Information Sciences Letters

Volume 11
Issue 4 *Jul. 2022*

Article 25

2022

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Ajjaz Maqbool
School of Mathematics, 542001 Indore, Devi Ahliya Vishwavidyalaya, (MP) India, ajjazmaqbool013@gmail.com

Chitranjan Sharma
Department of Mathematics, Holker science college Indore (MP) India, ajjazmaqbool013@gmail.com

Mushtaq.A Lone
Department of statistics, SKUAST-K, India, ajjazmaqbool013@gmail.com

Riyad Alshalabi
Applied Science University, East Al-Ekir., Kingdom of Bahrain, ajjazmaqbool013@gmail.com

Information Sciences Letters
An International Journal

http://dx.doi.org/10.18576/isl/110425

Intuitionistic Fuzzy Programming Technique to Solve Multi-Objective Transportation Problem

Ajjaz Maqbool[1,*] *, Chitranjan Sharma*[2]*, Mushtaq.A Lone*[3]*, and Riyad Alshalabi* [4]

[1]School of Mathematics, 542001 Indore, Devi Ahliya Vishwavidyalaya, (MP) India
[2]Department of Mathematics, Holker science college Indore (MP) India
[3]Department of statistics, SKUAST-K, India
[4]Applied Science University, East Al-Ekir., Kingdom of Bahrain

Received: 12 Feb. 2022, Revised: 22 Mar. 2022, Accepted: 14 May 2022.
Published online:1 Jul. 2022.

Abstract: This paper presents an explanation of the multi-objective transportation problem (MOTP) via Fuzzy programming algorithm and the goods is to be transported from origin to destination. The time and cost of transportation from origin i to destination j were recorded. Here we have considered MOTP with intuitionistic fuzzy numbers and completed the problem in both ways. Therefore, the optimal compromise solution will remain same both the exponential and linear membership function. For the solution the membership functions are used for such a problem. LINDO statistical software was used in the present facts analysis and is completed in two stages.

Keywords: Multi-objective-transportation problem, membership function, Fuzzy programing Technique.

1 Introduction

There are various practical applications of Transportation Problem which may be treated as special type of Linear programming problem.TP is one of the powerful frame wor k which ensures efficient movement and timely availability of the raw material. It is applicable in various real life human activity and is one of the best optimization method. At early it was observed that for determining the optimal shipping pattern, it is called transportation problem. So it deals with transportation of different types of goods from each of m origins $i = 1,2,3, \dots m$ to any of n destinations $j = 1,2,3, \dots n$. One must determine the amount x_{ij} to be transported from all the origin $i = 1,2,3, \dots m$ to all the destinations $j = 1,2,3, \dots n$ in such a way that the minimization of total cost . Every transportation problem is not single objective which are characterised by multi-objective function are considered here. A different type of linear programming problem were constraints are of equality type and all the objectives are conflicting with each other, are called MOTP. The multi-objective fuzzy linear programming technique, were the objectives are fuzzy in nature. The fuzzy linear programming technique for the multi-objective transportation problem gives an optimal compromise solution.

Diaz [1, 2] worked for finding the solution of multi-objective transportation problem by developed an algorithm. Isermann [3] also developed an algorithm for identifying all the non-dominated solution, for a linear multi-objective transportation problem. Ringuest and Rinks [4] developed two interactive algorithms for solving multi-objective transportation problem. Zimmermann [5] first applied suitable membership functions to solve linear programming problem with several objective functions. He showed that solutions obtained by fuzzy linear programmin g are always efficient. Bit et al. [6] applied the fuzzy progra mming technique with linear membership function to solve the multi-objective transportation problem. Hitchcock [7] studied and modelled simple transportation problem in shape of standard linear programming problem. In beginning of decision making parameters of MOTP are assumed to be fixed in values. But due many unsure situations like road conditions, traffic conditions, variation in diesel charges and many others. And a few different unpredicted elements like weather circumstance. Therefore, due to those reason conventional models are not valid. Zadeh [8] delivered the concept of fuzzy sets for dealing uncertainty. Later Bellman and Zadeh [9] used it for decision making of actual life issues. Verma et al. [10] carried out fuzzy programming approach to solve MOTP through a few non-linear membership functions. Das et al. [11] proposed solution techniques for the solution of MOT P with interval cost, source and destinations parameters. Li and Lai [12] have presented a fuzzy compromise programm ing approach to multi objective transportation problem. Wa hed [13] initiated the optimal compromise solution of MOT P and obtained solution is tested by using degree of closeness of the compromise solution to the ideal solution using family of distances. Leberling [14] used a special-

