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On Poisson Weighted Pranav Distribution Applicable to Count Data

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Abstract: This paper presents a new model for the computation of count data by combining the Poisson distribution and the Weighted Prav Distribution. The main objective of this paper is to provide a comprehensive analysis of the distribution's statistical properties and mathematical properties. Then, a parameter estimation procedure is performed to estimate the optimal distribution. Finally, a real data set is analyzed to evaluate the proposed model's suitability.

Keywords: Poisson distribution, weighted Pranav distribution, compound distribution, count data, maximum likelihood estimation

1 Introduction

The modeling and statistical analysis of count data are very important in various fields such as biology, engineering, and sociology. It is very common for people to deal with count data and make critical decisions. To improve the decision making process while dealing with it, we have a probability model that can be fitted to it. Compounding is a technique that can combine two distributions. In this type of technique, the resulting distribution will be either continuous or discrete. This work has been done in this area since 1920. [1] the relationship between a negative binomial distribution and a Poisson distribution was established through the use of a compounding mechanism. The rate parameter in the Poisson distribution was regarded as a gamma variate.. [3,4]proposed a number of compound distributions, such as the exponential and the compound gamma distribution. He also obtained the compound alpha distribution of Polya. [5, 6]introduced the use of count data models in medical research by developing a method that combines the advantages of both the inverse and the discrete version of Weibull distribution. The method takes into account the probability parameter as a randomly variable after the beta distribution. [2] developed a new method for calculating count data that combines the Lindley distribution and the Poisson distribution. This is useful in biological science.[7] A new compound probability model was developed by combining the negative binomial distribution with the generalized exponential distribution.[8] introduced a new discrete model that combines the Geeta distribution with the generalized Beta distribution. This allows him to classify various distributions into special cases.[9] introduced a new type of distribution called the Poisson-inverse Gaussian. It is an alternative to the negative Binomial.. [10] developed a new method for calculating the probability of a count data's occurrence in epileptic seizures. This new approach combines the Ishita distribution with the Poisson distribution.[13] developed a compounding model using Exponential and Negative Binomial distributions. By mixing these two models, he was able to show the flexibility of this model in fitting count data.. [11] clarified the difference between the negative binomial distribution and the Poisson distribution is that the former undercounts the number of zeros while the latter overstates them[15] introduced Poisson cousillindly regression model for analyzing over dispersed count data. [14] introduced Cosine geometric distribution for count data modeling. [16] Introduces a new generalization of Pranav distribution using weighting technique. In this paper, we present a new count data model that combines the Poisson and the Weighted Pranav distributions. It can be used to analyze the data generated by statistical techniques.

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2 Definition of Proposed Model(Poisson-Weighted Pranav Distribution)

If $Z|\nu \sim$P(ν), where ν being itself a random variable following two parameter Poisson weighted Pranav distribution with parameters ζ *and* η, then determining the distribution that results from marginalizing over ν will be known as compound Poison distribution with that of Weighted Pranav distribution, which is denoted by PWPD $(Z;\zeta,\eta)$. Our proposed model will be discrete as parent distribution is a discrete.

Theorem 2.1: The probability mass function(p.m.f.) of a Poisson Weighted Pranav Distribution i.e., PWPD$(Z;\zeta,\eta)$ is given by

$$P(Z=z)=\frac{\zeta^{\eta+4}(\eta+z)!}{z!\eta!(\zeta^4+(\eta+1)(\eta+2)(\eta+3))}\left[\frac{\zeta(1+\zeta)^3+(\eta+z+3)(\eta+z+2)(\eta+z+1)}{(1+\zeta)^{\eta+z+4}}\right];z=0,1,2,3,...,;\zeta,\eta>0$$

Proof: Using the definition, the p.m.f. of a PWPD $(Z;\zeta\, and\, \eta)$ can be obtained as

$$j(z|\nu)=\frac{e^{-\nu}\nu^x}{(z)!}\ ;z=0,1,2,3,...,;\nu>0$$

When its parameter ν follows WPD with p.d.f.

