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On the Geometry of Equiform Normal Curves in the Galilean Space $G^4$

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Abstract: In our article, we establish the definition of the Equiform Normal curves in Galilean space $G^4$. To obtain the position vector of an Equiform Normal curve in $G^4$, we have to solve an integro-differential equation in $\mu_2$, where $\mu_2$ is the position function of a space curve $\gamma(\sigma)$ in the direction of third vector $V_3$ of the Galilean space. Special cases of Equiform Normal curvatures are discussed. Finally, we prove that there is no equiform normal curve that is congruent to an Equiform Normal curve in $G^4$.

Keywords: Normal Curves, Curvature, Galilean Space, Equiform Geometry, integro-differential equation.

1 Introduction

We consider a Galilean space as the limiting case of a Pseudo-Euclidean space in which the isotropic cone degenerates into a plane [1]. There are many concepts and principles such as velocity, momentum, kinetic energy, laws of motion, and conservation laws in classical physics defined in Galilean space [1]. Besides, Galilean space has an influential role in non-relativistic physics. More details about Galilean space can be found in [2,3,4,5,6].

From the differential geometric point of view, the study of normal curves in Galilean space has its interest. Many researchers have derived many valuable and interesting results on normal curves in Galilean space, see [7,8].

Cayley-Klein space has an equiform geometry derived when the angles between planes and lines are preserved by the space’s similarity group [9].

In [10], equiform Darboux helices in Galilean space $G_3$, were defined and their explicit parameter equations were obtained. In [11] 4-dimensional Galilean space $G^4$, an equiform differential geometry of curves was created. They calculate the angle between the equiform Frenet vectors and their derivatives. Also, in [12], the authors introduced the Pseudo-Galilean space $G^4_1$ and study the equiform differential geometry of curves in this space.

In section 3, we present a summary of Frenet equations in equiform geometry in $G^4$. In section 4, firstly, in Galilean 4-space $G^4$, the equiform normal curve, and differentiate the equation of the position vector of this curve are defined. Next, the nonhomogeneous linear system of differential equations with variable coefficients is obtained, and then we solve this system. Secondly, with constant curvatures $K_1$, $K_2$, and $K_3$. We got the position vector of an equiform normal curve in $G^4$. Finally, the characterizations of equiform normal curves in Galilean space $G^4$ is introduced.

2 Basic Concepts

In this section, we introduce some basic definitions that are considered in this paper. For more basic concepts, see [4,5,11,13]. Given $\mathbf{a} = (a_1, a_2, a_3, a_4)$ and $\mathbf{b} = (b_1, b_2, b_3, b_4)$ be two vectors in $G^4$. Then the dot(scalar) product $g$ between the two vectors $\mathbf{a}$ and $\mathbf{b}$ in $G^4$ is defined by the following relation

$$g(\mathbf{a}, \mathbf{b}) = \begin{cases} a_1b_1, & \text{if } a_1 \neq 0 \text{ or } b_1 \neq 0, \\ a_2b_2 + a_3b_3 + a_4b_4, & \text{if } a_1 = 0 \text{ and } b_1 = 0. \end{cases}$$

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Also, the norm of the vector $\mathbf{a}$ with coordinates $(a_1, a_2, a_3, a_4)$ is given by $|a| = \sqrt{g(\mathbf{a}, \mathbf{a})}$.

For a given vectors $\mathbf{x}, \mathbf{y}, \mathbf{z}$ with the coordinates $(x_1, x_2, x_3, x_4)$, $(y_1, y_2, y_3, y_4)$, $(z_1, z_2, z_3, z_4)$, the cross product is defined by the relation

\[
x \times y \times z = \begin{vmatrix}
0 & e_3 & e_4 \\
e_1 & x_1 & x_3 \\
e_2 & x_2 & x_4 \\
y_1 & y_2 & y_4 \\
z_1 & z_2 & z_4 \\
\end{vmatrix},
\]

where $e_1$ are the standard basis vectors. Suppose that $\beta : I \subset \mathbb{R} \to G^4$ is a curve of $C^0$ defined in the Galilean space $G^4$ such that

\[
\beta(t) = (u(t), v(t), w(t), r(t)).
\]

when the curve $\beta$ is parametrized by the Galilean arc-length $s$, it will be written in the form

\[
\beta(s) = (s, v(s), w(s), r(s)).
\]

