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Some Properties of Entropy for an Extended Exponential Distribution(EED) Based on Order Statistics

Huda M Alshanbari[1], Rifat Nisa[2], Abd Al-Aziz Hosni El-Bagoury[3] and Javid Gani Dar[4,]*

[1]Department of Mathematical Sciences, Faculty of Science, Princess Nourah bint Abdulrahman University, Riyadh, Saudi Arabia
[2]P. G. Department of Statistics, University of Kashmir, Kashmir India
[3]Department of Mathematics, Faculty of Science, Tanta University, Tanta, Egypt
[4]Department of Mathematical Sciences, IUST, Awantipor, Kashmir, India

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Abstract: Residual and past residual entropy functions concatenated with uncertainty measurements are predominant factors in information theory. Many existing comparative analysis procedures are already asserted for determining the aging process of associated components like life testing problems and survival function. This paper focuses on proposing functions that are based upon the extended exponential distribution (EED). Existing past residual entropy function is examined on upper bounds of different order statistics and the results are analyzed for proposed Shannon's entropy and residual entropy functions.
Keywords: Entropy; Order Statistics; Past Entropy; Residual Entropy; Extended Exponential Distribution.

1 Introduction

The main investigation of uncertainty measures was adopted by Nyquist [1] and Hartley [2]. Later Shannon [3] done one extraordinary discovery that is the ability to actually quantify "amount of information" inherent in a probability distribution and this has paved way to countless discoveries that affect many attributes in our daily lives (e.g., cell phone and positioning system (G.P.S) technologies) etc. known as Shannon entropy (SE). For a continuous random variable (r.v) Y , Shannon's entropy is defined as:

$$SE(Y)= - E[\log f(y)] \qquad (1)$$

For various properties of the EED one should refer to Nadarajah and Haghighi [4]. Analytical expressions of the entropy of univariate distributions are discussed in [5,6,7]. Similarly, for the order statistics, the uncertainty measures have been studied by elite group of researchers. Several results are provided by Wong and Chen [8], Park [9], Lazo et. al, [10] and some characterizations of SE for order statistics. Some real world problems like robust statistical estimation, description of probability distribution of record values and order statistics, detection of outliers, case-study of censored samples has been successfully addressed by the theory of order statistics. Attributes of information pertaining to order statistics that are based on Kullback-Leibler [11] and measure using probability integral transformation are completely investigated by Ebrahimi et.al [12,13]. Several abstractions of Shannon's entropy are present in the collection of research papers of information theory. Accordingly, in this article we derive the entropy expression for EED. The SE measures amount of information (uncertainty) associated with a r.v Y . In the present communication, exact analytical expression of SE for an extended exponential distribution (EED) has been derived.

*Corresponding author e-mail: javinfo.tat@yahoo.co.in

2 Shannon Entropy of EED

Consider a r.v Y having EED with pdf and the cdf respectively given as:

$$f(y)=\mu\phi\left(1+\phi y\right)^{\mu-1}e^{1-(1+\phi y)^{\mu}} \tag{2}$$

and

$$F(y)=1-e^{1-(1+\phi y)^{\mu}} \tag{3}$$

respectively, for $y>0, \phi>0$ *and* $\mu>0$, where ϕ the scale and μ the shape parameter.

$$\log(f(y))=\log(\mu\phi)+(\mu-1)\log(1+\phi y)+\left(1-(1+\phi y)^{\mu}\right). \tag{4}$$

using equations (2) and (4) in equation (1):

$$SE(Y)=-\log(\mu\phi)-(\mu-1)E\left(\log\left(1+\phi y\right)\right)-E\left(1-(1+\phi y)\right)^{\mu} \tag{5}$$

we have to derive the expressions $E\left(\log\left(1+\phi y\right)\right)$ and $E\left(1-(1+\phi y)\right)^{\mu}$

