlocal operators; due to nonlinearity, a modified numerical scheme was suggested and used to present the numerical simulations. The suggested mathematical model was able to predict two to three waves of the spread in the near future.

1.4 ARDL model

ARDL models have been prominently used in econometrics for decades, and this tool has become an immensely popular method for assessing cointegration relationships between variables due to the work of Pesaran and Shin [12] and Pesaran et al. [13]. ARDLs are standard least squares regressions that include lags of both the dependent and explanatory variables as regressors (Greene [14]). In particular, if y_t is the dependent variable and $x_1, x_2, x_3 ... x_k$are k explanatory variables, a general ARDL$(p, q_1, q_2, q_3 ... q_k)$model is given by

$$y_t = \alpha_0 + \alpha_1 t + \sum_{i=1}^{p} w_i y_{t-1} + \sum_{j=1}^{k} \beta_{j,l_j} x_{j,t-1_j} + \varepsilon_t \tag{1}$$

where ε_t are the usual innovations, a_0 is the regression constant, and α_1, w_i and β_{j,l_j} are the coefficients associated with a linear trend, with lags of y_t, and with lags of the k regressors $x_{j,t}$ for j=1,2,...,k,respectively. An ARDL(p,q) model has p lags of the dependent variable and q lags of the independent variable:

$$y_t = \beta_0 + \beta_{t-1} + ... + \beta_p y_{t-p} + \alpha_0 x_1 + \alpha_1 x_{t-1} + \alpha_2 x_{t-2} + \alpha_q x_{t-q} + \mu_1 \tag{2}$$

where μ_t is a random "disturbance" term. the model is "autoregressive" in the sense that y_t is "explained" (in part) by lagged values of itself. it also has a "distributed lag" component in the form of successive lags of the successive lags of the "x" explanatory variable. Sometimes, the current value of x_t itself is excluded from the distributed lag part of the model's structure (Soharwardi, Khan, and Mushtaq [15]).

2 MATERIALS AND METHODS

2.1 Materials

The cumulative total number of COVID-19 infections and deaths as of 20^{th}January 2021, starting on 9^{th} March 2020, were collected from the official website maintained by the Tamil Nadu State Government, India. Several statistical methodologies were used to achieve the objectives of the present study. Here, the cumulative number of new COVID-19 cases is denoted by X (independent variable), and the cumulative total number of deaths due to COVID-19 is denoted by Y, which are the study variables. EViews Ver. 11 software was used to estimate the model parameters and error diagnostics and to study the stability of the estimated model.

2.2 Methods

To apply the ARDL model, the study variables should fulfil certain stationarity conditions. That is, the variables should be purely I(0), purely I(1) or I(0)/I(1) Alimi [6]. To test this, three different tests, viz., the Dickey and Fuller [16], Phillips and Perron [17] and Kwiatkowski et al. [18] tests, were used. The Akaike information criterion (AIC) was used to select the optimal lag. To test the normality of the residual, the Jarque-Bera test [19] is used. For testing for autocorrelation and serial correlation, the Ljung-Box test (Ljung and Box [20]) and the Breusch-Godfrey test (Breusch [21]; Godfrey [22]), respectively, were used. To test for heteroscedasticity, the Breusch-Pagan-Godfrey heteroscedasticity test (Godfrey [22]; Breusch and Pagan [23]) was used. Model stability was studied based on the CUSUM and CUSUMSQ tests (Brown et al. [24]). Finally, to test the cointegration (long-run relationship), the bounds test (Pesaran et al. [13]) was employed. Details of these methods have been omitted in this paper and are available extensively in the literature.

3 RESULTS AND DISCUSSION

In this section, we provide the empirical findings and their interpretations in sequence.

3.1 Unit root test

The results presented in Tables 1 and 2 reveal that since the p-values of the ADF and PP tests indicate a high level of significance ($p < 0.0000$), both the study variables, X and Y, are found to be stationary without differencing and are hence of order I(0).

As the KPSS statistics are non significant, the study variables, X and Y, are stationary without differencing.

