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Interactive Approach for Solving Multi-level Multi-objective Quadratic Fractional Programming Problems with Fuzzy Parameters in the Constraints

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Abstract: This paper presents an interactive approach for solving multi-level multi-objective quadratic fractional programming problems with fuzzy parameters in the constraints**.** Firstly, the concept of α-cut is applied to transform the fuzzy mathematical problem into a common deterministic one. Then, the quadratic fractional objective functions in each level are transformed into non-linear objective functions based on a proposed transformation. Secondly, the interactive approach was extended to solve such problem. Finally, an illustrative numerical example is introduced to demonstrate the applicability and performance of the proposed approach.

Keywords: Multi-level programming; Multi-objective programming; Quadratic fractional programming problems; Fuzzy sets; interactive approach.

1 Introduction

Hierarchical decision structures are prevalent in government systems, competitive economic organizations, supply chains, agriculture, biofuel production, and so on Baky [1]. The area of multi-level mathematical programming (MLMP) provides the art and science of making such decisions. Several mathematical models for such problems have been exhibited [2,3,4,5,6]. The fundamental idea of MLMP methodology is that the first-level decision maker (FLDM) decides his/her objectives and/or choices, hence, asks each inferior level of the association for their solutions, which obtained individually. The lower level decision makers' choices are then presented and altered by the FLDM in light of the general advantage for the association [1,2].

As of late, MLMP has been deeply deliberated and several methods have been exhibited for solving such problems [1,7,8,9,10]. An interactive algorithm for bi-level decision-making problem has been proposed by Shi and Xia [11]. Interactive fuzzy programming has been extended by Sakawa et al. [12] to thoroughly consider in MLMP problems with fuzzy parameters. The balance space approach was extended to solve MLMP problems by Abo-Sinna and Baky [2]. Baky [13] presented fuzzy goal programming (FGP) methodology to takle decentralized bi-level programming problems (BL-PP). A further extension of the FGP approach for BL-PP with fuzzy demands was considered by Baky et al. [14]. Chen and Chen utilized a fuzzy variable for relative satisfactions among leader- and –follower to solve the decentralized BL-PP [7]. Arora and Gupta exhibited interactive FGP methodology for BL-PP with the merits of dynamic programming [15].

Fractional programming deals with the optimization of one or more ratios of functions subject to set constraints. Over the past four decades, fractional programming has become one of the planning tools. It is routinely applied in engineering, business, finance, economics and other disciplines [16,17,18,19].

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During the past few decades, MLMP [1,2,7] as well as bi-level mathematical programming (BLMP) problems [13,15,16,20] have been deeply studied and many methodologies have been developed for solving such problems. The use of the concept of the membership function of fuzzy set theory to multi-level programming problems for satisfactory decisions was first presented by Lai in [21]. Sakawa et al. [19] developed interactive fuzzy programming for MLMP with fuzzy parameters. Also, Abo-Sinna and Baky [2] presented balance space approach for multi-level multi-objective programming problems. Baky [13] studied FGP algorithm for solving a decentralized bi-level multi-objective programming problem.

Pramanik and Roy [22] adopted fuzzy goals to specify the decision variables of higher-level DMs and proposed weighted/ unweighted FGP models for solving MLMP to obtain a satisfactory solution. Arora and Gupta [15] extended this approach by employing dynamic programming to solve the FGP model in sequence for bi-level programming problems. Multi-level decision-making problems were recently studied by Chen and Chen [7].

The optimization of ratios of criteria gives more insight into the situation than the optimization of each criterion. Indeed, in such situations, it is often a question of optimizing a ratio debt/equity, output/employee, actual cost/standard cost, profit/cost, inventory/sales, student/cost, doctor/patient, and so on subject to some constraints [12,12,17,19]. Such type of problems in large hierarchical organizations of complex and conflicting multi-objectives formulate ML-MOFP problems. Recently Lachhwani [9] proposed FGP approach presented by Baky [1] with some modifications for ML-MOFP problems. Baky et al. [17] presented fuzzy goal programming procedures to bi-level multi-objective fractional programming. Also, computer-oriented technique was extended by Helmy et al. [23] to solve a special class of ML-MOFP problems.

When ML-MOFP problems are being formulated, the parameters of objective functions and constraints are normally assigned by experts. In most real situations, the possible values of these parameters are imprecisely or ambiguously known to the experts. Therefore, it would be more appropriate for these parameters to be represented as fuzzy numerical data that can be represented by fuzzy numbers [12,14,24]. The resulting mathematical programming problem involving fuzzy parameters would be viewed as a more realistic version than the conventional one. From this viewpoint, the parameters involved in the objective functions and the constraints of the ML-MOFP problem are assumed to be characterized by fuzzy numbers.

The current research presents an interactive approach for solving multi-level multi-objective quadratic fractional programming problems with fuzzy parameters in the constraints.

The remainder of this paper is organized as follows: the next section introduces the formulation of the problem; section 4 gives the formulation of crisp set of constraints and solution concept; in section 5 an illustrative example will be introduced.

