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Enhancement of Atom-Field Transfer of Coherence in a Two-Photon Micromaser Assisted by a Classical Field

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Abstract: We investigate the transfer of coherence from atoms to a cavity field initially in a statistical mixture within a two-photon micromaser arrangement. The field is progressively modified from a maximum entropy state (thermal state) towards an almost pure state (entropy close to zero) due to its interaction with atoms sent across the cavity. We trace over the atomic variables, i.e., the atomic states are not measured and recorded by a detector after they leave the cavity. We show that by applying an external classical driving field it is possible to substantially increase the field purity without the need of previously preparing the atoms in a superposition of their energy eigenstates. We also discuss some of the nonclassical statistical properties of the resulting field.

Keywords: Micromaser, two-photon transitions, thermal state, quantum fluctuations, quantum coherence

1 Introduction

In the past years there have been important advances regarding the control over both isolated atoms and electromagnetic fields. It is of particular importance the generation of pure states of quantized the field, not only for the study of the foundations of quantum mechanics, but also for applications in quantum computation and quantum cryptography [1]. High-Q single-mode cavities have been shown to be adequate systems for the generation and manipulation of quantum states of light [2]. In general, atoms conveniently prepared are injected inside a cavity, and their coupling to the quantized cavity field can make possible, for instance, the generation of nonclassical states of the field [3]. Other quantum effects, e.g., the atomic dipole squeezing, may also occur in a system in which atoms are sent sequentially through a cavity (micromaser), as discussed in [4]. The cavity itself is normally cooled down to its vacuum state, and the field build up occurs by coherently adding photons to it. Nevertheless, the radiation field in most ordinary situations happens to be in a mixed state (in thermal equilibrium, for instance). It would be therefore interesting to investigate the build up of quantum states of light starting from mixed (thermal) states rather than the vacuum state. In previous papers it has been shown that in micromasers undergoing either one-photon [5] or two-photon [6] transitions, field states with a modest degree of purity may be generated from highly mixed states, even if one does not perform conditional measurements on the atoms after they leave the cavity. It has also been shown that two-photon transitions [6] are normally more efficient compared to one-photon transitions regarding the atom-field transfer of coherence process. However, the degree of mixedness of the resulting field in those schemes is still considerably high. It would be therefore interesting to seek for ways of improving the process of transfer of coherence from atoms to the cavity field. Here we consider a situation in which the cavity is driven by an external field, so that each incoming atom simultaneously interacts with the cavity (quantized) field as well as with the (classical) external driving field. We may find in the literature discussions about several aspects of the action of an external field on the atom-quantized field system. In fact its presence might give rise to some nonclassical effects, such as large time scale revivals (super-revivals) [7] and enhancement of quadrature squeezing [8], for instance. There is also a proposal of a scheme of single-photon states generation [9] using an external driving field. Here

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we study the influence of an external classical field on the process of transfer of coherence from atoms to the quantized cavity field in a two-photon micromaser. Our aim is the generation of quantum states of light as pure as possible starting from a mixed state. We find that the driving field brings us several advantages: a simpler preparation of the atomic states is required (excited state $|e\rangle$ instead of a coherent superposition $|e\rangle + |g\rangle$); the transfer of coherence process is faster than in the case in which the external field is not present; the resulting field is very close to a pure state and has nonclassical properties as well. We are considering an ideal cavity, although we are aware that cavity losses are an important source of decoherence that will for sure degrade the process of transfer of coherence. We are not including losses in our analysis because we would like to investigate the main features of the influence of the driving field. In order to characterize the generated field we make use of representations of the field in number (photon number distribution) and coherent (Q-function) states as well as the second order correlation function, $g^{(2)}(0)$. The paper is organized as follows: in section 2 we solve the model and calculate the field density matrix after the passage of N atoms through the cavity; in section 3 we discuss our numerical results concerning the degree of field purification using the linear entropy (mixedness parameter). We also analize some of the nonclassical statistical properties of the field, and in section 4 we present our conclusions.