Now, Frenet Serret vector fields of $\beta(s)$ is obtained by the next formulas

\[
T(s) = \beta'(s) = (1, v'(s), w'(s), r'(s)),
\]

\[
N(s) = \frac{1}{\kappa_1(s)} T'(s) = \frac{1}{\kappa_1(s)} (1, v''(s), w''(s), r''(s)),
\]

\[
B_1(s) = \frac{1}{\kappa_2(s)} (0, \frac{d}{ds}(v''(s)), \frac{d}{ds}(w''(s)), \frac{d}{ds}(r''(s))),
\]

\[
B_2(s) = \frac{\zeta}{\kappa_2(s)} T(s) \times N(s) \times B_1(s),
\]

where the coefficient $\zeta = \pm 1$, is chosen by the criterion $\det(T(s), N(s), B_1(s), B_2(s)) = 1$. Here $T(s)$, $N(s)$, $B_1(s)$ and $B_2(s)$ are the Tangent, the Normal, the First bi-normal, and the Second bi-normal vectors of $\beta(s)$.

Moreover, $k_1(s)$ and $k_2(s)$ are the first, and the second curvatures, that defined by the following formulas

\[
k_1(s) = \left|\beta''(s)\right|_{G^4} = \sqrt{(v''(s))^2 + (w''(s))^2 + (r''(s))^2},
\]

\[
k_2(s) = \left|N'(s)\right|_{G^4} = \sqrt{g(N'(s), N'(s))}.
\]

Also, the third curvature function of the curve $\beta(s)$ is given by $k_3(s) = g(B'_1(s), B_2(s))$ and the Frenet equations in $G^4$ are given by

\[
T'(s) = k_1(s) N(s),
\]

\[
N'(s) = k_2(s) B_1(s),
\]

\[
B'_1(s) = -k_2(s) N(s) + k_3(s) B_2(s),
\]

\[
B'_2(s) = -k_3(s) B_1(s).
\]

### 3 Frenet Equations in Equiform Geometry in $G^4$

Let $\gamma : I \subset \mathbb{R} \to G^4$ be a parametrized curve and $s$ is the arc length of $\gamma$, the equiform parameter ($\sigma$) of the parametrized curve $\gamma(s)$ will be defined by

\[
\sigma = \int ds / \rho,
\]

$\rho = \frac{1}{\kappa_1}$ is called the radius of curvature for the curve $\gamma(s)$. Suppose $h$ is a homothety with center at the origin and the coefficient $\lambda$. If we take $\tilde{\gamma} = h(\gamma)$, then $\tilde{s} = \lambda s$ and $\tilde{\rho} = \lambda \rho$, where $s$ and $\rho$ are the arc length parameter and the radius of curvature of $\gamma$ respectively. Hence $\sigma$ is an equiform invariant parameter of $\gamma$.

Let $\{V_1, V_2, V_3, V_4\}$ be an equiform invariant tetrahedron of the curve $\gamma$. The first derivatives of such vectors with respect to the euform parameter $\sigma$ are given by:

\[
\begin{align*}
\dot{V}_1 &= K_1 V_1 + V_2, \\
\dot{V}_2 &= K_1 V_2 + K_2 V_3, \\
\dot{V}_3 &= -K_2 V_2 + K_1 V_3 + K_3 V_4, \\
\dot{V}_4 &= -K_3 V_3 + K_1 V_4,
\end{align*}
\]

where $K_1 = \rho$, $K_2 = \frac{\kappa_2}{\kappa_1}$, $K_3 = \frac{\kappa_3}{\kappa_1}$ are the Equiform curvatures of the curve $\gamma$.