2 Problem Formulation

Consider the hierarchical system be composed of a p-level DM. Let the DM at the i^{th}-level denoted by DM_i controls over the decision variable $\boldsymbol{x_i} = (x_{i1}, x_{i2}, \cdots, x_{in_i}) \in R^{n_i},\ i = 1,2,\cdots,p$. where $\boldsymbol{x} = (\boldsymbol{x_1}, \boldsymbol{x_2}, \cdots, \boldsymbol{x_p}) \in R^n$ and $n = \sum_{i=1}^{p} n_i$ and furthermore it is assumed that

$$F_i(\boldsymbol{x_1}, \boldsymbol{x_2}, \cdots, \boldsymbol{x_p}) \equiv F_i(\boldsymbol{x}) : R^{n_1} \times R^{n_2} \times \cdots \times R^{n_p} \rightarrow R^{k_i}, \quad i=1,\cdots,p, \tag{1}$$

are the vector of quadratic fractional objective functions for $DM_i,\ i = 1,2,\cdots,p$. Mathematically, ML-MOQFP problem with fuzzy parameters in the constraints follows as [1,3,23]:

$[1^{st}$ ***Level***$]$

$$\underbrace{max}_{\boldsymbol{x_1}} \; F_1(\boldsymbol{x}) = \frac{\boldsymbol{x^T} Q^{1j} \boldsymbol{x} + \boldsymbol{c^{1j}} \boldsymbol{x} + \alpha^{1j}}{\boldsymbol{x^T} R^{1j} \boldsymbol{x} + \boldsymbol{d^{1j}} \boldsymbol{x} + \beta^{1j}}, \qquad j = 1,2,\cdots,k_1, \tag{2}$$

where $\boldsymbol{x_2}, \boldsymbol{x_3}, \cdots, \boldsymbol{x_p}$ solves

$[2^{nd}$ ***Level***$]$

$$\underbrace{max}_{\boldsymbol{x_2}} \; F_2(\boldsymbol{x}) = \frac{\boldsymbol{x^T} Q^{2j} \boldsymbol{x} + \boldsymbol{c^{2j}} \boldsymbol{x} + \alpha^{2j}}{\boldsymbol{x^T} R^{2j} \boldsymbol{x} + \boldsymbol{d^{2j}} \boldsymbol{x} + \beta^{2j}}, \qquad j = 1,2,\cdots,k_2, \tag{3}$$

$\vdots$

where $\boldsymbol{x_p}$ solves

$[p^{th}$ ***Level***$]$

$$\underbrace{max}_{\boldsymbol{x_p}} \ F_p(\boldsymbol{x}) = \frac{\boldsymbol{x^T} Q^{pj} \boldsymbol{x} + \boldsymbol{c^{pj}} \boldsymbol{x} + \alpha^{pj}}{\boldsymbol{x^T} R^{pj} \boldsymbol{x} + \boldsymbol{d^{pj}} \boldsymbol{x} + \beta^{pj}}, \qquad j = 1,2,\cdots,k_p, \tag{4}$$

subject to

$$\boldsymbol{x} \in \tilde{G} = \left\{ \boldsymbol{x} \in R^n \;\middle|\; \tilde{A}_1 \boldsymbol{x_1} + \tilde{A}_2 \boldsymbol{x_2} + \cdots + \tilde{A}_p \boldsymbol{x_p} \leq \widetilde{\boldsymbol{b}}, \quad \boldsymbol{x} = 0, \ \widetilde{\boldsymbol{b}} \in R^m \right\}, \tag{5}$$

where Q^{ij} is an $n \times n$ negative definite matrix, R^{ij} is an $n \times n$ positive semi-definite matrix $\boldsymbol{c}^{ij}$, $\boldsymbol{d}^{ij}$ are n-vectors, $\widetilde{\boldsymbol{A}}_{\boldsymbol{i}}$ is an $m \times n_i$, $i = 1,2,\cdots,p$ fuzzy matrices and $\widetilde{\boldsymbol{b}}$ is an m-vector of fuzzy parameters. It is customary to assume that $D_{ij}(\boldsymbol{x}) > 0, \ \forall \, \boldsymbol{x} \in \tilde{G}$, also α^{ij} and β^{ij} are constants and $\tilde{G}$ represents the multi-level convex constraints feasible choice set in the fuzzy environment.

3 Formulation of Crisp Set of Constraints and Solution Concept

Let $\mu_{\widetilde{\boldsymbol{A}}_{\boldsymbol{i}}}$, and $\mu_{\widetilde{\boldsymbol{b}}}$ be the membership functions which represents the fuzzy coefficients matrices $\widetilde{\boldsymbol{A}}_{\boldsymbol{i}}$ and the fuzzy numbers in the corresponding vector $\widetilde{\boldsymbol{b}}$ in model $(2-5)$ respectively. The α-cuts of $\widetilde{\boldsymbol{A}}_{\boldsymbol{i}}$ and $\widetilde{\boldsymbol{b}}$ are defined as [7,14,20,21,24]:

$$(\boldsymbol{A_i})_\alpha = \left\{ \boldsymbol{A_i} \in \left[(\boldsymbol{A_i})^L_\alpha, \ (\boldsymbol{A_i})^U_\alpha \right] \;\middle|\; \mu_{\widetilde{\boldsymbol{A}}_{\boldsymbol{i}}} \geq \alpha, \quad \boldsymbol{A_i} \in S\left(\widetilde{\boldsymbol{A}}_{\boldsymbol{i}}\right) \right\}, \tag{6}$$

$$(\boldsymbol{b})_\alpha = \left\{ \boldsymbol{b} \in \left[(\boldsymbol{b})^L_\alpha, \ (\boldsymbol{b})^U_\alpha \right] \;\middle|\; \mu_{\widetilde{\boldsymbol{b}}} \geq \alpha, \, \boldsymbol{b} \in S\left(\widetilde{\boldsymbol{b}}\right) \right\}, \tag{7}$$

where $S\left(\widetilde{\boldsymbol{b}}\right)$, and $S\left(\widetilde{\boldsymbol{A}}_{\boldsymbol{i}}\right)$ are the supports of the corresponding vectors and matrix of fuzzy numbers.