2 The model

We consider a two-photon micromaser in which the injected (three-level) atoms are coupled to the cavity field as well as to a classical driving field, in a configuration shown in figure 1. If the intermediate atomic level $|i\rangle$ is highly detuned from the field, i.e., if $\delta = E_i - (E_e + E_g)/2$ is large enough, it may be adiabatically eliminated so that we obtain the following effective hamiltonian [10]

$$\hat{H} = \hbar\omega\hat{a}^\dagger\hat{a} + \hbar\left[\frac{\omega_0}{2} + \chi(\hat{a}^\dagger + \varepsilon e^{-i\omega' t})(\hat{a} + \varepsilon e^{i\omega' t})\right]\sigma_z + \lambda\left[(\hat{a}^\dagger + \varepsilon e^{-i\omega' t})^2\sigma_- + (\hat{a} + \varepsilon e^{i\omega' t})^2\sigma_+\right], \quad (1)$$

where $\lambda = 2g^2/\delta$ is the effective coupling constant (being $g = g_{ig} = g_{ei}$ the coupling constants related to one-photon transitions), and $\sigma_+ = \sigma_{eg}$, $\sigma_- = \sigma_{ge}$ $\sigma_z = \sigma_{ee} - \sigma_{gg}$ are the atomic transition operators. The parameter $\chi = 2g^2/\delta$ is the dynamical Stark effect coefficient. The cavity field has frequency ω, and the classical driving field, of frequency ω' has an amplitude ε (dimensionless) taken real for simplicity. We assume that there is a detuning Δ between the cavity field and the atomic transition of frequency $\omega_0/2$, i.e., $\Delta = \omega_0 - 2\omega$, although the classical field itself is resonant with the atom, i.e., $\omega' = \omega_0/2$. We may write the evolution

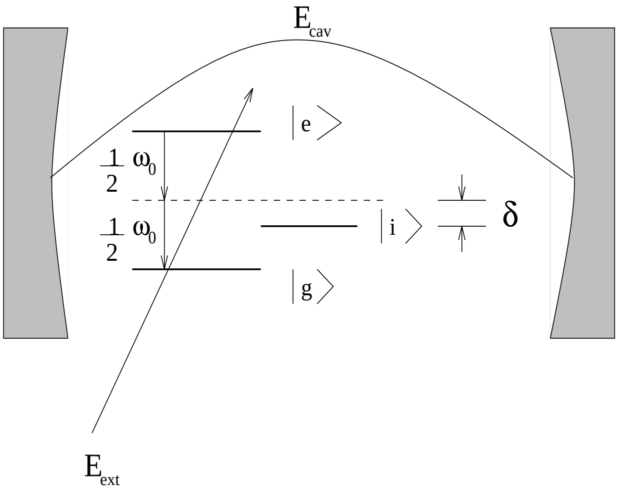

Fig. 1: Schematic representation of the two-photon micromaser setup.

operator relative to the hamiltonian in equation (1) as

$$\hat{U}(t) = \hat{D}^\dagger(\varepsilon^*)\hat{U}^\dagger_{tp}(t)\hat{D}(\varepsilon), \quad (2)$$

where

$$\hat{U}_{tp}(t) = \exp\left[-i\lambda t\left(\frac{\Delta}{2\lambda}\sigma_z + \frac{\chi}{\lambda}\hat{a}^\dagger\hat{a}\sigma_z + \hat{a}^{\dagger 2}\sigma_- + \hat{a}^2\sigma_+\right)\right], \quad (3)$$

is the evolution operator for the two-photon Jaynes-Cummings model and $\hat{D}(\varepsilon) = \exp(\varepsilon\hat{a}^\dagger - \varepsilon^*\hat{a})$ is Glauber's displacement operator.

We assume that the cavity field is initially in a thermal state with mean photon number $\bar{n}$

$$\hat{\rho}^f(0) = \sum_{n=0}^{\infty}\frac{\bar{n}^n}{(\bar{n}+1)^{n+1}}|n\rangle\langle n| \quad (4)$$

and the atom in a superposition of atomic energy eigenstates $|g\rangle$ and $|e\rangle$ states

$$|\phi_a\rangle = b|g\rangle + ae^{i\phi}|e\rangle, \quad (5)$$

where the coefficients a and b have been taken real for simplicity. Therefore the atom-field system is initially prepared in the product state $\hat{\rho}(0) = \hat{\rho}^f(0)\otimes|\phi_a\rangle\langle\phi_a|$.