Now using the definition of expectation and then substitution $\left(1+\phi y\right)=x,\ 1<x<\infty$ and integrating with the help of software mathematica, we get:

$$E\left(\log\left(1+\phi y\right)\right)=\frac{1}{\mu}0.596347 \tag{6}$$

and we derive the

$$E\left(1-(1+\phi y)^{\mu}\right)=-1 \tag{7}$$

Using the above substitution and using equations (6) and (7) in equation (5), we get:

$$SE(Y)=-\log(\mu\phi)-\frac{(\mu-1)}{\mu}0.596347+1$$

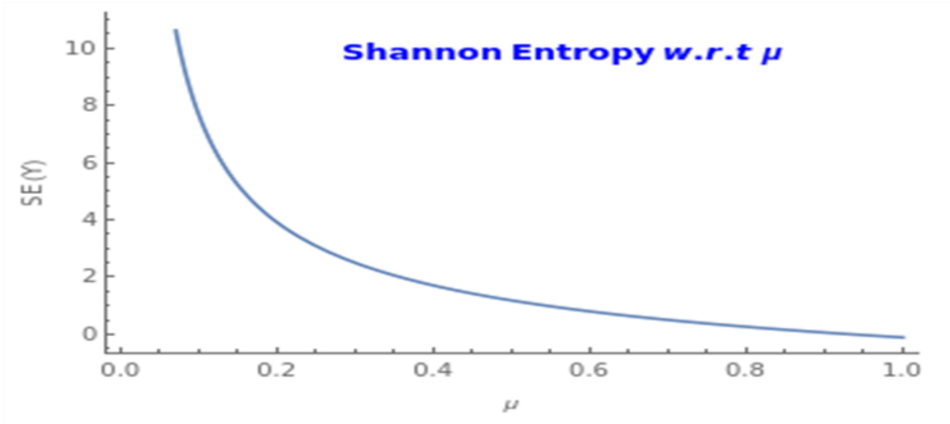

Fig1: Shannon' entropy for different values of μ.

It is clear that Shannon Entropy of EED with respect to μ is decreasing with the increase in the value of μ.

2.1 Preliminaries and Some Relations of Order Statistics

Assume that the random sample $Y_1, Y_2, \ldots, Y_n$ be from a distribution function (cdf) $F(y)$ with pdf $f(y)$. By arranging $Y_1, Y_2, \ldots, Y_n$ from the lowest to highest, order statistics of $Y_1, Y_2, \ldots, Y_n$ is defined as $Y_{1:n} \le Y_{2:n} \le \ldots \le Y_{n:n}$. The density of m^{th} order statistics refer to [14] is given by,

$$f_{m:n}(y) = \frac{n!}{(m-1)!(n-m)!}(F(y))^{m-1}(1-F(y))^{n-m} f(y) \tag{8}$$

For $m = 1,2,\ldots,n$

In recent past, order statistics and their moments established major interest and number of researchers including Saran and Pushkarna [15], Devendra Kumar et a1. [16] derived expressions of m^{th} order statistics for single moments, $E\left(Y_{m:n}^{(s)}\right) = \mu_{m:n}^{(s)}$ gave an explicit expressions for EED given in (2) for $0 < m < n$ and $s = 0,1,2,\ldots..,$

$$\mu_{m:n}^{(s)} = \frac{(m-1)!}{(n-1)!(m-n)!}\frac{e^{n-m+x+1}}{\phi^s}\sum_{x=0}^{m-1}\binom{m-1}{x}\sum_{k=0}^{m-1}\frac{(-1)^{x+s-k}}{(n-m+x+1)^{1+(k/\mu)}}\Gamma\left(\frac{k}{\mu}+1, n-m+x+1\right)$$

Mean and variance of order statistic for EED is obtained by putting $s = 1,2$, given as:

$$\mu_{m:n}^{(1)} = \frac{(m-1)!}{(n-1)!(m-n)!}\frac{e^{n-m+x+1}}{\phi}\sum_{x=0}^{m-1}\binom{m-1}{x}\left[(-1)^{x+1}\Gamma(1, n-m+x+1) + \frac{(-1)^{x+1}}{(n-m+x+1)^{1+(1/\mu)}}\Gamma\left(\frac{1}{\mu}+1, n-m+x+1\right)\right]$$

And

$$\sigma_{m:n}^{2} = \frac{(m-1)!}{(n-1)!(m-n)!}\frac{e^{n-m+x+1}}{\phi^2}\sum_{x=0}^{m-1}\binom{m-1}{x}\left[\frac{(-1)^{x+2}}{n-m+x+1}\Gamma(1, n-m+x+1) + \frac{2(-1)^{x+1}}{(n-m+x+1)^{1+(1/\mu)}}\Gamma\left(\frac{1}{\mu}+1, n-m+x+1\right)\right.$$

$$\frac{(-1)^x}{(n-m+x+1)^{1+(2/\mu)}}\Gamma\left(\frac{2}{\mu}+1, n-m+x+1\right)\Bigg]-\left[\mu_{m:n}^{(1)}\right]^2$$