Let $\alpha \in [0,1]$, be the grade of satisfaction associated with the set of constraints of the ML-MOQFP problem. The fuzzy constraints in (5) are to be understood with respect to the ranking relation $\sum_{j=1}^{n} \tilde{A}_{ij} x_j \leq_\alpha \tilde{b}_i$, between the fuzzy vectors. Thus, for $\alpha \in [0,1]$, the feasible set of the ML-MOQFP problem can be described as:

$$\boldsymbol{x} \in \boldsymbol{G}_\alpha = \left\{ \boldsymbol{x} \in R^n \;\middle|\; \begin{array}{l} \sum_{j=1}^{n} \left(\tilde{A}_{ij}\right)^U_\alpha x_j \leq \left(\tilde{b}_i\right)^U_\alpha, \ x_j = 0 \\ \sum_{j=1}^{n} \left(\tilde{A}_{ij}\right)^L_\alpha x_j \leq \left(\tilde{b}_i\right)^L_\alpha, \ i = 1,2,\cdots,m \end{array} \right\} \tag{8}$$

3.1 Nonlinear Model Development of ML-MOQFP Problem

Now, we make further extensions on the article of Lachhwani [9], to develop a methodology for obtaining the equivalent non-linear model of the ML-MOQFP problem. Since the MOQFP problem for the i^{th}-level DM may be written as [9,22]:

$$\underbrace{max}_{\boldsymbol{x_i}} F_i(\boldsymbol{x}) = \underbrace{max}_{\boldsymbol{x_i}} \left(f_{i1}(\boldsymbol{x}), \ f_{i2}(\boldsymbol{x}), \ \cdots, f_{ik_i}(\boldsymbol{x}) \right), \tag{9}$$

subject to

$$\boldsymbol{x} \in \boldsymbol{G}_\alpha = \left\{ \boldsymbol{x} \in R^n \;\middle|\; \begin{array}{l} \sum_{j=1}^{n} \left(\tilde{A}_{ij}\right)^U_\alpha x_j \leq \left(\tilde{b}_i\right)^U_\alpha, \ x_j = 0 \\ \sum_{j=1}^{n} \left(\tilde{A}_{ij}\right)^L_\alpha x_j \leq \left(\tilde{b}_i\right)^L_\alpha, \ i = 1,2,\cdots,m \end{array} \right\} \tag{10}$$

where

$$f_{ij}(\boldsymbol{x}) = \frac{Q^{ij}_1 \boldsymbol{x}^2_1 + Q^{ij}_2 \boldsymbol{x}^2_2 + \cdots + Q^{ij}_p \boldsymbol{x}^2_p + \boldsymbol{c}^{ij}_1 \boldsymbol{x}_1 + \cdots + \boldsymbol{c}^{ij}_p \boldsymbol{x_p} + \alpha^{ij}}{R^{ij}_1 \boldsymbol{x}^2_1 + R^{ij}_2 \boldsymbol{x}^2_2 + \cdots + R^{ij}_p \boldsymbol{x}^2_p + \boldsymbol{d}^{ij}_1 \boldsymbol{x}_1 + \cdots + \boldsymbol{d}^{ij}_p \boldsymbol{x_p} + \beta^{ij}} \qquad \forall \, i,j \tag{11}$$

Thus, for the sake of simplicity in this chapter we employ the representation of eq. (11) in order to deal with the ML-MOQFP problem. Let us take the transformation [5,6]:

$$y^{ij} = \frac{1}{R^{ij}_1 \boldsymbol{x}^2_1 + R^{ij}_2 \boldsymbol{x}^2_2 + \cdots + R^{ij}_p \boldsymbol{x}^2_p + \boldsymbol{d}^{ij}_1 \boldsymbol{x}_1 + \cdots + \boldsymbol{d}^{ij}_p \boldsymbol{x_p} + \beta^{ij}}, \tag{12}$$

which is equivalent to:

$$y^{ij}\left(R_1^{ij}\boldsymbol{x_1^2}+R_2^{ij}\boldsymbol{x_2^2}+\cdots+R_p^{ij}\boldsymbol{x_p^2}+\boldsymbol{d_1^{ij}x_1}+\cdots+\boldsymbol{d_p^{ij}x_p}+\beta^{ij}\right)=1, \tag{13}$$

So, each quadratic fractional objective function is transformed into the following nonlinear function:

$$f_{ij}(\boldsymbol{x},y)=\left(Q_1^{ij}\boldsymbol{x_1^2}+Q_2^{ij}\boldsymbol{x_2^2}+\cdots+Q_p^{ij}\boldsymbol{x_p^2}+\boldsymbol{c_1^{ij}x_1}+\cdots+\boldsymbol{c_p^{ij}x_p}+\alpha^{ij}\right)y^{ij} \tag{14}$$

Based on the equation (14), the nonlinear model of the MOQFP problem for i^{th} level decision maker is formulated as follows:

$$\underbrace{max}_{\boldsymbol{x_i}}\left[Q_1^{ij}\boldsymbol{x_1^2}+Q_2^{ij}\boldsymbol{x_2^2}+\cdots+Q_p^{ij}\boldsymbol{x_p^2}+\boldsymbol{c_1^{ij}x_1}+\boldsymbol{c_2^{ij}x_2}+\cdots+\boldsymbol{c_p^{ij}x_p}+\alpha^{ij}\right]y^{ij}, \tag{15}$$

subject to

$$y^{ij}\left[R_1^{ij}\boldsymbol{x_1^2}+R_2^{ij}\boldsymbol{x_2^2}+\cdots+R_p^{ij}\boldsymbol{x_p^2}+\boldsymbol{d_1^{ij}x_1}+\cdots+\boldsymbol{d_p^{ij}x_p}+\beta^{ij}\right]=1, \quad \forall i,j \tag{16}$$

$$\sum_{j=1}^{n}\left(\tilde{A}_{ij}\right)_\alpha^U x_j\leq\left(\tilde{b}_i\right)_\alpha^U, \;\; x_j=0 \tag{17}$$

$$\sum_{j=1}^{n}\left(\tilde{A}_{ij}\right)_\alpha^L x_j\leq\left(\tilde{b}_i\right)_\alpha^L, \;\; i=1,2,\cdots,m \tag{18}$$