Table 1: Results of the unit root tests (ADF & PP tests)(H_0: has a unit root test

Variables	Augmented Dickey-Fuller test		Phillips-Perron test	
	Level		Level	
	Intercept	Intercept Trend	Intercept	Intercept Trend
X	-4.3853** (0.0013)	-4.71353** (0.0030)	-4.7416** (0.0005)	-5.0926** (0.0011)
Y	-4.7416** (0.0005)	-5.09259** (0.0011)	-4.7588** (0.0005)	-5.0926** (0.0011)
	First Difference		First Difference	
X	-11.7059** (0.0000)	-13.0362** (0.0000)	-22.7091** (0.0001)	-23.4276** (0.0000)
Y	-12.0045** (0.0000)	-13.2693** (0.0000)	-25.7396** (0.0001)	-25.7634** (0.0000)
** 1% level of significance; figures in parentheses represent p-values.				

Table 2: Results of the unit root test (KPSS test) (H_0: variable is stationary

	Level	
	Intercept	Intercept Trend
X	0.2946 (0.73900)	0.1220 (0.2160)
Y	0.3465 (0.7390)	0.1181 (0.2160)
	First Difference	
X	0.5000 (0.7390)	0.5000 (0.2160)
Y	0.5000 (0.7390)	0.5000 (0.2160)
Figures in parentheses () represent p-values.		

3.2 Summary statistics

Figure 1 depicts the cumulative number of COVID-19-positive patients in different districts in Tamil Nadu through 20^{th} January 2021. The largest number of new COVID-19 infections was registered in Chennai (229,537), and the lowest registration was in Perumbalur (2,260). The overall cumulative number of cases in Tamil Nadu as of 20^{th} January 2021 was 830,015.

Figure 2 depicts the cumulative number of deaths due to COVID-19 in different districts in Tamil Nadu through 20^{th} January 2021. The largest number of deaths due to COVID-19 infections was registered in Chennai (4,076), and the lowest registration was in Perumbalur (21). The overall cumulative number of deaths due to COVID-19 in Tamil Nadu as of 20^{th} January 2021 was 12,288.

Figure 3 depicts the cumulative death rate due to COVID-19 in different districts in Tamil Nadu through 20^{th} January 2021. The highest death rates due to COVID-19 were registered in Madurai (2.19%) and Ramanathapuram (2.14%), and the lowest registration was in Nilgiris (2.1%). The overall cumulative death rate due to COVID-19 in Tamil Nadu as of 20^{th} January 2021 was nearly 2% .

3.3 Model selection

To choose the optimal lag values, p and q, AIC was calculated for the different values of p and q. The lower the AIC values, the better the lag values for p and q. Figure 4 illustrates that the AIC value is extremely low for lags p=2 and q=0. Accordingly, the ARDL (2,0) model is found to be the best among the 20 models investigated with different lag values.

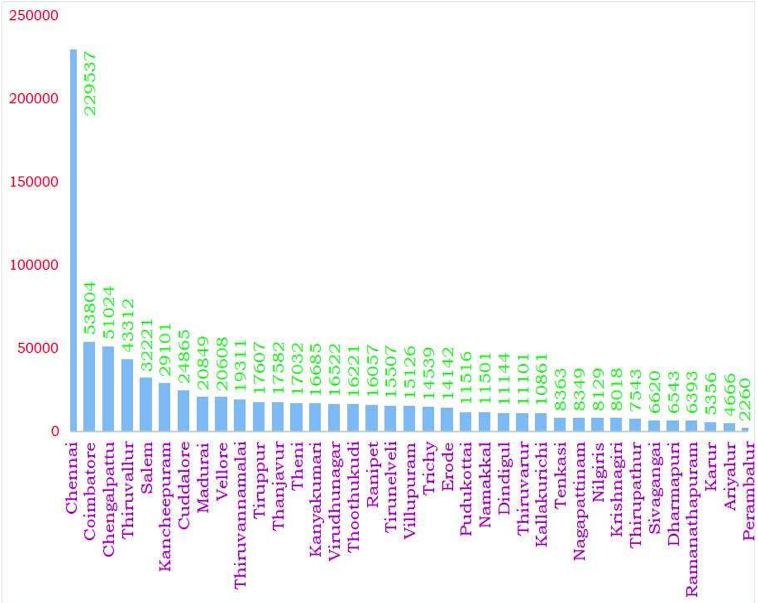

Fig. 1: Number of COVID-19 infections in all 37 district of Tamil Nadu, India.