The joint time evolved density operator will be $\hat{\rho}(t) = \hat{U}(t)^\dagger\hat{\rho}(0)\hat{U}(t)$. We are interested in keeping the interaction time as short as possible, in order to minimize the effects of dissipation. In our scheme there are no conditional measurements, i.e., the atomic state is not collapsed by a detector and the atoms just fly away after crossing the cavity. We are mostly interested in the dynamics of the field, so that we trace over the atomic degrees of freedom in order to obtain the field density operator at a time t, or $\hat{\rho}^f(t) = Tr_a\hat{\rho}(t)$. As in reference

[6], we may calculate the density operator in the number state basis, $\rho_N^f(n,n') \equiv \langle n|\hat{\rho}_N^f(t)|n'\rangle$, after N atoms prepared in superposition states as in equation (5) have crossed the cavity

$$\begin{aligned}\rho_N^c(n,n') = \sum_{m,m'} \rho_{N-1}^c(m,m') \sum_{j,j'} e_{j,m} e^*_{j',m'} \Big[\{ a^2 \alpha_j(\gamma)\alpha^\dagger_{j'}(\gamma) \\ + b^2 \alpha_j(\varepsilon)\alpha^\dagger_{j'}(\varepsilon) \} e_{j,n} e^*_{n',j'} \\ + b^2 \beta_j(\gamma)\beta_{j'}(\gamma)\sqrt{(j+2)(j+1)(j'+2)(j'+1)}\; e_{j+2,n} e^*_{n',j+2} \\ + a^2 \beta_j(\varepsilon)\beta_{j'}(\varepsilon)\sqrt{j(j-1)j'(j'-1)}\; e_{j-2,n} e^*_{n',j-2} \\ + iabe^{-i\phi}\alpha_j(\gamma)\beta_{j'}(\gamma)\sqrt{(j'+2)(j'+1)}\; e_{j,n} e^*_{n',j'+2} \\ + iabe^{i\phi}\alpha_j(\varepsilon)\beta'_j(\varepsilon)\sqrt{j'(j'-1)}\; e_{j,n} e^*_{n',j'-2} \\ - iabe^{i\phi}\beta_j(\gamma)\alpha^\dagger_{j'}(\gamma)\sqrt{(j+2)(j+1)}\; e_{j+2,n} e^*_{n',j'} \\ - iabe^{-i\phi}\beta_j(\varepsilon)\alpha^\dagger_{j'}(\varepsilon)\sqrt{j(j-1)}\; e_{j-2,n} e^*_{n',j'} \Big], \qquad (6)\end{aligned}$$

with $e_{j,n}$ and $e^*_{n',j'}$ given by

$$e_{(j,n)} = \langle j|\varepsilon;n\rangle = e^{\frac{-|\varepsilon|^2}{2}} \varepsilon^{j-n}\sqrt{\frac{n!}{j!}}\mathscr{L}_n^{j-n}(|\varepsilon|^2)$$

$$e^*_{(n',j')} = \langle n';\varepsilon|j'\rangle = e^{\frac{-|\varepsilon|^2}{2}} (\varepsilon^*)^{n'-j'}\sqrt{\frac{j'!}{n'!}}\mathscr{L}_{j'}^{n'-j'}(|\varepsilon|^2), \quad (7)$$

where $|\varepsilon;n\rangle = \hat{D}(\varepsilon)|n\rangle$ are displaced number states and $\mathscr{L}_n^{j-n}$ are the associated Laguerre polynomials. The coefficients $\alpha_n(\gamma)$ e $\beta_n(\varepsilon)$ are

$$\alpha_n(\gamma) = \cos(\gamma_n\lambda t) + i\frac{\sin(\gamma_n\lambda t)}{\gamma_n}\left(\frac{\Delta}{2\lambda} + \frac{\chi}{\lambda}(n+1)\right), \quad (8)$$

$$\alpha_n(\varepsilon) = \cos(\varepsilon_n\lambda t) + i\frac{\sin(\varepsilon_n\lambda t)}{\varepsilon_n}\left(\frac{\Delta}{2\lambda} + \frac{\chi}{\lambda}(n-1)\right), \quad (9)$$