Following the above discussion thus, the nonlinear model of the ML-MOQFP problem is formulated as follows [3,5]:
[1^{st} *Level*]

$$\underbrace{max}_{\boldsymbol{x_1}}\; F_1(\boldsymbol{x},y)=\underbrace{max}_{\boldsymbol{x_1}}\left(f_{11}(\boldsymbol{x},\,y),\; f_{12}(\boldsymbol{x},\,y),\;\cdots,f_{1k_1}(\boldsymbol{x},\,y)\right), \tag{19}$$

where $\boldsymbol{x_2},\boldsymbol{x_3},\cdots,\,\boldsymbol{x_p}$ solves
[2^{nd} *Level*]

$$\underbrace{max}_{\boldsymbol{x_2}}\; F_2(\boldsymbol{x},\,y)=\underbrace{max}_{\boldsymbol{x_2}}\left(f_{21}(\boldsymbol{x},\,y),\; f_{22}(\boldsymbol{x},\,y),\;\cdots,f_{2k_2}(\boldsymbol{x},\,y)\right), \tag{20}$$

$\vdots$

where, $\boldsymbol{x_p}$ solves
[p^{th} *Level*]

$$\underbrace{max}_{\boldsymbol{x_p}}\; F_p(\boldsymbol{x},\,y)=\underbrace{max}_{\boldsymbol{x_p}}\left(f_{p1}(\boldsymbol{x},\,y),\; f_{p2}(\boldsymbol{x},\,y),\;\cdots,f_{pk_p}(\boldsymbol{x},\,y)\right), \tag{21}$$

subject to

$$y^{ij}\left[R_1^{ij}\boldsymbol{x_1^2}+R_2^{ij}\boldsymbol{x_2^2}+\cdots+R_p^{ij}\boldsymbol{x_p^2}+\boldsymbol{d_1^{ij}x_1}+\cdots+\boldsymbol{d_p^{ij}x_p}+\beta^{ij}\right]=1, \quad \forall i,j \tag{22}$$

$$\sum_{j=1}^{n}\left(\tilde{A}_{ij}\right)_\alpha^U x_j\leq\left(\tilde{b}_i\right)_\alpha^U, \;\; x_j=0 \tag{23}$$

$$\sum_{j=1}^{n}\left(\tilde{A}_{ij}\right)_\alpha^L x_j\leq\left(\tilde{b}_i\right)_\alpha^L, \;\; i=1,2,\cdots,m \tag{24}$$

where

$$f_{ij}(\boldsymbol{x},y)=\left[Q_1^{ij}\boldsymbol{x_1^2}+Q_2^{ij}\boldsymbol{x_2^2}+\cdots+Q_p^{ij}\boldsymbol{x_p^2}+\boldsymbol{c_1^{ij}x_1}+\cdots+\boldsymbol{c_p^{ij}x_p}+\alpha^{ij}\right]y^{ij}, \tag{25}$$

and the system of constraints in (22)-(24), at an α-level denoted by $\boldsymbol{S_\alpha}$, which form a nonempty convex set.

3.2 Interactive Approach for ML-MOQFP Problem with Fuzzy Parameters in the constraints

After the crisp nonlinear model of the problem, equations (19)-(24), is developed at a desired value of α, then the interactive approach is used to solve the nonlinear model. In the interactive mechanism, as explained previously obtaining the preferred solution mainly based on the ε-constraint method and the concept of satisfactoriness [11].

3.2.1 The First Level Decision Maker Problem

The first level decision-making problem of the ML-MONP model follows as:

$$\underbrace{max}_{\boldsymbol{x_1}} \; F_1(\boldsymbol{x},y) = \underbrace{max}_{\boldsymbol{x_1}} \left(f_{11}(\boldsymbol{x},\, y),\, f_{12}(\boldsymbol{x},\, y),\, \cdots, f_{1k_1}(\boldsymbol{x},\, y)\right), \tag{26}$$

subject to

$$(x_1, x_2, \cdots, x_p, y) \in \boldsymbol{S}_\alpha. \tag{27}$$

To obtain the α-Pareto optimal solution of the FLDM; the MODM problem, model (26)-(27), is transformed into the following SODM problem:

$$\underbrace{max}_{\boldsymbol{x_1}} \; f_{1j}(\boldsymbol{x},\, y) \qquad (j = \ell), \tag{28}$$

subject to

$$f_{1j}(\boldsymbol{x},\, y) \geq \delta_{1j}, \qquad (j = 1, 2, \cdots, k_1), \qquad (j \neq \ell), \tag{29}$$

$$(y_1, y_2, \cdots, y_p, t) \in \boldsymbol{S}_\alpha. \tag{30}$$

So the solution of the first level is obtained by executing algorithm **I**, as $\left(x_1^*, x_2^*, \cdots, x_p^*\right) = \left(x_1^F, x_2^F, \cdots, x_p^F\right)$.
The preferred solution of the i^{th}LDM problem is obtained by the following algorithm:

Algorithm I:

Step 1.	Set the satisfactoriness s_{iv}, $(i = 1, 2, \cdots, p)$, $v = 1, 2, \cdots$. Let $s_i = s_{i0}$ at the beginning, and let $s_i = s_{i1}, s_{i2}, s_{i3}, \cdots$, $(i = 1, 2, \cdots, p)$ respectively.
Step 2.	Set up the ε-constraint problem $P(\varepsilon_{-\ell}(s_{iv}))$, if $P(\varepsilon_{-\ell}(s_{iv}))$ has no solution or has an optimal solution with $f_{\ell j}(\boldsymbol{x},\, y) < \delta_{\ell j}$, then go to step 1, to adjust $s = s_{i(v+1)} < s_{iv}$. Otherwise, go to step 3.
Step 3.	If the decision maker is satisfied with $\left(x_1^*, x_2^*, \cdots, x_p^*\right)$, then it is the preferred solution of the i^{th}LDM, go to step 5. Otherwise, go to step 4.
Step 4.	Adjust satisfactoriness, let $s_{i(v+1)} > s_{iv}$ and go to step 2.
Step 5.	Stop.