*Corresponding author e-mail: ajjazmaqbool013@gmail.com

kind nonlinear membership function for the vector maximum linear programming problem. He confirmed that the solution obtained by using fuzzy linear programming with this form of nonlinear membership function are always efficient. Dhingra and H. Moskowitz [15] defined other forms of the non-linear (exponential, quadratic and logarithmic) membership functions and carried out them to an optimal design problem. Verma, Biswal and Biswas [16] used the fuzzy programming technique with a few non-linear (hyperbolic and exponential) membership functions to solve a multi-objective transportation problem are always efficient. Recently. Recently, Antony et al [17] proposed method for the solution of transportation problem using triangular intuitionistic fuzzy numbers. Fuzzy programming technique used by Lone *et. al* [18] in willow wicker cultivation and Intuitionistic Fuzzy programming techniques was further discussed by several authors. Yeola and Jahav [19] proposed a method for solving MOTP by using fuzzy linear membership functions for different costs and the method is parallel to New Row Maxima Method. Boualam et al. [20] worked on field of maximum power point tracking methods for PV sydtem by using fuzzy logic controller. Usha Rani and Depak [21] study about the optimal agriculture production planning through land allocation by using FMOLP. Nemat Allah et al [22] they studied facility location problem with point and area destinations is a special kind of location problems. they also proved three fuzzy theorems which showed that the objective function is separable, convex and piece wise continuous. Further work has been done in the field of IFTP of type 2 by Singh and Yadav [23]. Kumar and Hussain [24] developed a method for real transportation based on intuitionistic fuzzy programming problem based on ranking method.

2 Mathematical Models

In this section, we use fuzzy programming technique to solve MOTP taking a membership and exponential function .A variable x_{ij} represents the quantity to be transported from origin $O_i$to destination D_j.A multi objective transporta tion problem may be stated mathematically

Maximize

$$Z_k = \sum_i^m \sum_j^n c_{ij}\, x_{ij},\ \ k = 1,2,\ldots K. \tag{1}$$

Subject to

$$\sum_j^n x_{ij} = a_i, i = 1,2,3..m, \tag{2}$$

$$\sum_i^m x_{ij} = b_j, j = 1,2,3..n. \tag{3}$$

$$x_{ij} \geq 0, for\ all\ i, j$$

Where the subscript on Z_k and superscript C_{ij}^k, denoted the K^{th} penalties.

Criterion: We assume that

$a_i > 0\ for\ all\ i, b_i > 0, for\ all\ j, C_{ij} > 0, for\ all\ i\ and\ j.$

$\sum_i^m a_i = \sum_j^n b_j$ (Equilibrium condition)

We treat equilibrium condition as iff condition for the existence of a feasible solution to the balanced condition lin ear transportation problem. A transportation problem has exactly mn variables $and\ m + n$ constraints.

3 Fuzzy Algorithm to Solve Multi-Objective Transportation Problem

Step 1: Construct the Fuzzy transportation problem.

Step 2: Solve multi-objective transportation problem k times taking one objective at a time derive corresponding values for every objective at each solution and we make payoff matrix as follows:

$$\begin{array}{c} \\ X_1 \\ X_2 \\ \vdots \\ X_k \end{array} \begin{array}{c} \begin{array}{cccc} Z_1 & Z_2 & \ldots & Z_K \end{array} \\ \begin{bmatrix} Z_{11} & Z_{12} & \ldots & Z_{1K} \\ Z_{21} & Z_{22} & \ldots & Z_{2K} \\ \vdots & \vdots & & \vdots \\ Z_{K1} & Z_{K2} & \ldots & Z_{KK} \end{bmatrix} \end{array}$$

where, $X^1, X^2, X^3, X^4 \ldots\ldots\ldots\ldots\ldots X^K$ are the isolated optimal solutions of the K different transpot ation problems for K different objective function $Z_{ij} = Z_j(X^i)$, $(i = 1,2,\ldots k\ \ and\ j = 1,2,\ldots k)$ be the ith row and jth column element of the pay-off matrix.