$$\beta_n(\gamma) = ia^{\dagger 2}\frac{\sin(\gamma_n\lambda t)}{\gamma_n}, \quad (10)$$

$$\beta_n(\varepsilon) = ia^2\frac{\sin(\varepsilon_n\lambda t)}{\varepsilon_n}, \quad (11)$$

with

$$\gamma_n^2 = \left(\frac{\Delta}{2\lambda} + \frac{\chi}{\lambda}(n+1)\right)^2 + (n+1)(n+2), \quad (12)$$

$$\varepsilon_n^2 = \left(\frac{\Delta}{2\lambda} + \frac{\chi}{\lambda}(n-1)\right)^2 + n(n-1). \quad (13)$$

A particularly important result is that in order to transfer coherence from atoms to the field, there is no need of having the atoms initially prepared in a superposition of their energy eigenstates [see equation (5)]. A simpler atomic preparation, e.g. $|e\rangle$, is sufficient, making unnecessary the previous passage of atoms in a Ramsey zone normally used to prepare superpositions of atomic states, as needed in the scheme discussed in [6]. If the atom is initially prepared in state $|e\rangle$ ($b = 0$), the resulting density matrix will be

$$\rho_N^f(n,n') = \sum_{m,m'} \rho_{N-1}^f(m,m') \sum_{j,j'} e_{j,m} e^*_{j',m'}$$
$$\times \Big[a^2 \alpha_j(\gamma)\alpha^\dagger_{j'}(\gamma) e_{j,n}\ \ e^*_{n',j'}$$
$$+ a^2 \beta_j(\varepsilon)\beta_{j'}(\varepsilon)\sqrt{j(j-1)j'(j'-1)} e_{j-2,n} e^*_{n',j-2} \Big]. \quad (14)$$

We note that its off-diagonal elements may be populated, i.e., $\rho_N^f(n,n')$ might be nonzero (for $n \neq n'$) even for an initial diagonal density matrix as the one in equation (4). Because of the presence of the classical field ($\varepsilon \neq 0$), there are now sums over j, j' and m, m', which make possible to have $\rho_N^f(n,n') \neq 0$. In the present scheme the action of the classical field somehow induces atomic quantum coherence simultaneously to the quantized field-atom interaction, differently from previously discussed schemes [5,6], which rely upon the previous preparation of the atomic state in a Ramsey zone before they enter the cavity. We would like to remark that as the field becomes less mixed, the atoms, initially prepared in pure states leave the cavity essentially in mixed states.

3 Results

3.1 Field state purification

We now adopt a procedure similar to the one used in reference [6]. Initially, we fix the values of the parameters Δ (detuning) and χ (Stark shift coefficient). We are going to look for a resulting field as pure as possible without having the field energy reduced. We consider the situation in which N atoms have already crossed the cavity, so that each atom is able to transfer some "amount of coherence" to the field. The field purity is normally quantified by the parameter known as linear entropy ζ

$$\zeta = 1 - Tr[(\rho^f)^2] = 1 - \sum_{n,n'} |\rho_N^f(n,n')|^2.$$

For pure states $\zeta = 0$, and for statistical mixtures of pure states we have $\zeta > 0$, behaviour similar to the von Neumann entropy $S = -Tr[\rho\ln(\rho)]$. We will numerically determine under which conditions it is possible to minimize ζ. The initial field is a (highly mixed) thermal state (4) having mean photon number $\bar{n} = 5$. Every atom entering the cavity is previously prepared in its excited state $|e\rangle$. We have calculated the field evolution for a range of times, in order to find the small interaction times which minimizes ζ (maximizes the field purity) after N atoms have crossed the cavity. For simplicity we have considered the same interaction time for each atom. The optimum interaction time in this case is $T \approx 8.9/\lambda$. In figure 2 we have a plot of the linear entropy ζ as a function of the total number of atoms N crossing the