3.2.2 The Second Level Decision Maker Problem

Secondly, the first level decision variable x_1^F should be included in the SLDM problem; hence, the problem of SLDM can be formulated as:

$$\underbrace{max}_{\boldsymbol{x_2}} \; F_2(\boldsymbol{x},y) = \underbrace{max}_{\boldsymbol{x_2}} \left(f_{21}(\boldsymbol{x},\, y),\, f_{22}(\boldsymbol{x},\, y),\, \cdots, f_{2k_2}(\boldsymbol{x},\, y)\right), \tag{31}$$

subject to

$$\left(x_1^F, x_2, \cdots, x_p\right) \in \boldsymbol{S}_\alpha. \tag{32}$$

The ε-constraint method is utilized to obtain the SODM as follows [11,25]:

$$\underbrace{max}_{\boldsymbol{x_2}} \; f_{2j}(\boldsymbol{x},\, y) \quad (j = \ell), \tag{33}$$

subject to

$$f_{2j}(\boldsymbol{x},\ y) \geq \delta_{2j}, \qquad (j = 1, 2, \cdots, k_2), \qquad (j \neq \ell), \tag{34}$$

$$\left(x_1^F, x_2, \cdots, x_p\right) \in \boldsymbol{S}_\alpha. \tag{35}$$

Solving model (33)-(35) to obtain the second level non-inferior solution $\left(x_1^F, x_2^S, \cdots, x_p^S\right)$ that is closest to the FLDM solution $\left(x_1^F, x_2^F, \cdots, x_p^F\right)$ by following algorithm **I**.

Therefore, we will test whether $\left(x_1^F, x_2^S, \cdots, x_p^S\right)$ is a preferred solution to the FLDM or it may be changed according to the following test [11,25]: If

$$\frac{\left\| F_1\left(x_1^F, x_2^F, \cdots, x_p^F\right) - F_1\left(x_1^F, x_2^S, \cdots, x_p^S\right) \right\|_2}{\left\| F_1\left(x_1^F, x_2^S, \cdots, x_p^S\right) \right\|_2} < \sigma^F \tag{36}$$

Then, $\left(x_1^F, x_2^S, \cdots, x_p^S\right)$ is a preferred solution to the FLDM, where σ^F is a small positive constant given by the FLDM.

3.2.3 The P^{th} Level Decision Maker Problem

Consequently, the decision variables controlled by the top levels $\left(x_1^F, x_2^S, \cdots, x_{(p-1)}^{(p-1)}\right)$ should be given to the P^{th} LDM problem; hence, the P^{th} LDM problem follows as:

$$\underbrace{max}_{x_p} \ F_p(\boldsymbol{x}, y) = \underbrace{max}_{x_p} \left(f_{p1}(\boldsymbol{x},\ y),\ f_{p2}(\boldsymbol{x},\ y),\ \cdots, f_{pk_p}(\boldsymbol{x},\ y)\right), \tag{37}$$

subject to

$$\left(x_1^F, x_2^S, \cdots, x_{(P-1)}^{(P-1)}, x_p\right) \in \boldsymbol{S}_\alpha. \tag{38}$$

Based on the ε-constraint method the SODM problem of the P^{th}LDM follows as:

$$\underbrace{max}_{x_p} \ f_{pj}(\boldsymbol{x},\ y) \qquad (j = \ell), \tag{39}$$

subject to

$$f_{pj}(\boldsymbol{x},\ y) \geq \delta_{pj}, \qquad (j = 1, 2, \cdots, k_p), \qquad (j \neq \ell), \tag{40}$$

$$\left(x_1^F, x_2^S, \cdots, x_{(P-1)}^{(P-1)}, x_p\right) \in \boldsymbol{S}_\alpha. \tag{41}$$

Solving model (39)-(41) the non-inferior solution of the P^{th}LDM, closest to the preferred solutions of the top levels $\left(x_1^F, x_2^S, \cdots x_{(P-1)}^{(P-1)}, x_p^p\right)$, is obtained by following algorithm **I**.

Now, we will test whether $\left(x_1^F, x_2^S, \cdots, x_p^p\right)$ is a preferred solution to the P^{th} LDM or it may be changed according to the following test: If

$$\frac{\left\| F_{(p-1)}\left(x_1^F, x_2^S, \cdots, x_p^{(p-1)}\right) - F_{(p-1)}\left(x_1^F, x_2^S, \cdots, x_p^p\right) \right\|_2}{\left\| F_{(p-1)}\left(x_1^F, x_2^S, \cdots, x_p^p\right) \right\|_2} < \sigma^{(p-1)} \tag{42}$$

Then, $\left(x_1^F, x_2^S, \cdots, x_p^p\right)$ is a preferred solution to the P^{th}LDM. Where $\sigma^{(p-1)}$ is a small positive constant given by the $(p-1)^{th}$LDM.

For the i^{th}LDM problem $\delta_{ij},\ b_{ij}$ *and* a_{ij} are defined as:

$$\delta_{ij} = (b_{ij} - a_{ij})\, s_i + a_{ij}, \qquad (i = 1, 2, \cdots, p), \qquad (j = 1, 2, \cdots, k_p), \tag{43}$$

$$b_{ij} = \underbrace{max}_{x \in \boldsymbol{S_\alpha}} f_{ij}(\boldsymbol{x},\ y), \qquad (i = 1,2,\cdots,p), \quad (j = 1,2,\cdots,k_p), \tag{44}$$

$$a_{ij} = \underbrace{max}_{x \in \boldsymbol{S_\alpha}} f_{ij}(\boldsymbol{x},\ y), \qquad (i = 1,2,\cdots,p), \quad (j = 1,2,\cdots,k_p), \tag{45}$$

where s_i is the satisfactoriness given by the i^{th} level decision maker [25].