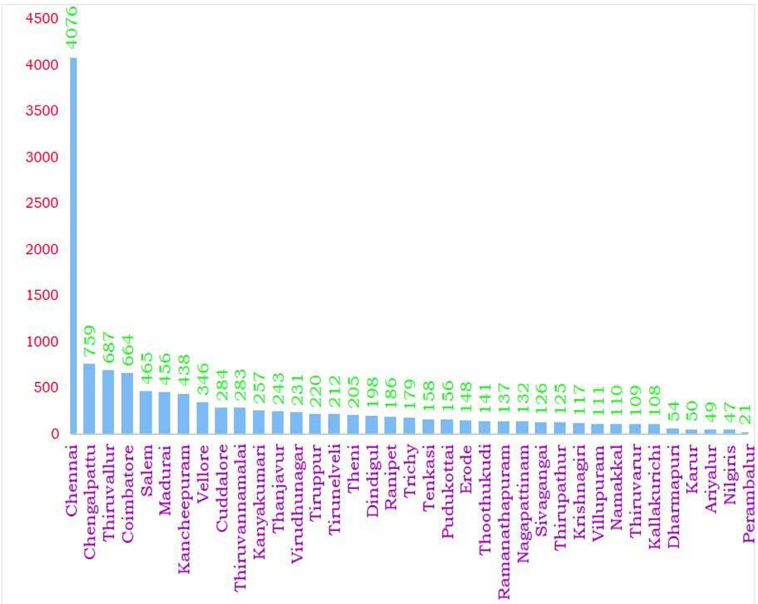

Fig. 2: Number of deaths due to COVID-19 in all 37 district of Tamil Nadu, India.

3.4 ARDL(2.0)Model

The ARDL (p=2, q=0) model was employed to study the short-run relationship between the cumulative number of COVID-19 cases, X, and the cumulative number of deaths due to COVID-19, Y. The findings are reported in Table 3. The results reveal that the overall goodness of fit of the model, as shown by the coefficient of determination, $R^2 = 99\%$, is extremely high and highly significant, implying that almost 99% of the variation in the dependent variable is explained by the model and the rest is explained by the error term. The value of the D-W statistic is nearly equal to two, which confirms that there are no spurious results. The estimated model is

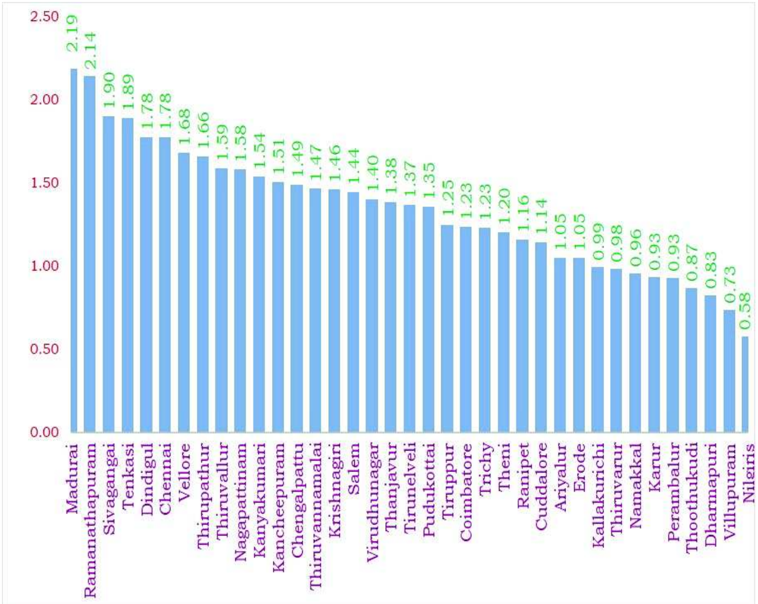

Fig. 3: Death rate due to COVID-19 in all 37 districts of Tamil Nadu, India.