Step 3: From step 2, set upper and lower bounds for each objective for degree of acceptance and degree of rejection corresponding to the set of solution.

For membership functions: Upper and Lower bound for membership function.

$$U_K^\mu = Max(Z_K(X_r))$$
$$L_K^\mu = Min\left(Z_K(X_r)\right),\quad 0 \leq r \leq K$$

where the upper bound U_K^μ and the lower bound for L_K^μ for the K^{th} objective function Z_k,

$k = 1,2,\ldots k$, U_K^μ is the highest acceptable level of achievement for objective k, L_K^μ is the aspired level of achievement for objective $k$and

$d_k = U_k^\mu - L_k^\mu$,the degradation allowance for objective k.``

Step 4. Consider the membership function as following linear functions: -

$$\mu_k\{Z_k(X)\} = \begin{cases} 1, & L_k^\mu \geq Z_k(X) \\ 1 - \frac{\left(U_k^\mu - Z_1(X)\right)}{d_k}, & L_k^\mu \leq Z_k(X) \leq U_k^\mu; \\ 0, & Z_k(X) \geq U_k^\mu \end{cases}$$

$$d_k = U_k^\mu - L_k^\mu. \tag{4}$$

Step 5. We find crisp model by using a LMF for the initial fuzzy model

If we use a LMF, the crisp model can be simplified as

Minimize α

Such that

$$Z_k(X) - \alpha d_k \le L_k^{\mu},$$
$$\sum_j^n x_{ij} = a_i, \quad i = 1,2,3..m,$$
$$\sum_i^m x_{ij} = b_j, \quad j = 1,2,3..n.$$
$$x_{ij} \ge 0, for\ all\ i\ and\ \alpha \ge 0$$

Above linear programming problem can be solved by using LONDO/TORA statistical software.

Step 6. Solve the crisp model by an appropriate mathematical programming algorithm.

Minimize α

$C_{ij}^k x_{ij} - \alpha(d_{k)} \le L_k^{\mu}$, $k = 1,2, \ldots..K$,

Subject to

$\sum_j^n x_{ij} = a_i$, $i = 1,2,3..m$,

$\sum_i^m x_{ij} = b_j$, $j = 1,2,3 \ldots \ldots \ldots \ldots$

$x_{ij} \ge 0, for\ all\ i, j$

We use another membership function such as Hyperbolic Tangent function can be formulated as

Minimize α

$\alpha \ge \frac{1}{2} + \frac{1}{2}\tanh\{\frac{U_k+L_k}{2} + Z_k\}\tau_k$,

where $\tau_k = \frac{S}{U_k - L_k}$, where S is no. of constraints.

$$\sum_j^n x_{ij} = a_i \quad i = 1,2,3..m,$$
$$\sum_i^m x_{ij} = b_j, \quad j = 1,2,3..n. \quad (5)$$
$$x_{ij} \ge 0, for\ all\ i\ and\ \alpha \ge 0$$

Step 7. An intuitionistic fuzzy optimization for MOLP problem with such an Exponential membership function for the kth objective function is defined as

$$\mu_k^e\{Z_k(X)\} = \begin{cases} 1, & L_k^{\mu} \ge Z_k(X) \\ e^{-\frac{1}{2}\left(\frac{Z_k - L_k^{\mu}}{d_k}\right)}, & L_k^{\mu} \le Z_k(X) \le U_k^{\mu}; \\ 0, & Z_k(X) \ge U_k^{\mu} \end{cases}$$

$$d_k = U_k^{\mu} - L_k^{\mu} \quad (6)$$

Where, $k = 1,2,3, \ldots \ldots k$.