$$h(\nu;\zeta)=\frac{\nu^{\eta}\zeta^{\eta+4}(\zeta+\nu^3)e^{-\zeta\nu}}{\eta!(\zeta^4+(\eta+1)(\eta+2)(\eta+3))}\ ;\nu>0,\zeta,\eta>0$$

We have

$$P(Z=z)=\int_0^{\infty}g(z|\nu).h(\nu;\zeta)d\nu$$

$$P(Z=z)=\int_0^{\infty}\frac{e^{-\nu}\nu^z}{(z)!}\frac{\nu^{\eta}\zeta^{\eta+4}(\zeta+\nu^3)e^{-\zeta\nu}}{\eta!(\zeta^4+(\eta+1)(\eta+2)(\eta+3))}d\nu$$

$$P(Z=z)=\frac{\zeta^{\eta+4}}{z!\eta!(\zeta^4+(\eta+1)(\eta+2)(\eta+3))}\left(\int_0^{\infty}\zeta e^{-(1+\zeta)\nu}\nu^{\eta+z}d\nu+\int_0^{\infty}e^{-(1+\zeta)\nu}\nu^{\eta+z+3}d\nu\right)$$

$$P(Z=z)=\frac{\zeta^{\eta+4}(\eta+z)!}{z!\eta!(\zeta^4+(\eta+1)(\eta+2)(\eta+3))}\left[\frac{\zeta(1+\zeta)^3+(\eta+z+3)(\eta+z+2)(\eta+z+1)}{(1+\zeta)^{\eta+z+4}}\right];z=0,1,2,3,...,;\zeta,\eta>0 \quad (1)$$

which is the p.m.f. of PWPD.

3 Special cases

Case 1 : If we put $\eta=0$, then Poisson Weighted Pranav Distribution reduces to Poisson Pranav Distribution with p.m.f. as

$$f(z;\eta)=\frac{\zeta^4}{(\zeta^4+6)}\left[\frac{\zeta(1+\zeta)^3+(z+1)(z+2)(z+3)}{(1+\zeta)^{z+4}}\right]$$

Case 2: If we put $\eta=1$, then Poisson Weighted Pranav Distribution reduces to Poisson Size Biased Pranav Distribution with p.m.f. as

$$f(z;\eta)=\frac{\zeta^5(z+1)}{(\zeta^4+24)}\left[\frac{\zeta(1+\zeta)^3+(z+4)(z+3)(z+2)}{(1+\zeta)^{z+5}}\right]$$

4 Moments

4.1 Factorial Moments

Using (1), the rth factorial moment about origin of the PPD (1) can be obtained as

$$\mu'_{(s)}=E[E(Z^{(s)}|\nu)],\text{where}Z^{(s)}=Z(Z-1)(Z-2)...(Z-s+1)$$

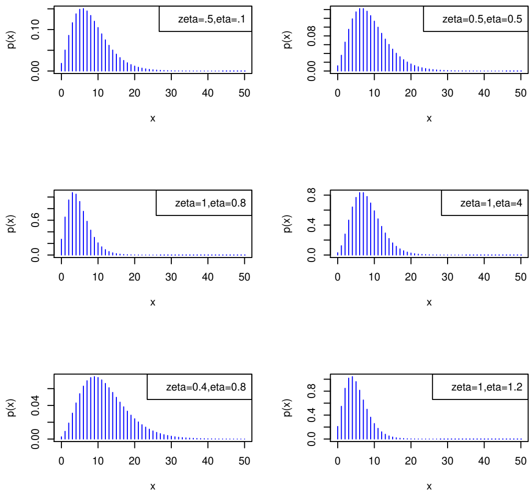

Fig. 1: Probability mass function (pmf) plot for different values of ζ and η

$$\mu'_{(s)} = \int_0^\infty \left[\sum_{z=0}^{\infty} z^{(s)} \frac{e^{\nu}\nu^{z}}{(z)!}\right] \frac{\nu^{\eta}\zeta^{\eta+4}}{\eta!(\zeta^4+(\eta+1)(\eta+2)(\eta+3))}(\zeta+\nu^3)e^{-\zeta\nu}d\nu$$

$$\mu'_{(s)} = \frac{\zeta^{\eta+4}}{\eta!(\zeta^4+(\eta+1)(\eta+2)(\eta+3))}\int_0^\infty \left[\nu^s\left(\sum_{z=s}^{\infty}\frac{e^{-\nu}\lambda^{z-s}}{(z-s)!}\right)\right]\nu^{\eta}(\zeta+\nu^3)e^{-\zeta\nu}d\nu$$