4 Interactive Algorithm for ML-MOQFP Problem with Fuzzy Parameters in the Constraints

Following the discussion in the previous sections, the proposed interactive algorithm will be constructed for solving ML-MOQFP problem with fuzzy parameters in the constraints as follows:

Step 1. Formulate the crisp set of constraints for the ML-MOQFP problem at the given α-level, model (8).
Step 2. Formulate the ML-MONP model (19)-(24), of the ML-MOQFP problem.
Step 3. Calculate the individual maximum and minimum values for each objective function $f_{ij}(\boldsymbol{x},y)$.
Step 4. Set r=0.
Step 5. Execute the steps presented in Algorithm **I** to obtain a set of preferred solutions for the FLDM problem equations (28)-(30). The FLDM puts these solutions in order according to the following format: $\left(x_1^r,\cdots,x_p^r\right),\cdots,\left(x_1^{r+n},\cdots,x_p^{r+n}\right)$. Preferred ranking $\left(x_1^r,\cdots,x_p^r\right) \succ \left(x_1^{r+1},\cdots,x_p^{r+1}\right) \succ \cdots \succ \left(x_1^{r+n},\cdots,x_p^{r+n}\right)$.
Step 6. Given $x_1^F = x_1^r$, to the SLDM problem. Solve the SLDM problem, equations (33)-(36), following Algorithm **I** and obtain $\left(x_2^s,\ x_3^s,\cdots,x_p^s\right) = \left(x_2^*,x_3^*,\cdots,x_p^*\right)$.
Step 7. If $\frac{\left\|F_1\left(x_1^F,x_2^F,\cdots,x_p^F\right)-F_1\left(x_1^F,x_2^S,\cdots,x_p^S\right)\right\|_2}{\left\|F_1\left(x_1^F,x_2^S,\cdots,x_p^S\right)\right\|_2} < \sigma^F$, then go to Step 8. Otherwise go to Step 11.
Step 8. Given $\left(x_1^F,x_2^S,\cdots,x_{(P-1)}^{(P-1)}\right)$ to the P^{th}LDM problem, solve the P^{th}LDM problem model (39)-(41), following Algorithm **I** to obtain $\left(x_1^F,x_2^S,\cdots,x_{(P-1)}^{(P-1)},x_P^p\right)$.
Step 9. If $\frac{\left\|F_{(p-1)}\left(x_1^F,x_2^S,\cdots,x_p^{(p-1)}\right)-F_{(p-1)}\left(x_1^F,x_2^S,\cdots,x_p^p\right)\right\|_2}{\left\|F_{(p-1)}\left(x_1^F,x_2^S,\cdots,x_p^p\right)\right\|_2} < \sigma^{(p-1)}$, then go to Step 10. Otherwise go to Step 11.
Step 10. If the FLDM is satisfied with $\left(x_1^F,x_2^S,\cdots,x_p^p\right)$ and $F_1\left(x_1^F,x_2^S,\cdots,x_p^P\right)$, then $\left(x_1^F,x_2^S,\cdots,x_p^p\right)$ is the preferred solution of the ML-MOQFP problem, go to Step 12. Otherwise go to Step 11.
Step 11. Let $r = r+1$, and go to Step 7.
Step 12. Stop.

5 Illustrative Example

The following ML-MOQFP problem with fuzzy parameters in the constraints, is given to demonstrate the proposed interactive approach.

$[1^{st}\ \boldsymbol{Level}]$

$$\underbrace{max}_{\boldsymbol{x_1}}\left(f_{11} = \frac{-x_1^2-2x_2^2-x_3^2+5x_2+10}{x_1^2+3x_2+5},\quad f_{12} = \frac{-x_1^2-x_2^2-4x_3^2+5x_1+12}{x_1^2+3x_2+5}\right),$$

where $\boldsymbol{x_2},\boldsymbol{x_3}$ solves

$[2^{nd}\ \boldsymbol{Level}]$

$$\underbrace{max}_{\boldsymbol{x_2}}\left(f_{21} = \frac{-x_1^2-2x_2^2-2x_3^2+5x_3+6}{x_2^2+3x_1+1},\quad f_{22} = \frac{-3x_1^2-x_2^2-x_3^2+7x_3+8}{x_2^2+3x_1+1}\right),$$

where $\boldsymbol{x_3}$ solves

[3^{rd} ***Level***]

$$\underbrace{max}_{x_3}\left(f_{31}=\frac{-x_1^2-4x_2^2-x_3^2+6x_2+7}{x_3^2+5x_2+2},\quad f_{32}=\frac{-x_1^2-2x_2^2-x_3^2+9}{x_3^2+5x_2+2}\right),$$

subject to

$$\tilde{4}x_1+\tilde{7}x_2+\tilde{2}x_3\leq\widetilde{30},$$

$$\tilde{3}x_1-\tilde{0}x_2+\widetilde{14}x_3\leq\widetilde{18},$$

$$\tilde{7}x_2+\tilde{8}x_3\geq\widetilde{12},$$

Here, the fuzzy numbers are assumed to be $\mathscr{LR}$-fuzzy numbers and are given as follows:

$\tilde{4}=(4,2,1)_{LR},\tilde{7}=(7,4,2)_{LR},\tilde{2}=(2,2,3)_{LR},\tilde{3}=(3,2,2)_{LR},\tilde{0}=(0,1,2)_{LR},\widetilde{14}=(14,4,2)_{LR},\tilde{8}=(8,4,2)_{LR},\widetilde{30}=(30,5,10)_{LR},\widetilde{18}=(18,3,4)_{LR},\widetilde{12}=(12,2,8)_{LR}$