2.2 Order Statistics of EED

The pdf of the m^{th} order statistic of EED is defined as:

$$f_{m:n}(Y)=\frac{n!}{(m-1)!(n-m)!}\mu\phi\left(1+\phi y\right)^{\mu-1}e^{1-(1+\phi y)^{\mu}}\left[1-e^{1-(1+\phi y)^{\mu}}\right]^{m-1}\left[e^{1-(1+\phi y)^{\mu}}\right]^{n-m} \tag{9}$$

Put $m=n$ in equation (9), we get the pdf of the largest order statistics $Y_{(n)}$ as:

$$f_{n:n}(Y)=n\mu\phi\left(1+\phi y\right)^{\mu-1}e^{1-(1+\phi y)^{\mu}}\left[1-e^{1-(1+\phi y)^{\mu}}\right]^{n-1} \tag{10}$$

Put $m=1$ in equation (9), we get the pdf of the smallest order statistics $Y_{(1)}$ as:

$$f_{1:n}(Y)=n\mu\phi\left(1+\phi y\right)^{\mu-1}e^{1-(1+\phi y)^{\mu}}\left[e^{1-(1+\phi y)^{\mu}}\right]^{n-1} \tag{11}$$

Now, let $W_1, W_2, \ldots, W_n$ be a random sample from $U(0,1)$ with the order statistics $Z_1 < Z_2 < \ldots < Z_n$. The density of $Z_m, m=1,2,\ldots,n$ is:

$$f_{Z_m}(Z)=\frac{1}{B(m,n-m+1)}Z^{m-1}(1-Z)^{n-m}, \qquad Z\in(0,1).$$

where $B(m,n-m+1)=\dfrac{(m-1)!(n-m)!}{n!}$

The SE of the beta distribution is:

$$SE_n(Z_m)=-(m-1)\left[\psi(m)-\psi(n+1)\right]-(n-m)\left[\psi(n-m+1)-\psi(n+1)\right]+\log B(m,n-m+1) \tag{12}$$

Where $\psi(x)=\dfrac{d\log\Gamma x}{dx}$ and $\psi(n+1)=\psi(n)+\dfrac{1}{n}$

$$SE(V_m)=SE_n(Z_m)-E_{g_m}\left[\log f_Y(F_Y^{-1}(Z_m))\right] \tag{13}$$

Using the substitution in equation (13) $Z_m=F_Y(y_m)$ and $Y_m=F_Y^{-1}(Z_m), m=1,2,\ldots,n$ is the probability integral transformations, the entropies of order statistics are obtained as:

For evaluating $SE(V_m)$, we have $F_Y^{-1}(Z_m)=\dfrac{1}{\phi}[(1-\log(1-Z))^{\frac{1}{\mu}}-1]$ and the expectation expression in (13) we get:

$$E_{g_m}\left[\log f_Y(F_Y^{-1}(Z_m))\right]=\left[\log(\mu\phi)+\left[\psi(n-m+1)-\psi(n+1)\right]\right.$$
$$\left.+\sum_{i=0}^{m-1}\sum_{k=0}^{\infty}\binom{m-1}{k}\frac{1}{B(m,n-m+1)(k-1)(n-m+i+1)^k}(-1)^{i+k}\Gamma k\right] \quad (14)$$

using (12) and (14) in equation (13), we get:

$$SE(Y_m)=-(m-1)[\psi(m)-\psi(n+1)]-(n-m+1)\left[\psi(n-m+1)-\psi(n+1)\right]-\left[\log(\mu\phi)\right.$$
$$+\log B(m,n-m+1)-\sum_{i=0}^{k-1}\sum_{k=0}^{\infty}\binom{m-1}{i}\frac{1}{B(m,n-m+1)(k-1)(n-m+i+1)^k}(-1)^{i+k}\Gamma k \quad (15)$$

For $m=1, SE(Z_1)=1-\log(n)-\frac{1}{n}$.

$$SE(Y_1)=1-\log(n\mu\phi)-\sum_{k=0}^{\infty}\frac{(-1)^k\Gamma k}{(k-1)n^{(k-1)}}$$

For the sample maximum $m=n, SE_n(Z_n)=1-\log(n)-\frac{1}{n}$.

$$SE(Y_n)=1.5772-\frac{1}{n}-\log(n\mu\phi)+\psi(n+1)-n\sum_{i=0}^{n-1}\sum_{k=0}^{\infty}\binom{n-1}{i}\frac{(-1)^{i+k}\Gamma k}{(k-1)(i+1)^k} \quad (16)$$

where $-\psi(1)=0.5772$ is the Euler's constant.