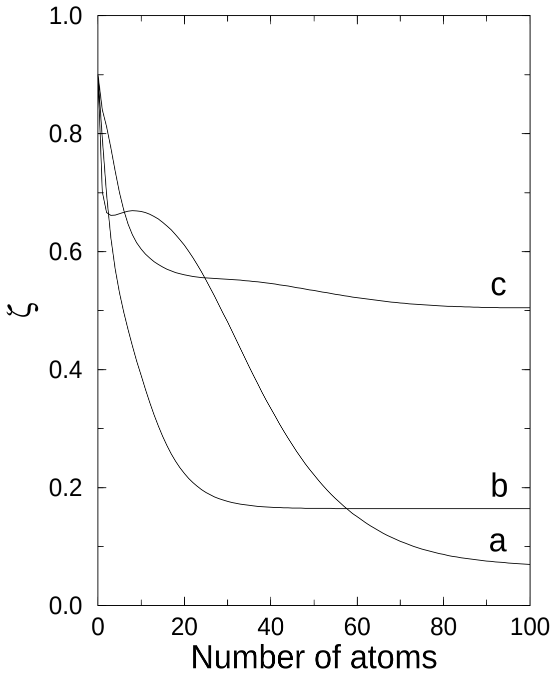

Fig. 2: Field linear entropy ζ as a function of the number of atoms, for different classical external fields (a) $\varepsilon = 1.0$, (b) $\varepsilon = 2.0$ and (c) $\varepsilon = 3.0$. The atom is initially in its excited state ($b = 0$), the field in a thermal state having $\bar{n} = 5$, and $\chi/\lambda = \Delta/\lambda = 1$.

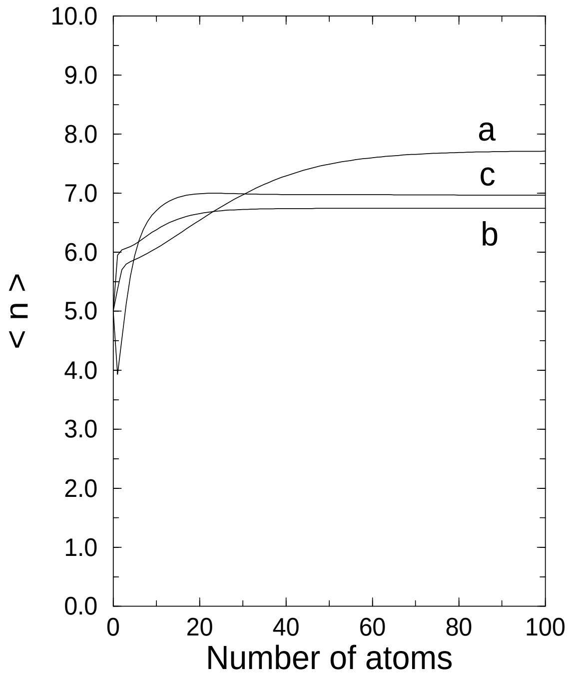

Fig. 3: Mean photon number in the cavity as a function of the number of atoms. The same parameters as in figure 2.

cavity, for different values of the classical field amplitude ε. We note that the linear entropy is very sensitive to ε. For instance, for $\varepsilon = 2.0$, the field purity considerably increases for a relatively small number of atoms. In this case ζ reaches a minimum value $\zeta_{min} \approx 0.18$. If $\varepsilon = 1.0$, a considerably purer field is generated, ($\zeta_{min} \approx 0.07$), although it takes longer for ζ to reach its minimum value after ~ 100 atoms have crossed the cavity, as shown in figure 2. In both cases there is an increase in the mean photon number of the cavity field (figure 3).

3.2 Nonclassical properties

The resulting field after the process of transfer of coherence occurs is not only an almost pure state, but also has nonclassical features, for instance, statistical properties significantly different from the original thermal state. We are now going to calculate quantities which are relevant to characterize the generated state.

