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Proposed Approach for Solving Stochastic Vector Optimization Problem with Random parameters in the Constraints

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Abstract: This paper introduces an efficient approach for stochastic vector optimization problem (SVOP) with random parameters in the right-hand side of the constraints. The proposed technique uses the scalarization concept to transform SVOP to a stochastic single objective optimization problem (SSOP) based on the nonnegative weighted sum approach. The statistical inference methods should be applied to convert SSOP into its equivalent deterministic single objective optimization problem (DSOP). The resulting problem can be solved as linear or nonlinear programming problem to obtain the efficient solutions. Finally, an illustrated example is given to verify the validity of the proposed approach.

Keywords: Stochastic Optimization, Chance Constraint, Statistical Inference, Scalarization Techniques, Efficient Solutions.

1 Introduction

We have to make decisions in most of the real-life issues based on uncertain data or information. To deal with such uncertainty, we use Stochastic programming as a form of optimization based on the theory of probability.
Saad and Farag [1] discussed the stochastic multicriteria integer programming problem with random variables in both the objective function and in the constraints. A comparison between the solution of stochastic multiobjective programming problems by using different criteria and multiobjective approach is performed, [2]. Bayoumi et al.[3] presented an stochastic approach for solving stochastic bi-criteria programming problem with random parameters in the objective function and the right hand side of the constraints, the stochastic parameters in the constraints were normally distributed. Kassem [4] deals with multiobjective nonlinear programming problems with random variables in the objective functions from parametric point of view. Widyan in [5] introduced an approach to treat the stochastic multiobjective optimization using the expected value criteria. Many earlier publications reviewed different aspects in stochastic optimization programming problems [6]. The stochastic multicriteria optimization problems have been taken into consideration by utilizing the probabilistic programming approach as well as chance-constrained approach [7,8].

In this work, we consider a stochastic bi-objective programming problem (SBOPP) with continuous random variables belong to the uniform distribution in the right-hand side of the constraints. Mathematical and statistical methods depend on chance constrained approach have been used to determine the efficient solution of the stochastic problem.

2 Problem Formulation

Consider the following SBOPP with stochastic parameters in the right hand side of the constraints which is formulated as:

$$\min_{x\in D} Q(X) = \{q_1(x), q_2(x)\} \tag{1}$$

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Where,
$D(x,\nu)=\{x\in R^n; g_i\leq \nu_i, i=1,2,3,...,m\}$, x is an n-dimensional decision variable column vector, $\nu_i, i=1,2,3,...,m$ are random variables belong to continuous uniform distribution $U(a_1,a_2), a_1,a_2\in R$ such that $a_1\leq \nu_i\leq a_2, D: R^n\rightarrow R^m$ are the constraint vectors, and $Q: R^n\rightarrow R^2$ are the objective functions.
Problem (1) has stochastic parameters in the constraints; therefore, we cannot apply the solution approach to solve it directly as classical deterministic mathematical programming problems.

3 Scalarization Technique

Non-negative Weighted sum approach is used to transform problem (1) to stochastic single objective optimization problem as follows

$$\min_{x\in D} Q(x)=\{\omega(q_1(x))+(1-\omega)(q_2(x))\} \tag{2}$$

Where $0\leq\omega\leq 1$

Theorem 3.1. Let the problem (1) be convex. if $x^*\in D$ is pareto optimal, then there exists ω such that $0\leq\omega\leq 1$; x^* is a solution of the weighting problem (2) [9].

Corollary 3.1. From the previous theorem we can say that the set of all efficient solutions for problem (1) can be determined.

4 The Proposed Approach

As mentioned above in section 2 the random variables are considered in the right hand side of the constraints; $D(x,\nu)=\{x\in R^n; g_i(x)\leq \nu_i, i=1,2,3,\ldots,m\}, \nu_i, i=1,2,...,m$ are independent stochastic parameters uniformly distributed, such that $a_1\leq\nu_i\leq a_2, i=1,2,3,...,m$, the constraints can be rewritten using chance constrained methods under the conditions of level of significance that assign a small value of probability, say $\gamma_i, 0\leq\gamma_i\leq 1, i=1,2,3,...,m$, then problem (2) will be transformed into a deterministic one.