These formulas can be written in matrix form as follows:

\[
\begin{bmatrix}
\dot{V}_1 \\
\dot{V}_2 \\
\dot{V}_3 \\
\dot{V}_4
\end{bmatrix} =
\begin{bmatrix}
K_1 & 0 & 0 & 0 \\
0 & K_1 & K_2 & 0 \\
0 & -K_2 & K_1 & K_3 \\
0 & 0 & -K_3 & K_1
\end{bmatrix}
\begin{bmatrix}
V_1 \\
V_2 \\
V_3 \\
V_4
\end{bmatrix}.
\]

The Equiform curvatures are defined as:

\[
K_1 = \frac{1}{\rho^2} < V_i, V_i >, \quad i = 1, 2, 3, 4,
\]

\[
K_2 = \frac{1}{\rho^2} < V_2, V_3 > = -\frac{1}{\rho^2} < V_2, V_3 >,
\]

\[
K_3 = \frac{1}{\rho^2} < V_3, V_4 > = -\frac{1}{\rho^2} < V_3, V_4 >
\]

or

\[
K_j = \begin{cases} 
\frac{1}{\rho^2} g(V_j, V_j), & i = 1, 2, 3, 4, \text{ for } j = 1, \\
\frac{1}{\rho^2} g(V_j, V_{j+1}), & i = 1, 2, 3, 4, \text{ for } j = 2, 3.
\end{cases}
\]

### 4 Equiform Normal Curves in $G^4$

**Definition 1.** Suppose that $\gamma : I \subset \mathbb{R} \to G^4$ is a parametrized curve in $G^4$. If the orthogonal components of $V_1(\sigma)$ contains a fixed point for all $\sigma \in I$, the curve $\gamma$ is called an Equiform Normal curve.
For an equiform normal curve \( \gamma(\sigma) \) in \( G^4 \), the position vector is defined by

\[
\gamma(\sigma) = \mu_1(\sigma)V_2(\sigma) + \mu_2(\sigma)V_3(\sigma) + \mu_3(\sigma)V_4(\sigma),
\]
where \( \mu_1(\sigma), \mu_2(\sigma), \) and \( \mu_3(\sigma) \) are differentiable functions of \( \sigma \). Differentiating equation (3), we obtain

\[
\frac{d\gamma(\sigma)}{d\sigma} = \mu_1(K_1V_2 + K_2V_3) + \frac{d\mu_1}{d\sigma}V_2 + \mu_2(-K_2V_2 + K_1V_3) + K_3V_4 + \frac{d\mu_2}{d\sigma}V_3 + \mu_3(-K_3V_3 + K_4V_4) + \frac{d\mu_3}{d\sigma}V_4,
\]

\[
\frac{d\gamma(\sigma)}{d\sigma} = (K_1\mu_1 - K_2\mu_2 + \frac{d\mu_1}{d\sigma})V_2(\sigma)
\]

\[
+ (K_2\mu_1 + K_1\mu_2 - K_3\mu_3 + \frac{d\mu_1}{d\sigma})V_3(\sigma)
\]

\[
+ (K_3\mu_2 + K_4\mu_3 - \frac{d\mu_2}{d\sigma})V_4(\sigma)
\]

\[
= \alpha_1(\sigma)V_2(\sigma) + \alpha_2(\sigma)V_3(\sigma) + \alpha_3(\sigma)V_4(\sigma),
\]
where \( \alpha_1(\sigma), \alpha_2(\sigma) \) and \( \alpha_3(\sigma) \) are differentiable functions of \( \sigma \).