3 Residual Entropy

In the event that an element is understood to possess survived to age t_Θ, at that time Shannon's Entropy isn't applicable so as to evaluate the uncertainty of the remaining period of time of the system. Therefore, Ebrahimi and Pellerey [17] characterized the residual entropy (RE) that evaluates the uncertainty in such cases. For a random period of time Y of system at time t_Θ, the RE is outlined as:

$$RE(Y;t_\Theta)=-\int_{t_\Theta}^{\infty} f_{t_\Theta}(Y)\log f_{t_\Theta}(Y)\,dy.$$

where $f_{t_\Theta}(Y)$ is the pdf of the variable $Y_{t_\Theta}=\left(Y-t_\Theta \big/ Y>t_\Theta\right)$ and is:

$$f_{t_\Theta}(Y)=\left\{\frac{f(y)}{\bar{F}(t_\Theta)}, if\ Y>t_\Theta\right\}$$

$$RE(Y;t_\Theta)=-\frac{1}{\bar{F}(t_\Theta)}\int_{t_\Theta}^{\infty} f(y)\log(f(y))\,dy+\frac{\log(f(y))}{\bar{F}(t_\Theta)}\int_{t_\Theta}^{\infty} f(y),\quad t_\Theta>0 \quad (17)$$

Where $\bar{F}(t_\Theta)$ is the survival function of *Y*.

3.1 Residual Entropy of EED

Analogous to equations (2), (4) and (17) after solving the RE using a r.vY that follows an extended exponential distribution is derived as:

$$RE\left(Y;t_{\Theta}\right)=\log(\mu\phi)-(\mu-1)e^{1-(1+\phi t_{\Theta})^{\mu}}\log(1+\phi t_{\Theta})-\frac{(\mu-1)}{\mu}\frac{1}{e^{-(1+\phi t_{\Theta})^{\mu}}}\Gamma\left(0,(1+\phi t_{\Theta})^{\mu}\right)-1 \quad (18)$$

3.2 Residual Entropy of Order Statistics Analogous to expression (17), the RE of order statistics $Y_{m,n}$ as:

$$RE\left(Y_{m,n};t_{\Theta}\right)=-\int_{t_{\Theta}}^{\infty}\frac{f_{m,n}(y)}{\bar{F}_{m,n}(t_{\Theta})}\log\frac{f_{m,n}(y)}{\bar{F}_{m,n}(t_{\Theta})}dy,\ \ t_{\Theta}>0 \quad (19)$$

It is clear that the by putting $m=1$, we obtain RE of first order statistics , then using probability integral transformation $Z=F_{Y}(y)$ in equation (19) and using pdf and cdf of order statistics, we have:

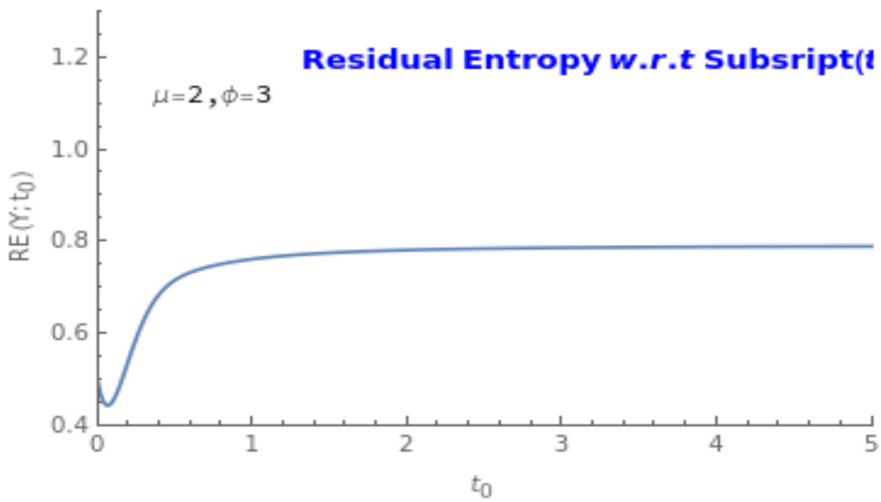

Fig 2: Residual entropy for smallest order statistics for $\mu=2,\phi=3$

$$RE\left(Y_{1,n};t_{\Theta}\right)=\left(\frac{n-1}{n}\right)-\log(n)+\log\left(\bar{F}(t_{\Theta})\right)-\left(\frac{n}{\bar{F}^{n}(t_{\Theta})}\right)\int_{F(t_{\Theta})}^{1}(1-z)^{n-1}\log f\left(F^{-1}(z)\right)dz \quad (20)$$