Taking u=z-s, we get

$$\mu'_{(s)} = \frac{\zeta^{\eta+4}}{\eta!(\zeta^4+(\eta+1)(\eta+2)(\eta+3))}\int_0^\infty \left[\nu^r\left(\sum_{u=0}^{\infty}\frac{e^{-\nu}\nu^{u}}{u!}\right)\right]\nu^{\eta}(\zeta+\nu^3)e^{-\zeta\nu}d\nu$$

$$\mu'_{(s)'} = \left[\frac{\zeta^4(\eta+s)!+(\eta+s+3)!}{\zeta^s\eta!(\zeta^4(\eta+1)(\eta+2)(\eta+3))}\right] \tag{2}$$

Taking s=1,2,3,4 in (2), the first four factorial moments about origin of Poisson-Pranov Distribution can be obtained as

$$\mu'_{(1)} = \frac{\zeta^4(\eta+2)!+(\eta+4)!}{\zeta\eta!(\zeta^4+(\eta+1)(\eta+2)(\eta+3))}$$

$$\mu'_{(2)} = \frac{\zeta^4(\eta+2)!+(\eta+5)!}{\zeta^2\eta!(\zeta^4+(\eta+1)(\eta+2)(\eta+3))}$$

$$\mu'_{(3)} = \frac{\zeta^4(\eta+3)!+(\eta+6)!}{\zeta^3\eta!(\zeta^4+(\eta+1)(\eta+2)(\eta+3))}$$

$$\mu'_{(4)} = \frac{\zeta^4(\eta+3)!+(\eta+6)!}{\zeta^3\eta!(\zeta^4+(\eta+1)(\eta+2)(\eta+3))}$$

4.2 Moments about origin (Raw moments)

The first four moments about origin , using the relationship between factorial moments about origin and the moments about origin, of PPD (1) can be obtained as

$$\mu'_1 = \frac{\zeta^4(\eta+2)!+(\eta+4)!}{\zeta\eta!(\zeta^4+(\eta+1)(\eta+2)(\eta+3))}$$

$$\mu_2' = \frac{(\zeta^4(\eta+2)! + (\eta+5)! + \zeta^5(\eta+1)! + \zeta(\eta+4)!}{\zeta^2\eta!(\zeta^4 + (\eta+1)(\eta+2)(\eta+3))}$$

$$\mu_3' = \frac{\zeta^4(\eta+3)! + (\eta+6)! + 3(\zeta^5(\eta+2)! + (\eta+5)!) + \zeta^6(\eta+1)! + \zeta^2(\eta+4)!}{\zeta^3\eta!(\zeta^4 + (\eta+1)(\eta+2)(\eta+3))}$$

$$\mu_4' = \frac{\zeta^4(\eta+4)! + (\eta+7)! + 6(\zeta^5(\eta+3)! + \zeta(\eta+6)!) + 7(\zeta^6(\eta+2)! + \zeta^2(\eta+3)!) + \zeta^7(\eta+1)! + \zeta^3(\eta+4)!}{\zeta^4\eta!(\zeta^4 + (\eta+1)(\eta+2)(\eta+3))}$$

4.3 Moments about the Mean (Central moments)

Using the relationship $\mu_s = E(Y-\mu_1')^s = \sum_{J=0}^{s} \binom{s}{J} \mu_k'(-\mu_1')^{s-J}$ we get, the moments about the mean of the PWPD (1) can be obtained as

$$\mu_2 = \frac{(\zeta^4(\eta+2)! + (\eta+5)! + \zeta^5(\eta+1)! + \zeta(\eta+4)!)(\eta!(\zeta^4 + (\eta+1)(\eta+2)(\eta+3))) - (\zeta^4(\eta+1)! + (\eta+4)!)^2}{\zeta^2(\eta!)^2(\zeta^4 + (\eta+1)(\eta+2)(\eta+3))^2}$$