**Theorem 1.** Let \( \gamma(\sigma) \) be an Equiform Normal curve in \( G^4 \) with Equiform curvatures \( K_1, K_2 \) and \( K_3 \). The position vector of \( \gamma(\sigma) \) is defined if \( K_1, K_2 \) and \( K_3 \) satisfy the following integro-differential equation in \( \mu_2 \)

\[
\frac{d}{d\sigma} \left[ \mu_2 \exp \left( \int K_1(\sigma) d\sigma \right) \right] + \frac{3}{2} \sum_{j=2}^{3} K_j(\sigma) \int_0^\sigma K_j(\theta) \mu_2(\theta) \exp \left( \int K_1(\theta) d\theta \right) d\theta = C_0K_3(\sigma) - C_1K_2(\sigma).
\]

**Proof.** The system of differential equations that constitutes the normal curvatures is given by

\[
\frac{d\mu_1}{d\sigma} + K_1(\sigma)\mu_1 = K_2(\sigma)\mu_2 + \alpha_1(\sigma),
\]

\[
\frac{d\mu_2}{d\sigma} + K_1(\sigma)\mu_2 = -K_2(\sigma)\mu_1 + \alpha_2(\sigma),
\]

\[
\frac{d\mu_3}{d\sigma} + K_1(\sigma)\mu_3 = -K_3(\sigma)\mu_2 + \alpha_3(\sigma).
\]

The system (4)-(6) is a nonhomogeneous linear system with variable coefficients. First, we solve the corresponding homogeneous system, then obtain a particular solution for the nonhomogeneous term. The associated homogeneous system is given by

\[
\frac{d\mu_1}{d\sigma} + K_1(\sigma)\mu_1 = K_2(\sigma)\mu_2,
\]

\[
\frac{d\mu_2}{d\sigma} + K_1(\sigma)\mu_2 = -K_2(\sigma)\mu_1 + K_3(\sigma)\mu_3,
\]

\[
\frac{d\mu_3}{d\sigma} + K_1(\sigma)\mu_3 = -K_3(\sigma)\mu_2.
\]

Our objective is to convert this system into one differential equation in one unknown function. First, we multiply the equations (7), (8) and (9) by the integrating factor \( \exp(\int K_1(\sigma) d\sigma) \), then we have

\[
\frac{d}{d\sigma} \left[ \mu_1 \exp \left( \int K_1(\sigma) d\sigma \right) \right] = K_2(\sigma)\mu_2 \exp \left( \int K_1(\sigma) d\sigma \right),
\]

\[
\frac{d}{d\sigma} \left[ \mu_2 \exp \left( \int K_1(\sigma) d\sigma \right) \right] = (-K_2(\sigma)\mu_1 + K_3(\sigma)\mu_3)
\]

\[
\exp \left( \int K_1(\sigma) d\sigma \right),
\]

\[
\frac{d}{d\sigma} \left[ \mu_3 \exp \left( \int K_1(\sigma) d\sigma \right) \right] = -K_3(\sigma)\mu_2 \exp \left( \int K_1(\sigma) d\sigma \right).
\]

By adding the first and third equations of equation (10), one gets

\[
\frac{d}{d\sigma} \left[ (\mu_1 + \mu_3) \exp \left( \int K_1(\sigma) d\sigma \right) \right] + \frac{3}{2} \sum_{j=2}^{3} K_j(\sigma) \int_{\theta}^{\sigma} K_j(\theta) \mu_2(\theta) \exp \left( \int K_1(\theta) d\theta \right) d\theta = C_0K_3(\sigma) - C_1K_2(\sigma).
\]

Next, we discuss some cases of equation (12).

**First case**

If the curvatures \( K_2 \) and \( K_3 \) are considered to be functions in \( \sigma \), while \( K_1 \) is constant and \( \sigma_0 = 0 \). We study the case when \( K_2(\sigma) = e^{a\sigma} \) and \( K_3(\sigma) = e^{b\sigma} \) where \( a, b \in \mathbb{R} \) and \( K_1 = k_1 \in \mathbb{R} \). Let \( \gamma(\sigma) = \mu_2(\sigma)e^{\frac{b\sigma}{a}} \), then equation (12) becomes

\[
\frac{d\gamma}{d\theta} + \int_0^\theta \left( e^{a(\sigma+\theta)} + e^{b(\sigma+\theta)} \right) y(\theta) d\theta = C_0e^{a\sigma} - C_1e^{b\sigma}.
\]