3.2.1 Second order coherence function

Quantum states of light may be classified according to their optical coherence properties. It is considered to be a quantum regime if the one-mode second order coherence function $g^{(2)}(0)$

$$g^{(2)}(0) = \frac{\langle \hat{a}^\dagger \hat{a}^\dagger \hat{a}\hat{a} \rangle}{\langle (\hat{a}^\dagger \hat{a})^2 \rangle}$$

is less than one. For a thermal field $g^{(2)}(0) = 2$, and for a coherent field $g^{(2)}(0) = 1$. In our case we have verified that $g^{(2)}(0)$ decreases, although the exclusive quantum regime is never reached, as $g^{(2)}(0)$ is always greater than one. This may be seen in figure 4, where we have plotted $g^{(2)}(0)$ as a function of the number of atoms crossing the cavity.

3.2.2 Representation in Fock and coherent state basis

The Projection of the field state in the Fock basis (photon number distribution, or $P_n = \langle n|\hat{\rho}|n\rangle$), may turn evident some nonclassical properties. In this case we note a clear departure from the thermal (geometrical) distribution as shown in figure 5, where it it shown the photon number distribution of the resulting field state after $N = 100$ atoms have crossed the cavity. The photon number distribution of such a field, shown in figure 5, is very different from the distribution of a thermal state; we note strong oscillations in P_n, an evidence of nonclassical behaviour which might be associated to the two-photon interaction in our scheme. However the photon number distribution gives us just partial information about the field. A more complete characterization may be obtained from quasiprobability functions in the coherent state basis, which are basically representations of the density operator $\hat{\rho}$ in terms of functions. A convenient quasiprobability is the Q-function, defined as [11]

$$Q(x,y) = \frac{1}{\pi}\langle \beta|\hat{\rho}|\beta\rangle,$$

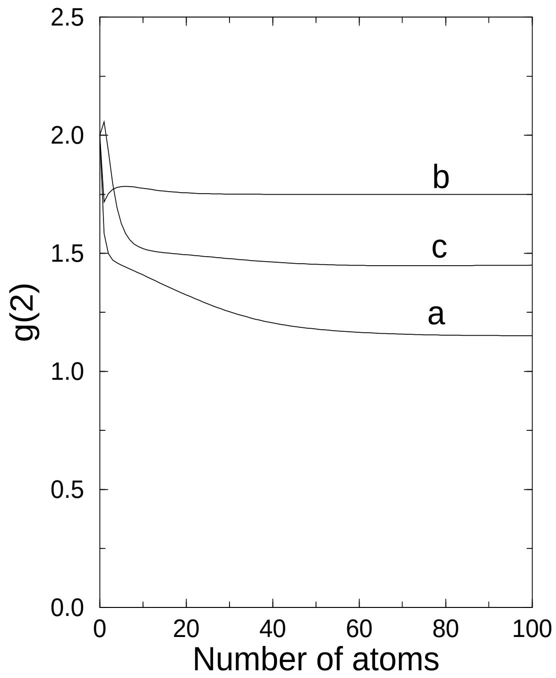

Fig. 4: Second order correlation function of the cavity field as a function of the number of atoms. The same parameters as in figure 2.

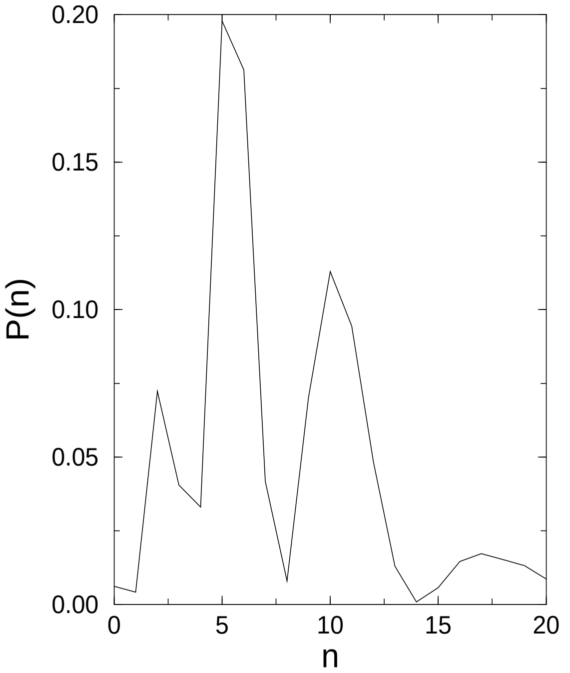

Fig. 5: Photon number distribution of the cavity field after $N =$ atoms have crossed the cavity and with $\varepsilon = 1.0$.