The chance constrained programming with random variables can be stated as

$$P(g_i(x)\leq\nu_i)\geq\gamma_i, i=1,2,3,...,m \tag{3}$$

Where $g_i(x)=\sum_{j=1}^{n}a_{ij}x_j, i=1,2,3,...,m$
and,

$$f_\nu(\nu_i)=\begin{cases}\frac{1}{a_2-a_1}, & a_1\leq\nu_i\leq a_2\\ 0, & else\ where.\end{cases} \tag{4}$$

By applying the chance constrained approach, the set of stochastic constraints in (3) will be transformed into their deterministic ones as follows:

$$P(\sum_{j=1}^{n}a_{ij}x_j\leq\nu_i)\geq\gamma_i, i=1,2,3,...,m \tag{5}$$

Then,

$$P(\nu_i\leq g_i(x))\leq 1-\gamma_i \tag{6}$$

The cumulative probability distribution function (CPDF) for the continuous random variable ν is defined as:

$$F_\nu(\nu)=P(V\leq\nu)=\int_{-\infty}^{\nu}f(w)dw \tag{7}$$

The cumulative distribution function of the uniform distribution in (4) is given by

$$F(\nu)=\begin{cases}0 & ;\nu\leq a_1\\ \frac{\nu-a_1}{a_2-a_1} & ;a_1\leq\nu\leq a_2\\ 1 & ;\nu\geq a_2.\end{cases} \tag{8}$$

Therefore, (6) will be as follows

$$F_{\nu}(g_i(x)) \leq 1-\gamma_i, \quad i = 1,2,3,...,m \tag{9}$$

Then,

$$g_i(x) \leq F^{-1}(\delta_i), \quad i = 1,2,3,...,m \tag{10}$$

Where $\delta_i = 1-\gamma_i$ and $F^{-1}(.)$ is the inverse distribution function of the continuous random variable $\nu_i, i = 1,2,...,m$. Consequently, the equivalent deterministic bi-criteria optimization problem for the model (1) and (2) can be written as:

$$\min_{x\in D} Q(x) = \{\omega(q_1(x)) + (1-\omega)(q_2(x))\} \tag{11}$$

Subject to

$$\begin{aligned} g_i(x) - F^{-1}(\delta_i) \leq 0, \quad & i = 1,2,3,...,m \\ x_j \geq 0, \quad & j = 1,2,3,...,n. \end{aligned}$$

For any value of $\omega \in [0,1]$, then using Lingo software [10], the optimal solution of problem (11) is considered an efficient solution for problem (1). In addition, the set of all efficient solutions of SVOP can be determined by decomposing the parametric space of ω [11].

5 Illustrative Example

To illustrate our suggested approach, the stochastic bi-objective optimization problem is considered as follows:

$$\min\{4x_1 + 7x_2,\ 10x_1 - 8x_2\}$$

subject to

$$\begin{aligned} 6x_1 + 5x_2 &\leq \nu_1 \\ 10x_1 + 2x_2 &\leq \nu_2 \\ -13x_1 + 15x_2 &\leq \nu 3 \\ x_1, x_2 &\geq 0 \end{aligned}$$

Where ν_1, ν_2 and ν_3 are independent random variables uniformly distributed on the intervals $[10,20], [5,10]$ and $[20,30]$ respectively. The tolerance of decision maker are $\gamma_1 = 0.5, \gamma_2 = 0.4$ and $\gamma_3 = 0.8$.
Thus $F^{-1}(0.5) = 15, F^{-1}(0.6) = 8$ and $F^{-}1(0.2) = 22$. Therefore, using the proposed approach, the equivalent deterministic bi-objective optimization model can be demonstrated as follows

$$\min \omega(4x_1 + 7x_2) + (1-\omega)(10x_1 - 8x_2)$$

subject to

$$\begin{aligned} 6x_1 + 5x_2 - 15 &\leq 0 \\ 10x_1 + 2x_2 - 8 &\leq 0 \\ -13x_1 + 15x_2 - 22 &\leq 0 \\ x_1, x_2 &\geq 0 \end{aligned}$$

For different values of weights ω. For this purpose the Lingo package [10] is used, the subsets of efficient and non-dominated solutions are displayed in Table (1).

Table1. Subset of efficient and non-dominated solutions

ω	x_1	x_2	$q_1(x)$	$q_2(x)$	Q(x)
0.3	0.80000	0.00000	3.20000	8.00000	6.56000
0.7	0,43182	1.84091	14.61365	-10.40908	7.10682

The tests can be continued with other values of γ level of significance until the decision-maker is convinced with the most suitable solution.

6 Conclusion

This paper introduces a suggested approach for treating stochastic bi-criteria optimization problem (SBCPP) with random parameters uniformly distributed in the right hand side of the constraints. The chance constrained method used to transform the stochastic model to deterministic one. Using continuous uniform distribution enable us to transform the stochastic model into deterministic one more easily than other distribution functions. For further research, we suggest to apply the proposed approach in this paper on multiobjective stochastic problem with random parameters in both the objective functions and the constraints with different distribution functions.

Conflict of interest

The authors declare that they have no conflict of interest.

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