By applying the operator \( \Theta(D_\sigma) = (D_\sigma - a)(D_\sigma - b) \) (where \( D_\sigma = \frac{d}{d\sigma} \)), on equation (13), one gets

\[
\left(D_\sigma^3 - (a+b)D_\sigma^2 + (ab + e^{2a\sigma} + e^{2b\sigma})D_\sigma - (2(a-b)e^{2a\sigma} + (2a-b)e^{2b\sigma}) \right)y = 0,
\]

when \( a = b \), the solution of equation (14) is given by

\[
y(\sigma) = \hat{C}_1 \cos \left( \frac{\sqrt{2} e^{a\sigma}}{a} \right) + \hat{C}_2 \sin \left( \frac{\sqrt{2} e^{a\sigma}}{a} \right) + \hat{C}_3 G_{1,1}^{2,1}(\frac{e^{2a\sigma}}{2a^2}; 0, 0.0.-1/2) e^{a\sigma}.
\]
where \( G_{3,1}^{2,0}(\frac{\sigma}{\sqrt{2}}, 0, 1/2) \) is the Meijer G-function and defined by [14]

\[
G_{p,q}^{m,n}(a_1, a_2, \ldots, a_m; b_1, b_2, \ldots, b_n; z) = \frac{1}{2\pi i} \int_L \left( \prod_{j=1}^m (1 - a_j + s) \right) \left( \prod_{j=m+1}^n (1 - b_j - s) \right) s^q e^{zs} ds.
\]

**Second case**

If \( K_2 \) and \( K_3 \) are constants, while \( K_1(\sigma) \) is an arbitrary function in \( \sigma \). By substituting into equation (12) and differentiating both sides, we have

\[
\frac{d^2}{d\sigma^2} \left[ \mu_1(\sigma)e^{\int K_1(\sigma)d\sigma} + (K_2^2 + K_3^2)(\mu_2(\sigma)e^{\int K_1(\sigma)d\sigma}) \right] = 0. \quad (15)
\]

Hence, the solution of equation (15) is given by

\[
\mu_1(\sigma) = e^{-\int K_1(\sigma)d\sigma} \left( C_1 \cos \sqrt{K_2^2 + K_3^2} + C_2 \sin \sqrt{K_2^2 + K_3^2} \right). \quad (16)
\]

Consequently, we can obtain the corresponding solutions of \( \mu_1(\sigma) \) by integrating the first equation of system (10) and \( \mu_1(\sigma) \) from equation (11). Hence we calculate the solution of the homogeneous system (7)-(9). Based on the variation of parameters, a particular solution of the non-homogeneous system (4)-(6) is obtained.

**Third case**

When all the curvatures \( K_1, K_2, \) and \( K_3 \) are constants, it is treated as in the second case considering \( K_1 \) is a constant. Then the solutions \( \mu_1(\sigma), \mu_2(\sigma), \) and \( \mu_3(\sigma) \) of the homogeneous system (4)-(6) are given by

\[
\mu_1(\sigma) = e^{-\int K_1(\sigma)d\sigma} \left( C_1 \sin \sqrt{K_2^2 + K_3^2} + C_2 \cos \sqrt{K_2^2 + K_3^2} \right)
\]

\[
\mu_2(\sigma) = e^{-\int K_1(\sigma)d\sigma} \left( C_1 \cos \sqrt{K_2^2 + K_3^2} + C_2 \sin \sqrt{K_2^2 + K_3^2} \right)
\]

\[
\mu_3(\sigma) = e^{-\int K_1(\sigma)d\sigma} \left( C_1 \sin \sqrt{K_2^2 + K_3^2} - C_2 \cos \sqrt{K_2^2 + K_3^2} - \frac{e^{i\phi}}{K_1} \right)
\]

**Corollary 1.** Let \( \gamma(\sigma) \) be an Equiform Normal curve in \( G^4 \) with non vanishing constant Equiform curvatures \( K_1, K_2, \) and \( K_3 \) for the homogeneous system (4)-(6). Then the following statements are satisfied.