The ML-MOQFP problem is transformed into ML-MONP model based on the proposed transformation as follows:
[1^{st} ***Level***]

$$\underbrace{max}_{x_1}\begin{pmatrix} f_{11}(x,y)=\left(-x_1^2-2x_2^2-x_3^2+5x_2+10\right)y_1, \\ f_{12}(x,y)=\left(-x_1^2-x_2^2-4x_3^2+5x_1+12\right)y_1, \end{pmatrix},$$

where x_2,x_3 solves
[2^{nd} ***Level***]

$$\underbrace{max}_{x_2}\begin{pmatrix} f_{21}(x,y)=\left(-x_1^2-2x_2^2-2x_3^2+5x_3+6\right)y_2, \\ f_{22}(x,y)=\left(-3x_1^2-x_2^2-x_3^2+7x_2+8\right)y_2, \end{pmatrix}$$

where x_3 solves
[3^{rd} ***Level***]

$$\underbrace{max}_{x_3}\begin{pmatrix} f_{31}(x,y)=\left(-x_1^2-4x_2^2-x_3^2+6x_2+7\right)y_3, \\ f_{32}(x,y)=\left(-x_1^2-2x_2^2-x_3^2+9\right)y_3, \end{pmatrix},$$

subject to

$$\left(x_1^2+3x_2+5\right)y_1=1,$$

$$\left(x_2^2+3x_1+1\right)y_2=1,$$

$$\left(x_3^2+5x_2+2\right)y_3=1,$$

$$3.6x_1+6.2x_2+1.6x_3\leq 29,$$

$$4.2x_1+7.4x_2+2.6x_3\leq 32,$$

$$2.6x_1-0.4x_2+13.2x_3\leq 17.4,$$

$$3.4x_1+0.2x_2+14.4x_3\leq 18.8,$$

$$6.2x_2+7.2x_3\geq 11.6,$$

$$7.4x_2+8.4x_3\geq 13.6,$$

The individual maximum and minimum values are given in Table 1

Table 1: The individual maximum and minimum values

	$f_{11}(\boldsymbol{x},\boldsymbol{y})$	$f_{12}(\boldsymbol{x},\boldsymbol{y})$	$f_{21}(\boldsymbol{x},\boldsymbol{y})$	$f_{22}(\boldsymbol{x},\boldsymbol{y})$	$f_{31}(\boldsymbol{x},\boldsymbol{y})$	$f_{32}(\boldsymbol{x},\boldsymbol{y})$
$max\ f_{ij}(\boldsymbol{x},\boldsymbol{y})$	1.632	1.426	7.83	7.88	1.29	1.281
$min\ f_{ij}(\boldsymbol{x},\boldsymbol{y})$	-0.321	-0.562	-1.59	-2.67	-1.77	-1.2

Formulate and solve SODM problem of the FLDM (28)-(30):

$$max\ f_{11}(x,y) = \left(-x_1^2 - 2x_2^2 - x_3^2 + 5x_2 + 10\right) y_1,$$
$$subject\ to$$
$$\left(-x_1^2 - x_2^2 - 4x_3^2 + 5x_1 + 12\right) y_1 \geq 1.2272,$$
$$\left(x_1^2 + 3x_2 + 5\right) y_1 = 1,$$
$$\left(x_2^2 + 3x_1 + 1\right) y_2 = 1,$$
$$\left(x_3^2 + 5x_2 + 2\right) y_3 = 1,$$
$$3.6x_1 + 6.2x_2 + 1.6x_3 \leq 29,$$
$$4.2x_1 + 7.4x_2 + 2.6x_3 \leq 32,$$
$$2.6x_1 - 0.4x_2 + 13.2x_3 \leq 17.4,$$
$$3.4x_1 + 0.2x_2 + 14.4x_3 \leq 18.8,$$
$$6.2x_2 + 7.2x_3 \geq 11.6,$$
$$7.4x_2 + 8.4x_3 \geq 13.6,$$

where $\delta_{12} = (b_{12} - a_{12}) s_1 + a_{12} = 1.2272$, so the solution of the FLDM is $\left(x_1^F, x_2^F, x_3^F\right) = (0.295,\ \ 0.694,\ 1.014)$ and $s_1 = 0.9$, $\sigma^F = 0.15$ are given by the FLDM.

Secondly, the SLDM formulate its SODM problem (33)-(45):

$$max\ f_{21}(x,y) = \left(-x_1^2 - 2x_2^2 - 2x_3^2 + 5x_3 + 6\right) y_2,$$
$$subject\ to$$
$$\left(-3x_1^2 - x_2^2 - x_3^2 + 7x_2 + 8\right) y_2 \geq 3.66,$$
$$\left(x_1^2 + 3x_2 + 5\right) y_1 = 1,$$
$$\left(x_2^2 + 3x_1 + 1\right) y_2 = 1,$$
$$\left(x_3^2 + 5x_2 + 2\right) y_3 = 1,$$
$$x_1 = 0.295;$$
$$3.6x_1 + 6.2x_2 + 1.6x_3 \leq 29,$$
$$4.2x_1 + 7.4x_2 + 2.6x_3 \leq 32,$$
$$2.6x_1 - 0.4x_2 + 13.2x_3 \leq 17.4,$$
$$3.4x_1 + 0.2x_2 + 14.4x_3 \leq 18.8,$$
$$6.2x_2 + 7.2x_3 \geq 11.6,$$
$$7.4x_2 + 8.4x_3 \geq 13.6,$$

where $\delta_{22} = (b_{22} - a_{22}) s_2 + a_{22} = 3.66$, so the SLDM solution is $\left(x_1^F, x_2^S, x_3^S\right) = (0.295,\ 0.443,\ 1.23)$ and $s_2 = 0.6$, $\sigma^S = 0.05$ are given by the SLDM.