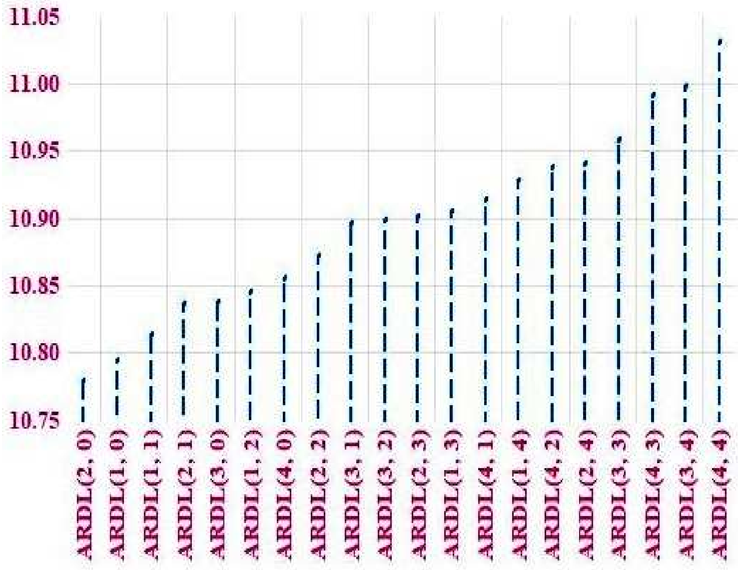

Fig. 4: Selection of the appropriate model based on the AIC

$$y_t = -44.08078^{**} - 0.057429^{**} y_{t-1} - 0.021385 y_{t-2} + 0.018039^{**} x_1 \tag{3}$$

(** indicates $p < 0.0000$)

where y_t is the number of deaths due to COVID-19 at time t and x_t is the number of COVID-19-infected new cases, which is highly significant, indicating that if the number of COVID-19-infected new cases increases by one unit, the death rate will be increased by 2%. One lag period is also negative and highly significant, indicating the decrease in trends in death due to COVID-19.

Table 3: Results of the estimated ARDL(2,0) Model

Variable	Coefficient	Std. Error	t-Statistic	Prob.*
Y(-1)	-0.057429	0.014209	-4.041756	0.0003
Y(-2)	-0.021385	0.013647	-1.567008	0.1273
X	0.018039	0.000250	72.26581	0.0000
C	-44.08078	11.54557	-3.817983	0.0006
R-squared	0.994436	Mean dependent var		328.0000
Adjusted R-squared	0.993898	S.D. dependent var		670.9483
S.E. of regression	52.41287	Akaike info criterion		10.86339
F-statistic	1846.872	Durbin-Watson stat		1.892714
Prob(F-statistic)	0.000000			

Table 4: Results of the autocorrelation test

Autocorrelation	Partial Correlation		AC	PAC	Q-Stat	Prob*
. |. |	. |. |	1	0.050	0.050	0.0942	0.759
.*|. |	.*|. |	2	-0.166	-0.169	1.1715	0.557
. |. |	. |. |	3	0.023	0.043	1.1937	0.755
. |. |	. |. |	4	0.009	-0.023	1.1973	0.879
. |*. |	. |*. |	5	0.127	0.144	1.8927	0.864
. |. |	. |. |	6	0.032	0.012	1.9395	0.925
.*|. |	.*|. |	7	-0.158	-0.120	3.0931	0.876
. |. |	. |. |	8	-0.048	-0.036	3.2047	0.921
. |** |	. |*. |	9	0.216	0.188	5.5271	0.786
. |. |	. |. |	10	0.004	-0.042	5.5280	0.853
. |. |	. |. |	11	-0.010	0.053	5.5339	0.903
. |. |	. |. |	12	0.010	0.015	5.5400	0.937
. |. |	. |. |	13	-0.055	-0.030	5.7198	0.956
. |. |	.*|. |	14	-0.033	-0.098	5.7864	0.972
. |. |	. |. |	15	0.042	0.028	5.8997	0.981
.*|. |	.*|. |	16	-0.126	-0.115	6.9884	0.973

Table 5: Results of the Breusch-Godfrey serial-correlation LM test of the residuals

F-statistic	0.513799	Prob. F(2,29)	0.6036
Obs*R-squared	1.197763	Prob. Chi-Square(2)	0.5494

3.5 Test for Normality of the residuals

The errors are normally distributed, since the Jarque-Bera test statistic's value is non-significant (p=0.670828). To ensure the consistency of the ARDL (2,0) model, the following residual diagnostic tests are carried out.

3.6 Ljung-Box test for autocorrelation

The results of the Ljung-Box autocorrelation test (Ljung and Box [20]) presented in Table 4 indicate that the p-values of the Q statistics are clearly greater than 0.05 and strongly suggest the absence of autocorrelation in the model error.

3.7 Breusch-Godfrey serial correlation LM test (Breusch [21]; Godfrey [22])

Usually, when an analysis involves time-series data, the possibility of autocorrelation is high. Therefore, it is necessary to test the residuals for autocorrelation using the Breusch-Godfrey LM test. The results presented in Table 5 reveal that the null hypothesis of no serial correlation can be accepted since the p-value for the test is greater than 0.05, and hence, there is no serial correlation.