$$\mu_3 = \frac{\begin{array}{l}(\zeta^4(\eta+3)! + (\eta+6)! + 3(\zeta^5(\eta+2)! + (\eta+5)!) + \zeta^6(\eta+1)! + \zeta^2(\eta+4)!)(\eta!)^2(\zeta^4 + (\eta+1)(\eta+2)(\eta+3))^2 - \\ 3(((\zeta^4(\eta+2)! + (\eta+5)! + \zeta^5(\eta+1)! + \zeta(\eta+4)!)(\zeta^4(\eta+2)! + (\eta+4)!))(\eta!(\zeta^4(\eta+1)(\eta+2)(\eta+3)))\end{array}}{\zeta^3(\eta!)^3(\zeta^4 + (\eta+1)(\eta+2)(\eta+3))^3}$$

$$\mu_4 = \frac{\begin{array}{l}(\zeta^4(\eta+4)! + (\eta+7)! + 6(\zeta^5(\eta+3)! + \zeta(\eta+6)!) + 7(\zeta^6(\eta+2)! + \zeta^2(\eta+3)!) + \zeta^7(\eta+1)! + \zeta^3(\eta+4)!) \\ (\zeta^4(\eta!)^4(\zeta^4 + (\eta+1)(\eta+2)(\eta+3))^4) - 4((\zeta^4(\eta+3)! + (\eta+6)! + 3(\zeta^5(\eta+2)! + (\eta+5)!) + \zeta^6(\eta+1)! \\ +\zeta^2(\eta+4)!)(\zeta^4(\eta+2)! + (\eta+4)!)(\eta!)^2(\zeta^4 + (\eta+1)(\eta+2)(\eta+3))^2) + 6(((\zeta^4(\eta+2)! + (\eta+5)! \\ +\zeta^5(\eta+1)! + \zeta(\eta+4)!)(\zeta^4(\eta+2)! + (\eta+4)!)^2) - 3(\zeta^4(\eta+2)! + (\eta+4)!)^4\end{array}}{\zeta^4(\eta!)^4(\zeta^4 + (\eta+1)(\eta+2)(\eta+3))^4}$$

5 Coefficient of variation(C.V), Coefficient of skewness($\sqrt{\beta_1}$), Coefficient of kurtosis(β_2) and Index of Dispersion(γ)

The Coefficient of variation, $\sqrt{\beta_1}$, β_2 and γ of the PWPD are thus obtained as

$$C.V. = \frac{\sigma}{\mu_1'} = \frac{\sqrt{(\zeta^4(\eta+2)! + (\eta+5)! + \zeta^5(\eta+1)! + \zeta(\eta+4)!)(\eta!(\zeta^4 + (\eta+1)(\eta+2)(\eta+3))) - (\zeta^4(\eta+1)! + (\eta+4)!)^2}}{\zeta^4(\eta+2)! + (\eta+4)!}$$

$$\sqrt{\beta_1} = \frac{\mu_3}{\mu_2{}^{3/2}} = \frac{\begin{array}{l}(\zeta^4(\eta+3)! + (\eta+6)! + 3(\zeta^5(\eta+2)! + (\eta+5)!) + \zeta^6(\eta+1)! + \zeta^2(\eta+4)!)(\eta!)^2 \\ (\zeta^4 + (\eta+1)(\eta+2)(\eta+3))^2 - 3(((\zeta^4(\eta+2)! + (\eta+5)! + \zeta^5(\eta+1)! + \zeta(\eta+4)!) \\ (\zeta^4(\eta+2)! + (\eta+4)!))(\eta!(\zeta^4(\eta+1)(\eta+2)(\eta+3)))\end{array}}{((\zeta^4(\eta+2)! + (\eta+5)! + \zeta^5(\eta+1)! + \zeta(\eta+4)!)(\eta!(\zeta^4 + (\eta+1)(\eta+2)(\eta+3))) - (\zeta^4(\eta+1)! + (\eta+4)!)^2)^{3/2}}$$