Now, the FLDM test function, equation (36), will be utilized to decide whether the solution $(0.295,\ \ 0.433,\ 1.23)$ is acceptable or not:

$$\frac{\left\|F_1(0.295,\ 0.694,\ 1.014) - F_1(0.295, 0.443, 1.22)\right\|_2}{\left\|F_1(0.295, 0.443, 1.22)\right\|_2}$$

$$= \frac{\|(1.5889,\ 1.2266) - (1.5933, 1.1128)\|_2}{\|(1.5933, 1.1128\ )\|_2} = 0.059 < 0.15$$

Finally, the PLDM formulate it's SODM problem (39)-(41) as:

$$max\ f_{31}(x,y) = \left(-x_1^2 - 4x_2^2 - x_3^2 + 6x_2 + 7\right) y_3,$$

$$subject\ to$$

$$\left(-x_1^2 - 2x_2^2 - x_3^2 + 9\right) y_3 \geq 0.984,$$

$$\left(x_1^2 + 3x_2 + 5\right) y_1 = 1,$$

$$\left(x_2^2 + 3x_1 + 1\right) y_2 = 1,$$

$$\left(x_3^2 + 5x_2 + 2\right) y_3 = 1,$$

$$x_1 = 0.295,$$

$$x_2 = 0.443,$$

$$3.6x_1 + 6.2x_2 + 1.6x_3 \leq 29,$$

$$4.2x_1 + 7.4x_2 + 2.6x_3 \leq 32,$$

$$2.6x_1 - 0.4x_2 + 13.2x_3 \leq 17.4,$$

$$3.4x_1 + 0.2x_2 + 14.4x_3 \leq 18.8,$$

$$6.2x_2 + 7.2x_3 \geq 11.6,$$

$$7.4x_2 + 8.4x_3 \geq 13.6,$$

where $\delta_{32} = (b_{32} - a_{32}) s_3 + a_{32} = 0.984$, so the PLDM solution is $\left(x_1^F, x_2^S, x_3^T\right) = (0.295,\ \ 0.443,\ 1.23)$ and $s_3 = 0.6$, is given by the PLDM.

Now, the SLDM test function, equation (42), will be utilized to decide whether the solution $(0.295, 0.443, 1.23)$ is acceptable or not:

$$\frac{\|F_2(0.295, 0.443, 1.23) - F_2(0.295, 0.443, 1.23)\|_2}{\|F_2(0.295, 0.443, 1.23)\|_2} = 0 < 0.05$$

So $\left(x_1^F, x_2^S, x_3^T\right) = (0.295, 0.443, 1.23)$ is the preferred solution to the ML-MOQFP problem.

Using Lingo software package , the compromise solution of the ML-MOQFP problem with fuzzy parameters in the constraints using the proposed interactive algorithm is obtained as $(x_1, x_2,\ x_3) = (0.295,\ 0.443,\ 1.23)$ with objective function values $f_{11}(\boldsymbol{x}) = 1.593,\ \ f_{12}(\boldsymbol{x}) = 1.113,\ \ f_{21}(\boldsymbol{x}) = 4.154,\ \ f_{22}(\boldsymbol{x}) = 7.034,\ \ f_{31}(\boldsymbol{x}) = 1.27,\ \ f_{32}(\boldsymbol{x}) = 1.223$, and their corresponding membership function $\mu_{11} = 0.98,\ \mu_{12} = 0.843, \mu_{21} = 0.61,\ \mu_{22} = 0.92,\ \ \mu_{31} = 0.99,\ \mu_{32} = 0.97$.

The comparison among the results obtained by the proposed FGP approach in [5], the proposed interactive approach and the method of Lachhwani [9] for solving the ML-MOQFP problem with fuzzy parameters in the constraints is given in Table 2. The results show that the values of the objective functions and the membership functions obtained by the proposed FGP approach [5], the proposed interactive approach and the latter method presented by Lachhwani in [9] are close to one another.

Table 2: The comparison among the FGP approach in [5], the interactive approach and the method of Lachhwani in [9].

FGP approach [5]		Interactive approach		Lachhwani [9]	
$f_{11} = 1.527$	$\mu_{11} = 0.95$	$f_{11} = 1.593$	$\mu_{11} = 0.98$	$f_{11} = 1.516$	$\mu_{11} = 0.94$
$f_{12} = 1.325$	$\mu_{12} = 0.95$	$f_{12} = 1.113$	$\mu_{12} = 0.843$	$f_{12} = 1.33$	$\mu_{12} = 0.95$
$f_{21} = 2.54$	$\mu_{21} = 0.438$	$f_{21} = 4.154$	$\mu_{21} = 0.61$	$f_{21} = 2.248$	$\mu_{21} = 0.41$
$f_{22} = 3.55$	$\mu_{22} = 0.589$	$f_{22} = 7.034$	$\mu_{22} = 0.92$	$f_{22} = 3.739$	$\mu_{22} = 0.61$
$f_{31} = 1.22$	$\mu_{31} = 0.977$	$f_{31} = 1.27$	$\mu_{31} = 0.99$	$f_{31} = 1.164$	$\mu_{31} = 0.959$
$f_{32} = 1.02$	$\mu_{32} = 0.895$	$f_{32} = 1.223$	$\mu_{32} = 0.97$	$f_{32} = 0.926$	$\mu_{32} = 0.857$

6 Conclusion

In this paper the interactive approach was exhibited for solving ML-MOQFP problems with fuzzy parameters in the constraints. Based on the α-level properties and partial order relation, a numerical general model is constructed. Firstly, the quadratic fractional objective functions in each level are transformed into nonlinear objective functions based on a proposed transformation, thus the ML-MOQFP problem transformed into ML-MONP problem. Then, the stocktickerFGP and the interactive approach are used to solve ML-MONP problem.

Conflict of interest

The authors declare that they have no conflict of interest.

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