Table 6: Characteristics of the Breusch-Pagan-Godfrey heteroscedasticity test.

F-statistic	0.532243	Prob. F(3,31)	0.6636
Obs*R-squared	1.714453	Prob. Chi-Square(3)	0.6337

3.8 Breusch-Pagan-Godfrey heteroscedasticity test

To ensure consistency, the study further employed the Breusch-Pagan-Godfrey heteroscedasticity test, and the results are presented in Table 6. The results reveal that the null hypothesis of no heteroscedasticity is accepted, as the test is non-significant (the p-value is greater than 5%).

3.9 Actual and Fit of the model

The actual and fitted plot (Figure 5) of the ARDL (2,0) model shows that the fit of the model is good enough in terms of explaining the cumulative total deaths Actual and fitted forecast.

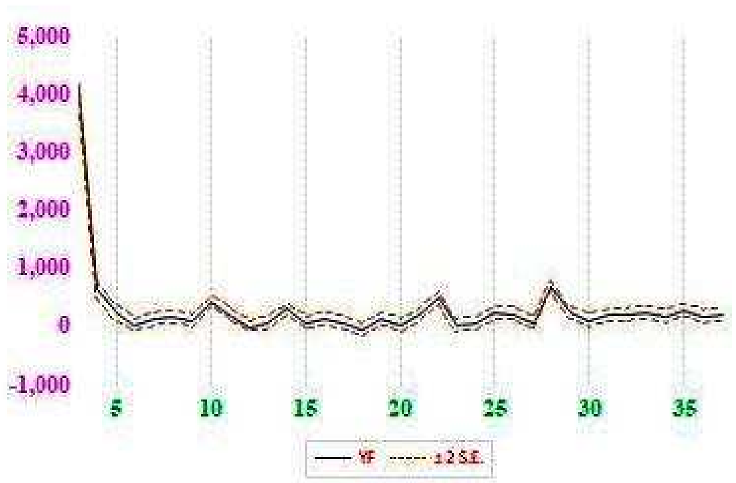

Fig. 5: Actual and fitted forecast plot.

3.10 Model stability

To check the robustness of our results, structural stability tests of the parameters of the long-run results are performed by the CUSUM and CUSUMSQ tests (Brown et al. [24]). This same procedure has been utilised by Pesaran and Pesaran [5] and Mohsen et al. [25] to test the stability of long-run coefficients. A graphical representation of the CUSUM and CUSUMSQ statistics is shown in Figures 6 and 7, respectively. The plots of both the CUSUM and CUSUMSQ are within the boundaries (shown by the dotted red lines) of the 5% significance level, and these statistics confirm the model's stability.

Fig. 6: CUSUM stability test.

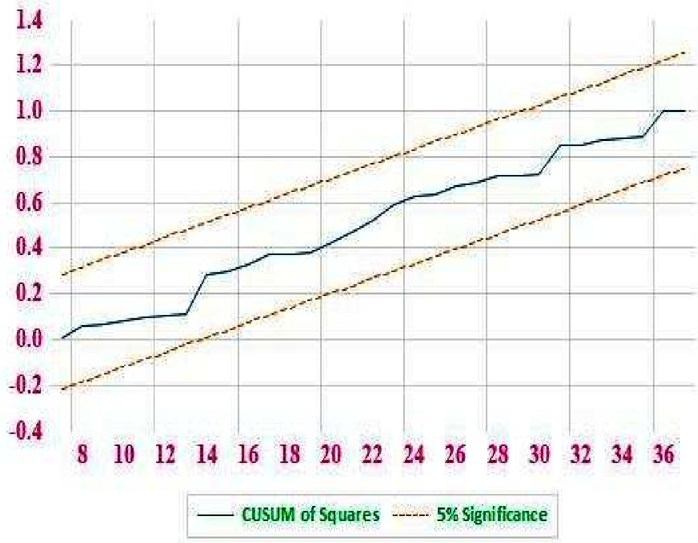

Fig. 7: CUSUMSQ stability test.

3.11 Bounds test for co integration

The bounds test developed by Pesaran et al. [13] is employed to test the cointegration (long-run relationship) between the study variables X and Y and is presented in Table 7. The test results reveal that there exists a cointegration relationship between X and Y, as the bounds test statistic is greater than the upper bound from I(1) (F-statistics = 2592.07 > 5.584), and it is highly significant, which implies the possibility of a log-run relationship between the study variables X and Y.