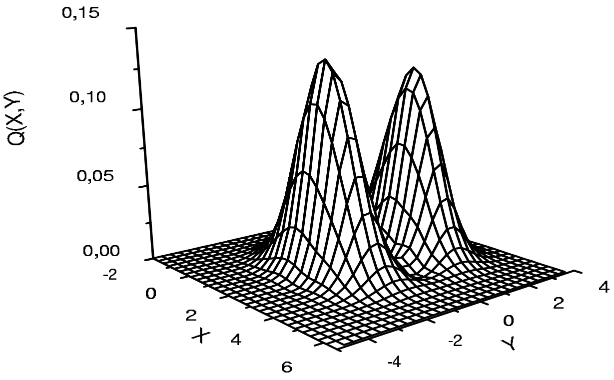

Fig. 6: Q function of the cavity field after $N =$ atoms have crossed the cavity and with $\varepsilon = 1.0$.

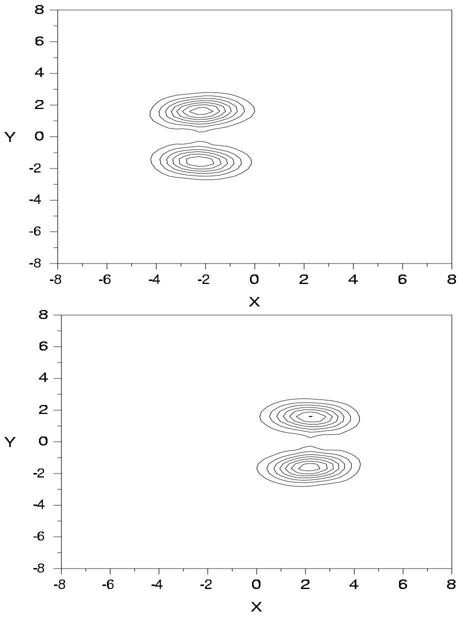

Fig. 7: Contour plots of the cavity field Q function after $N = 100$ atoms have crossed the cavity with $\varepsilon = -1.0$ (in the left) and $\varepsilon = 1.0$ (in the right).

where $|\beta\rangle$ is a coherent state having amplitude $\beta = x + iy$. In figure 6 we have the Q-function of the field after the passage of $N = 100$ atoms. We note a double peaked structure, which resembles two superposed deformed gaussians, which is displaced from the origin. As a matter of fact such a displacement may be attributed to the action of the classical field. This is clearly seen if we change the sign of the classical field amplitude ε; for $\varepsilon = -1.0$, for instance, the Q-function is displaced towards the opposite direction in phase space, as shown in figure 7, where we have the contour plots of the Q-function for the case of $\varepsilon = -1.0$ compared to the case in which $\varepsilon = 1.0$.

4 Conclusion

We have presented a scheme comprising a two-photon micromaser in which the process of transfer of coherence from atoms to a field in a thermal state may be substantially improved by coupling the atoms to a classical external field. We have shown that the continuous action of the external field has important consequences: the degree of purity of the field is considerably increased in a relatively short time and the atoms do not need to be prepared in superpositions of their internal states. The resulting field state is displaced from the origin in phase space, a direct consequence of the action of the external classical field. The field state has also a nonclassical character; we have verified strong oscillations in the photon number distribution of the field, a distinctive nonclassical feature, although there is no anti-bunching. A field with linear entropy as low as $\zeta \approx 0.07$ could be generated, in contrast to the case with no assistance of a classical field [6], which yields $\zeta \approx 0.53$. An important feature in our approach is that the interaction times have been kept as short as possible, so that a time of $\approx 10^{-2}$ sec is long enough to have around 100 atoms crossing the cavity. This is convenient if one wants to minimize the destructive effects of field dissipation, which normally leads to loss of coherence. Here we wanted mainly to capture the main features of the model, e.g., the action of a classical external field, and we decided to investigate the dissipation effects elsewhere. It is though apparent that some kind of competition process between the atom-field transfer of coherence and dissipation-induced loss of coherence will be established if cavity losses are taken into account.

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