4 Data Analysis

This section shall discuss how fuzzy programming technique to solve MOTP using the algorithm. Study the total cost of transportation and time. The data for cost and time of supply of product from source to destination are written in the form of equations.

$$Maximize\ Z_1 = 1X_{11} + 2X_{12} + 7X_{13}\ 7X_{14} + X_{21} + 9X_{22} + 3X_{23} + 4X_{24} + 8X_{31} + 9X_{32} + 4X_{33} + 6X_{34}$$

$$Maximize\ Z_2 = 4\,X_{11} + 4X_{12} + 3X_{13} + 5X_{21} + 8X_{22} + 9X_{23} + 10X_{24} + 6X_{31} + 2X_{32} + 5X_{33} + 1X_{34}$$

Such that

$$X_{11} + X_{12} + X_{13} + X_{14} = 18$$
$$X_{21} + X_{22} + X_{23} + X_{24} = 19$$
$$X_{31} + X_{32} + X_{33} + X_{34} = 17$$
$$X_{11} + X_{21} + X_{31} = 11$$
$$X_{12} + X_{22} + X_{32} = 3$$
$$X_{13} + X_{23} + X_{33} = 14$$
$$X_{14} + X_{24} + X_{34} = 16$$
$$X_{ij} \ge 0, \quad i = 1,2,3\ and\ j = 1,2,3.$$

The optimal compromise solution of the problem is represented as

$X^1 = \{\ X_{13} = 14,\ X_{14} = 4, X_{22} = 3,\ X_{24} = 6, X_{31} = 11\ and\ X_{34} = 6\ \}\ and\ rest\ all\ x_{ij}\ are\ zeros.$

$Z_1(X_1) = 301$

The optimal solution of the problem is given by

$X^2 = \{\ X_{11} = 5, X_{12} = 3, X_{23} = 3, X_{24} = 16, X_{31} = 6, X_{33} = 11,\ \}\ and\ rest\ all\ x_{ij}\ are\ zeros.$

$Z_2(X_2) = 310$

Similarly $Z_1(X_2) = 176\ and\ Z_2(X_1) = 214.$

Pay -off table is

	Z1	Z2
X1	301	214
X2	176	310

From this we get $U_1^{\mu} = 301$, $U_2^{\mu} = 310$, $L_1^{\mu} = 176, and\ L_2^{\mu}$=214

The membership functions is given by

$$\mu_1\{Z_1(X)\} = \begin{cases} 1, & 176 \ge Z_1(X) \\ 1 - \frac{(301 - Z_1(X))}{d_{k1}}, & 176 \le Z_1(X) \le 301 \\ 0, & Z_1(X) \ge 301 \end{cases}$$

$$d_{k1} = 125$$

$$\mu_2\{Z_2(X)\} = \begin{cases} 1, & 214 \ge Z_2(X) \\ 1 - \frac{310 + Z_2(X)}{d_{k2}}, & 214 \le Z_2(X) \le 310 \\ 0, & Z_2(X) \ge 310 \end{cases}$$

$$d_{k2} = 96$$

We find an equivalent crisp model

Minimize α

$$Z_1(X) - 125a \le 176$$
$$Z_2(X) - 96a \le 214$$

Solve the crisp model

$1X_{11} + 2X_{12} + 7X_{13} + 7X_{14} + X_{21} + 9X_{22} + 3X_{23} + 4X_{24} + 8X_{31} + 9X_{32} + 4X_{33} + 6X_{34} - 125\alpha \leq 176$

$4X_{11} + 4X_{12} + 3X_{13} + 4X_{14} + 5X_{21} + 8X_{22} + 9X_{23} + 10X_{24} + 6X_{31} + 2X_{32} + 5X_{33} + 1X_{34} - 96\alpha \leq 214$

$X_{11} + X_{12} + X_{13} + X_{14} = 18$
$X_{21} + X_{22} + X_{23} + X_{24} = 19$
$X_{31} + X_{32} + X_{33} + X_{34} = 17$
$X_{11} + X_{21} + X_{31} = 11$
$X_{12} + X_{22} + X_{32} = 3$
$X_{13} + X_{23} + X_{33} = 14$
$X_{14} + X_{24} + X_{34} = 16$

The optimal compromise solution of the problem by using LINDO Software is represented as

$$\{X_{12} = 1.098, X_{13} = 3.90, X_{14} = 13, X_{21} = 11, X_{23} = 8, X_{32} = 1.90, X_{33} = 12, X_{34} = 3 \text{ and rest all } X_{ij} \text{ are zeros.}\}$$

$Z_1^* = 238.496$ $Z_2^* = 261.82$

$\alpha = 0.50$

If we use another membership function, then an equivalent crisp model for the fuzzy model can be further formulated as

Minimize α

$$\alpha \geq \frac{1}{2} + \frac{1}{2}\tanh\{\frac{U_k+L_k}{2} + Z_k\}\tau_k,$$

where $\tau_k = \frac{7}{U_k - L_k}$.