The results presented in Table 8 and 9 reveal that the long-run relationship is Y = 0.0167** X – 40.8604. There is a positive, significant long-run relationship between X and Y. A one-unit increase in the cumulative number of daily confirmed COVID-19 cases leads to an increase in the cumulative number of deaths due to COVID-19 by 2%. The term

Table 7: ARDL bounds test and long-run dynamics

Model Specification	F-Statistic	k	Critical Value Bounds				Conclusion
			Lower Bound I(0)		Upper Bound I(1)		
ARDL(2,0)	2592.07**	1	1%	4.94	1%	5.58	Cointegrated
			5%	3.62	5%	4.16	
			10%	3.02	10%	3.51	
** Indicates significance at the 1% level.							

Table 8: Conditional Error Correction Regression

Conditional Error Correction Regression				
Variable	Coefficient	Std. Error	t-Statistic	Prob.
C	-44.08078	11.54557	-3.817983	0.0006
ECM(-1)	-1.078814	0.017403	-61.99016	0.0000
X**	0.018039	0.000250	72.26581	0.0000
D(Y(-1))	0.021385	0.013647	1.567008	0.1273
** Variable interpreted as Z = Z(-1) + D(Z).				

Table 9: Levels Equation

Levels Equation				
Case 2: Restricted Constant and No Trend				
Variable	Coefficient	Std. Error	t-Statistic	Prob.
X	0.016721	0.000324	51.55048	0.0000
C	-40.86042	11.00489	-3.712932	0.0008
EC = Y - (0.0167*X - 40.8604)				

ECM (-1) corresponds to the lagged error from the long-term equilibrium equation. The coefficient expresses the degree to which the variable Y regresses towards the long-term target. It is negative (-1.07881) and highly significant, thus reflecting a relatively quick adjustment towards the long-term target.

4 CONCLUSION

The results of this study reveal that the largest number of new COVID-19 infections was registered in Chennai (229,537), and the lowest number of new infections was reported in Perumbalur (2,260). The overall cumulative number of cases in Tamil Nadu as of 20^{th} January 2021 was 830,015. The largest number of deaths due to COVID-19 infections was registered in Chennai (4,076), and the lowest registration was in Perumbalur (21). The overall cumulative number of deaths due to COVID-19 in Tamil Nadu as of 20^{th} January 2021 was 12,288. The highest death rates due to COVID-19 were registered in Madurai (2.19%) and Ramanathapuram (2.14%), and the lowest death rate was in Nilgiris (2.1%). The overall cumulative death rate due to COVID-19 in Tamil Nadu as of 20^{th} January 2021 was nearly 2%. The ARDL (p = 2, q = 0) model is highly significant, and the value of the coefficient of determination, R^2 = 99%, implies that almost 99% of the variation in the dependent variable is explained by the model and that the rest is explained by the error term. The value of the D-W statistic is nearly equal to two, which confirms that there are no spurious results. The bounds test results reveal that a log-run relationship between the study variables exists. The error correction term is negative and highly significant, reflecting a relatively quick adjustment to the long-term target.

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Conflict of Interest

The authors declare that they have no conflict of interest.