1. \( 1 < \gamma(\sigma), V_2(\sigma) > = \rho_2 e^{-\int K_1(\sigma)d\sigma} \left( C_1 \sin \sqrt{K_2^2 + K_3^2} - C_2 \cos \sqrt{K_2^2 + K_3^2} \right) \),

2. \( 2 < \gamma(\sigma), V_2(\sigma) > = \rho_2 e^{-\int K_1(\sigma)d\sigma} \left( C_1 \cos \sqrt{K_2^2 + K_3^2} + C_2 \sin \sqrt{K_2^2 + K_3^2} \right) \),

3. \( 3 < \gamma(\sigma), V_2(\sigma) > = \rho_2 e^{-\int K_1(\sigma)d\sigma} \left( C_1 \sin \sqrt{K_2^2 + K_3^2} - C_2 \cos \sqrt{K_2^2 + K_3^2} - \frac{e^{i\phi}}{K_1} \right) \),

where \( C_1, C_2, \) and \( C_3 \) are constants.

**Proof.** Let \( \gamma(\sigma) \) be an Equiform Normal curve in \( G^4 \) with non vanishing constant Equiform curvatures \( K_1, K_2, \) and \( K_3 \). Then the Equiform curve \( \gamma(\sigma) \) can be written in the following form

\[
\gamma(\sigma) = \frac{K_2e^{-\int K_1(\sigma)d\sigma}}{\sqrt{K_2^2 + K_3^2}} \left( C_1 \sin \sqrt{K_2^2 + K_3^2} - C_2 \cos \sqrt{K_2^2 + K_3^2} \right) V_2(\sigma) + \frac{e^{\int K_1(\sigma)d\sigma}}{\sqrt{K_2^2 + K_3^2}} \left( C_1 \cos \sqrt{K_2^2 + K_3^2} + C_2 \sin \sqrt{K_2^2 + K_3^2} \right) V_1(\sigma)
\]

Taking the dot product of the both sides with \( V_2(\sigma), V_3(\sigma), \) and \( V_4(\sigma) \), we will obtain the result.

**Theorem 2.** Let \( \gamma(\sigma) \) be an Equiform Normal curve in \( G^4 \) with non vanishing constant Equiform curvatures \( K_1, K_2, \) and \( K_3 \) for the homogeneous system (4)-(6). Then there aren’t any curves that are congruent to \( \gamma(\sigma) \).

**Proof.** Let us define \( \zeta(\sigma) \) as

\[
\zeta(\sigma) = \gamma(\sigma) - \mu_1(\sigma)V_2(\sigma) - \mu_2(\sigma)V_3(\sigma) - \mu_3(\sigma)V_4(\sigma).
\]

By differentiate equation (18) with respect to \( \sigma \), we will obtain the following result

\[
\zeta(\sigma) = V_1(\sigma) - \mu_1(\sigma)V_2(\sigma) - \mu_2(\sigma)V_3(\sigma) - \mu_3(\sigma)V_4(\sigma).
\]

By using equation (1), we have

\[
\zeta(\sigma) = V_1(\sigma) + [-\mu_1(\sigma) - K_1\mu_1(\sigma) + K_2\mu_2(\sigma)]V_2(\sigma) + [-\mu_2(\sigma) - K_2\mu_2(\sigma) - K_3\mu_3(\sigma)]V_3(\sigma) + [-\mu_3(\sigma) - K_3\mu_3(\sigma) - K_1\mu_3(\sigma)]V_4(\sigma).
\]

Using equation (17) gives

\[
\zeta(\sigma) = V_1(\sigma) + \left( \frac{\sqrt{K_2^2 + K_3^2} - 1}{K_3\sqrt{K_2^2 + K_3^2}} \right) K_1\mu_2(\sigma) e^{-\int K_1(\sigma)d\sigma} C_1 V_3(\sigma)
\]

i.e., \( \zeta(\sigma) \) not equal zero, whence \( \zeta(\sigma) \) is not a constant vector. Thus, \( \gamma(\sigma) \) does not congruent to a Normal curve.

**Conflict of Interest**

All authors declare that there is no conflict regarding the publication of this paper.

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