Where

$$\left(\frac{n}{\bar{F}^{n}(t_{\Theta})}\right)\int_{F(t_{\Theta})}^{1}(1-z)^{n-1}\log f\left(F^{-1}(z)\right)dz=\log(\mu\phi)+\frac{\Gamma\left(2,-n\log\bar{F}(t_{\Theta})\right)}{n\bar{F}^{n}(t_{\Theta})}$$

$$+\frac{(\mu-1)}{\mu}\sum_{k=1}^{\infty}(-1)^{k+1}\frac{\Gamma\left(k+1,-n\log\bar{F}(t_{\Theta})\right)}{(k)\bar{F}^{n}(t_{\Theta})(n)^{k}} \quad (21)$$

using (21) in (20), we get:

$$RE\left(Y_{1,n};t_\Theta\right)=\left[\left(\frac{n-1}{n}\right)-\log(n\mu\phi)+\log\left(\bar{F}(t_\Theta)\right)-\frac{\Gamma\left(2,-n\log\bar{F}(t_\Theta)\right)}{n\,\bar{F}^n(t_\Theta)}\right.$$
$$\left.-\frac{(\mu-1)}{\mu}\sum_{k=1}^{\infty}(-1)^{k+1}\frac{\Gamma\left(k+1,-n\log\bar{F}(t_\Theta)\right)}{(k)\bar{F}^{\,n}(t_\Theta)(n)^k}\right] \tag{22}$$

3.3 Residual Entropy of Order Statistics for EED

Analogous to equations (2), (3), (10) and (22), then we derive the expression for the RE of order statistics for a r.vY using an extended exponential distribution as:

$$RE\left(Y_{1,n};t_\Theta\right)=\left[\left(\frac{n-1}{n}\right)-\log(n\mu\phi)+1-(1+\phi t_\Theta)^\mu-\frac{\Gamma\left(2,-n\left(1-(1+\phi t_\Theta)^\mu\right)\right)}{n\left(e^{1-(1+\phi t_\Theta)^\mu}\right)^n}\right.$$

$$\left.-\frac{(\mu-1)}{\mu}\sum_{k=1}^{\infty}(-1)^{k+1}\frac{\Gamma\left(k+1,-n\left(1-(1+\phi t_\Theta)^\mu\right)\right)}{(k)\left(e^{1-(1+\phi t_\Theta)^\mu}\right)^n(n)^k}\right] \tag{23}$$

4 Reversed Residual(Past) Entropy

It is affordable to consider that in varied circumstances uncertainty isn't really identified with future yet can likewise advert to past. For instance, if Y denotes the period of time of a system, Y at time t_Θ, a system is determined solely at certain pre assigned times, it's determined to be down, then the uncertainty of the system life relies i.e., on that moment in $(0,t_\Theta)$, it's failing. In view of the thought, Dicrescenzo and Longobardi [18] have contemplated the past entropy (PE) over $(0,t_\Theta)$ and is characterized as:

$$PE\left(Y;t_\Theta\right)=-\int_0^{t_\Theta}\frac{f(y)}{F(t_\Theta)}\log\frac{f(y)}{F(t_\Theta)}\,dy \tag{24}$$

Where $F(t_\Theta)$ is the distribution function of Y.

4.3 Reversed Residual (Past) Entropy of Order Statistics for EED

Extended exponential distribution as:

$$PE\left(Y_{n,n},t_\Theta\right)=\left(\frac{n-1}{n}\right)+\log\left(1-e^{1-(1+\phi t_\Theta)^\mu}\right)-\log(n\mu\phi)+\frac{n}{\left(1-e^{1-(1+\phi t_\Theta)^\mu}\right)^n}\sum_{i=1}^{n-1}\binom{n-1}{i}(-1)^i$$