$$\beta_2 = \frac{\mu_4}{\mu_2{}^2} = \frac{\begin{array}{l}(\zeta^4(\eta+4)! + (\eta+7)! + 6(\zeta^5(\eta+3)! + \zeta(\eta+6)!) + 7(\zeta^6(\eta+2)! + \zeta^2(\eta+3)!) + \zeta^7(\eta+1)! + \zeta^3(\eta+4)!) \\ (\zeta^4(\eta!)^4(\zeta^4 + (\eta+1)(\eta+2)(\eta+3))^4) - 4((\zeta^4(\eta+3)! + (\eta+6)! + 3(\zeta^5(\eta+2)! + (\eta+5)!) + \zeta^6(\eta+1)! \\ +\zeta^2(\eta+4)!)(\zeta^4(\eta+2)! + (\eta+4)!)(\eta!)^2(\zeta^4 + (\eta+1)(\eta+2)(\eta+3))^2) + 6(((\zeta^4(\eta+2)! + (\eta+5)! \\ +\zeta^5(\eta+1)! + \zeta(\eta+4)!)(\zeta^4(\eta+2)! + (\eta+4)!)^2) - 3(\zeta^4(\eta+2)! + (\eta+4)!)^4\end{array}}{((\zeta^4(\eta+2)! + (\eta+5)! + \zeta^5(\eta+1)! + \zeta(\eta+4)!)(\eta!(\zeta^4 + (\eta+1)(\eta+2)(\eta+3))) - (\zeta^4(\eta+1)! + (\eta+4)!)^2)^2}$$

$$\gamma = \frac{\sigma^2}{\mu_1'} = \frac{(\zeta^4(\eta+2)! + (\eta+5)! + \zeta^5(\eta+1)! + \zeta(\eta+4)!)(\eta!(\zeta^4 + (\eta+1)(\eta+2)(\eta+3))) - (\zeta^4(\eta+1)! + (\eta+4)!)^2}{(\zeta^4(\eta+2)! + (\eta+4)!)(\zeta\eta!(\zeta^4 + (\eta+1)(\eta+2)(\eta+3)))}$$

6 Recurrence Relation Between Probabilities

The PWPD can be written as

$$P(Z=z)=\frac{\zeta^{\eta+4}(\eta+z)!}{z!\eta!(\zeta^4+(\eta+1)(\eta+2)(\eta+3))}\left[\frac{\zeta(1+\zeta)^3+(\eta+z+3)(\eta+z+2)(\eta+z+1)}{(1+\zeta)^{\eta+z+4}}\right]$$

$$P(Z=z+1)=\frac{\zeta^{\eta+4}(\eta+z+1)!}{(z+1)!\eta!(\zeta^4+(\eta+1)(\eta+2)(\eta+3))}\left[\frac{\zeta(1+\zeta)^3+(\eta+z+4)(\eta+z+3)(\eta+z+2)}{(1+\zeta)^{\eta+z+5}}\right]$$

Dividing P(Z=z+1) by P(Z=z), we find the recurrence relation between probabilities

$$\frac{P(Z=z+1)}{P(Z=z)}=\frac{(\eta+z+1)(\zeta(1+\zeta)^3+(\eta+z+4)(\eta+z+3)(\eta+z+2))}{(z+1)(1+\zeta)(\zeta(1+\zeta)^3+(\eta+z+3)(\eta+z+2)(\eta+z+1))}$$

$$P(Z=z+1)=\frac{(\eta+z+1)(\zeta(1+\zeta)^3+(\eta+z+4)(\eta+z+3)(\eta+z+2))}{(z+1)(1+\zeta)(\zeta(1+\zeta)^3+(\eta+z+3)(\eta+z+2)(\eta+z+1))}P(Z=z)$$

7 Estimation of Parameters

In this section, we estimate the unknown parameter of the PWPD by using method of maximum likelihood estimation.