Which further implies that

$$\tau_k Z_k - \tanh^{-1}(2\alpha - 1) \leq \frac{U_k+L_k}{2}\tau_k$$

Minimize w

$\frac{7}{125}$ $(1X_{11} + 2X_{12} + 7X_{13} + 7X_{14} + X_{21} + 9X_{22} + 3X_{23} + 4X_{24} + 8X_{31} + 9X_{32} + 4X_{33} + 6X_{34}) - w \leq \frac{7}{125}(\frac{477}{2})$

$\frac{7}{96}$ $(4X_{11} + 4X_{12} + 3X_{13} + 4X_{14} + 5X_{21} + 8X_{22} + 9X_{23} + 10X_{24} + 6X_{31} + 2X_{32} + 5X_{33} + 1X_{34}) - w \leq \frac{7}{96}(\frac{524}{2})$

$X_{11} + X_{12} + X_{13} + X_{14} = 18$
$X_{21} + X_{22} + X_{23} + X_{24} = 19$
$X_{31} + X_{32} + X_{33} + X_{34} = 17$
$X_{11} + X_{21} + X_{31} = 11$
$X_{12} + X_{22} + X_{32} = 3$
$X_{13} + X_{23} + X_{33} = 14$
$X_{14} + X_{24} + X_{34} = 16$

Solving above problem the optimal solution is given by

$$\{X_{12} = 1.098, X_{13} = 3.90, X_{14} = 13, X_{21} = 11, X_{23} = 8, X_{32} = 1.90, X_{33} = 12, X_{34} = 3 \text{ and rest all } X_{ij} \text{ are zeros.}\}$$

$Z_1^* = 238.496$ $Z_2^* = 261.82$

$\alpha = 0.64$, Where $w = \tanh^{-1}(2\alpha - 1)$ and

$w = 0.29$

$$\mu_1^e\{Z_1(X)\} = \begin{cases} 1, & 176 \geq Z_1(X) \\ e^{-\frac{1}{2}\left(\frac{Z_1-176}{d_k}\right)} & 176 \leq Z_1(X) \leq 301 \\ 0, & Z_1(X) \geq 301 \end{cases}$$

$d_k = 125$

$$\mu_2^e\{Z_2(X)\} = \begin{cases} 1, & 214 \geq Z_1(X) \\ e^{-\frac{1}{2}\left(\frac{Z_2-214}{d_k}\right)} & 214 \leq Z_2(X) \leq 310 \\ 0, & Z_2(X) \geq 310 \end{cases}$$

$d_k = 96$

If we use the exponential membership functions, an equivalent crisp model can be formulated as

Minimize α

Subject to

$$\alpha \leq e^{-\frac{1}{2}\left(\frac{Z_1-176}{d_k}\right)} \text{ and } \alpha \leq e^{-\frac{1}{2}\left(\frac{Z_2-214}{d_k}\right)}$$

The problem is solved by LINDO

$$\{X_{12} = 1.098, X_{13} = 3.90, X_{14} = 13, X_{21} = 11, X_{23} = 8, X_{32} = 1.90, X_{33} = 12, X_{34} = 3 \text{ and rest all } X_{ij} \text{ are zeros.}\}$$

$\alpha = 0.60$

5 Conclusions

In this paper 3- types of functions are used to solve intuitionistic Fuzzy Multi-objective transportation Problem. Problem is completed in 3-steps, in first step which gives optimal compromise solution of the problem. If we use the second type of membership function the crisp model become linear. We use different exponential membership function again the optimal compromise solution does not change significantly. The method is very clear and easy to handle real life transportation problems because in some cases due to limited storage capacity, minimum quantity received by the destinations. In this Situation, a single shipment is not possible. Therefore, items are shipped to destinations from the origins into different stages. Initially, the minimum quantity may be shipped from origins to the destinations. The said method is based on intuitionistic fuzzy sets. Therefore, it will be easy to handle in real transportation problems.

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