Analogous to equations (2), (3), (9) and (24), wederive the expression for the RE of order statistics for a r.vY using an

$$\left[\frac{\gamma\left(2,-(i+1)\log\left(e^{1-(1+\phi t_\Theta)^\mu}\right)\right)}{(i+1)^2}-\left(\frac{\mu-1}{\mu}\right)\sum_{k=1}^{\infty}(-1)^{k+1}\frac{\gamma\left(k+1,-(i+1)\log\left(e^{1-(1+\phi t_\Theta)^\mu}\right)\right)}{(i+1)^{k+1}}\right]$$

4.4 An Upper Bound to the Past Entropy of Order Statistics

We exhibit the upper bound for the PE of order statistics under the condition that $f_{m,n} \le 1$

$$PE\left(Y_{m,n};t_\Theta\right)=-\int_0^{t_\Theta}\frac{f_{m,n}(y)}{F_{m,n}(t_\Theta)}\log\frac{f_{m,n}(y)}{F_{m,n}(t_\Theta)}dy$$

For $t_\Theta>0$, $\log F_{m,n}(t_\Theta)\le 0$. We obtain:

$$PE\left(y_{m,n};t_\Theta\right)\le\frac{SE\left(y_{m,n}\right)}{F_{m,n}(t_\Theta)} \quad (25)$$

Substituting $m=1$ in (25) and using equations (2) and (6) and probability integral transformation, we obtain:

$$PE\left(y_{1,n};t_\Theta\right)\le\frac{-1}{F_{1,n}(t_\Theta)}\left[\log(n\mu\phi)+n(n-1)\sum_{i=0}^{n-1}\sum_{l=2}^{\infty}(-1)^{i+1}\frac{1}{(l-1)(i+l)}\right.$$

$$\left.+\,n\sum_{i=0}^{n-1}\sum_{k=1}^{\infty}\binom{n-1}{i}\left((-1)^{i+1}\frac{1}{(k+i+1)k^2}\right)+\left(\frac{\mu-1}{\mu}\right)\sum_{k=1}^{\infty}(-1)^{k+1}\frac{\Gamma k}{n^k}\right]$$

$$PE\left(y_{1,n};t_\Theta\right)\le-\frac{1}{\left(1-\left(e^{1-(1+\phi t_\Theta)^\mu}\right)^n\right)}\left[\log(n\mu\phi)+n(n-1)\sum_{i=0}^{n-1}\sum_{l=2}^{\infty}(-1)^{i+1}\frac{1}{(l-1)(i+l)}\right.$$

$$\left.+\;n\sum_{i=0}^{n-1}\sum_{k=1}^{\infty}\binom{n-1}{i}\left((-1)^{i+1}\frac{1}{(k+i+1)k^2}\right)+\left(\frac{\mu-1}{\mu}\right)\sum_{k=1}^{\infty}(-1)^{k+1}\frac{\Gamma k}{n^k}\right]\Bigg]$$

Substituting $m=n$ in equation (25) and using equations (2) and (6) and probability integral transformation, we obtain:

$$PE\left(y_{n,n};t_\Theta\right)\le\frac{-1}{F_{n,n}(t_\Theta)}\left[\log(n\mu\phi)-\frac{(n-1)}{n}+\right.$$

$$\left.\left(\frac{n(\mu-1)}{\mu}\sum_{i=0}^{n-1}\binom{n-1}{i}(-1)^{i+1}\left(\sum_{k=1}^{\infty}\frac{\Gamma k}{(i+1)^{k+1}}+\frac{1}{(i+1)^2}\right)\right)\right]$$

$$PE\left(y_{n,n};t_\Theta\right)\le-\frac{1}{\left(1-e^{1-(1+\phi t_\Theta)^\mu}\right)^n}\left[\log(n\mu\phi)-\frac{(n-1)}{n}\left(\frac{n(\mu-1)}{\mu}\sum_{i=0}^{n-1}\binom{n-1}{i}(-1)^{i+1}\left(\sum_{k=1}^{\infty}\frac{\Gamma k}{(i+1)^{k+1}}+\frac{1}{(i+1)^2}\right)\right)\right]$$

5 Conclusions

We have taken together and considered the SE of order statistics based on Extended exponential distribution (EED). We have studied some basic results of SE, RE and PE of order statistics of EED. We also propose the result of an upper bound of the PE function. The hypothetical results got in this article can be utilized to go additional and investigate the applications in various orders where the uncertainty happens and to find the generalized entropy (Renyi Entropy, Verma Entropy etc.)

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Conflict of interest: The authors declare that they have no conflict of interest.

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