References

[1] X. Jiang, B. Zhao and J. Cao, Statistical Analysis on COVID-19, *Bio-medical Journal of Scientific and Technical Research*, **26**, 19716-19727 (2020).
[2] N. Al-Rousan and H. Al-Najjar, Data Analysis of Coronavirus COVID-19 Epidemic in South Korea based on Recovered and Death Cases, *Journal of Medical Virology*, **92**, 1603-1608 (2020), https://doi.org/10.1002/jmv.25850.
[3] D. Gondauri, E. Mikautadze and M. Batiashvili, Research on COVID-19 Virus Spreading Statistics based on the Examples of the Cases from Different Countries, *Electron Journal of General Medicine*, **17**, 1-4 (2020), DOI: 10.29333/ejgm/7869.
[4] C.W.J. Granger, Some Recent Development in A Concept of Causality, *Journal of Econometrics*, **39**, 7-21 (1988).
[5] M.H. Pesaran and B. Pesaran, Working with Microfit 4.0: *Interactive Econometric Analysis*, Oxford University Press, Oxford, UK (1997).
[6] R.S. Alimi, ARDL Bounds Testing Approach to Cointegration: A Re-Examination of Augmented Fisher Hypothesis in an Open Economy, *Asian Journal of Economic Modelling*, **2**, 103-114 (2014).
[7] R.R. Moawad, Financial Development and Economic Growth: ARDL Model, *International Multilingual Journal of Science and Technology*, **4**, 625-632 (2019).
[8] V. Agiwal, J. Kumar and K. Shangodoyin, A Bayesian Analysis of Complete Multiple Breaks in A Panel Autoregressive (CMB-PAR(1)) Time Series Model, *Statistics in Transition New Series*, **21**, 133-149 (2020) ,DOI: 10.21307/stattrans-2020-059.
[9] X. Liu, X. Zheng and B. Balachandran,COVID-19: Data-Driven Dynamics, Statistical and Distributed Delay Models, and Observations, *Nonlinear Dynamics*, **101**, 1527-1543 (2020), https://doi.org/10.1007/s11071-020-05863-5.
[10] A. Adiga,D. Dubhashi,B. Lewis,M. Marathe, S. Venkatramanan and A. Vullikanti Mathematical Models for COVID-19 Pandemic: A Comparative Analysis, *Journal of the Indian Institute of Science*, **100**, 793-807 (2020), https://doi.org/10.1007/s41745-020-00200-6.
[11] A. Atangana, and S.I. Araz, Modeling and Forecasting the Spread of COVID-19 with Stochastic and Deterministic Approaches: Africa and Europe, *Advances in Difference Equations*, **57** (2021), https://doi.org/10.1186/s13662-021-03213-2.
[12] M.H. Pesaran and Y. Shin, An Autoregressive Distributed-Lag Modelling Approach to Cointegration Analysis, *Econometrics and Economic Theory in the 20th Century: The Ragnar Frisch Centennial Symposium*, **31**, 371-413 (1998). http://dx.doi.org/10.1017/CCOL0521633230.011.
[13] M.H. Pesaran, Y. Shin and R.J. Smith, Bounds Testing Approaches to the Analysis of Level Relationships, *Journal of Applied Economics*, **16**, 289-326 (2001).
[14] W.H. Greene,*Econometric Analysis*, Pearson Prentice Hall, Upper Saddle River, NJ (2008).
[15] M.A. Soharwardi, R.E.A. Khan and S. Mushtaq, Long-run and short-run relationship between financial development and income inequality in Pakistan. *Journal of ISOSS*, **4(2)**,105-112 (2018).
[16] D.A. Dickey and W.A. Fuller, Distribution of the Estimators for Autoregressive Time Series with a Unit Root, *Journal of the American Statistical Association*, **74**, 427-431 (1979).
[17] P.C.B. Phillips and P. Perron, Testing for a Unit Root in Time Series Regression, *Biometrika*, **75**, 335-346 (1988).
[18] D. Kwiatkowski,P.C.B. Phillips, P.Schmidt and Y. Shin,, Testing the Null Hypothesis of Stationarity Against the Alternative of a Unit Root, *Journal of Econometrics*, **54**, 159-178 (1992).
[19] C.M. Jarque and A.K.Bera, Efficient Tests for Normality, Homoscedasticity and Serial Independence of Regression Residuals, *Economics Letters*, **6**, 255-259 (1980).
[20] G.M. Ljung and G.E.P.Box, The Likelihood Function of Stationary Autoregressive-Moving Average Models, *Biometrika*, **66**, 265-270 (1979).
[21] T.S. Breusch, Testing for Auto correlation in Dynamic Linear Models, *Australian Economic Papers*, **17**, 334-355 (1978).
[22] L.G. Godfrey, Testing Against General Autoregressive and Moving Average Error Models when the Regressors Include Lagged Dependent Variables, *Econometrica*, **46**, 1293-1301 (1978).
[23] T.S. Breusch and A.R.Pagan, A Simple Test for Heteroscedasticity and Random Coefficient Variation, *Econometrica*, **47**, 1287-1294 (1979).
[24] R.L. Brown,J. Durbin and J.M. Evans, Techniques for Testing the Constancy of Regression Relationships over Time, *Journal of the Royal Statistical Society. Series B (Methodological)*, **37**, 149-192 (1975).
[25] Mohsen, Bahmani-Oskooee and R.W.Ng, Long run demand for money in Hong Kong : An application of the ARDL model, *International Journal of Business and Economics*, **1(2)**, 147-155 (2002).