7.1 Method of Maximum Likelihood Estimation

Method of Maximum Likelihood Estimation is simple and most efficient method of estimation. Let$Z_1,Z_2,X_3,\ldots,Z_n$ be the random size of sample n draw from PWPD, then the likelihood function of PWPDis given as

$$L(z|\zeta,\eta)=\frac{\theta^{n(\alpha+4)}\prod_{i=1}^{n}\Gamma\alpha+x+1[\theta(1+\theta)^3+(\alpha+x+3)(\alpha+x+2)(\alpha+x+1)}{\prod_{i=1}^{n}x_i(\Gamma\alpha+1)^n[\theta^4+(\alpha+1)(\alpha+2)(\alpha+3)](1+\theta)^{n(\alpha+4)+\sum_{i=1}^{n}x_i}}$$

$$\log L=n(\alpha+4)\log\theta+\sum_{i=1}^{n}\log\Gamma\alpha+x+1+\sum_{i=1}^{n}\log(\theta(1+\theta)^3+(\alpha+x+3)(\alpha+x+2)(+x+1))-\sum_{i=1}^{n}\log x_i- n\log\Gamma\alpha+1-n\log(\theta^4(\alpha+1)(\alpha+2)(\alpha+3))-(n\alpha+n4+\sum_{i=1}^{n}x_i)\log(1+\theta)$$

$$\frac{\delta}{\delta\zeta}\log L=\frac{n(\alpha+4)}{\zeta}+\sum_{i=1}^{n}\frac{(1+\theta)^3+3\theta(1+\theta)^2}{(\theta(1+\theta)^3+(\alpha+x+1)(\alpha+x+2)(\alpha+x+3))}-\frac{4n\theta^3}{(\theta^4+(\alpha+1)(\alpha+2)(\alpha+3))}-\frac{n\alpha+n4+\sum_{i=1}^{n}x_i}{(1+\theta)}=0$$

$$\frac{\delta}{\delta\eta}\log L=n\log\theta+\sum_{i=1}^{n}[\psi(\alpha+x+1)-\psi(\alpha+1)]+\sum_{i=1}^{n}\frac{3\alpha^2+6x\alpha+12\alpha+9x+2x^2+9}{(\theta(1+\theta)^3+(\alpha+x+1)(\alpha+x+2)(\alpha+x+3))}-n\log(1+\theta) -\frac{n(3\alpha^2+10\alpha+11)}{(\theta^4+(\alpha+1)(\alpha+2)(\alpha+3))}=0$$

$$\frac{\delta}{\delta\zeta^2}\log L=\frac{-n(\alpha+4)}{\zeta^2}+\sum_{i=1}^{n}\frac{((\theta(1+\theta)^3+(\alpha+x+1)(\alpha+x+2)(\alpha+x+3))(3(1+\theta)^2+6(1+\theta))-((1+\theta)^3+3\theta(1+\theta)^2)^2)}{(\theta(1+\theta)^3+(\alpha+x+1)(\alpha+x+2)(\alpha+x+3))^2} +\frac{(n\alpha+4n+\sum x_i)}{(1+\theta)^2}-\frac{12n\theta^2(\theta^4+(\alpha+1)(\alpha+2)(\alpha+3))-16n\theta^6}{(\theta^4+(\alpha+1)(\alpha+2)(\alpha+3))^2}$$

$$\frac{\delta}{\delta\eta^2}\log L=\sum_{i=1}^{n}\left(\psi'(\alpha+x+1)-\psi'(\alpha+1)\right)-\frac{n(6\alpha+10)(\theta^4+(\alpha+1)(\alpha+2)(\alpha+3))-n(3\alpha^2+10\alpha+11)^2}{(\theta^4+(\alpha+1)(\alpha+2)(\alpha+3))^2} +\sum_{i-1}^{n}\frac{((\theta(1+\theta)^3+(\alpha+x+1)(\alpha+x+2)(\alpha+x+3))(6\alpha+6x+12)-(3\alpha^2+6x\alpha+12\alpha+9x+2x^2+9)^2}{(\theta(1+\theta)^3+(\alpha+x+1)(\alpha+x+2)(\alpha+x+3))^2}$$

$$\frac{\delta^2\log L}{\delta\zeta\delta\eta}=\frac{n}{\theta}+\frac{4n\theta^3(3\alpha^2+10\alpha+11)}{(\theta^4+(\alpha+1)(\alpha+2)(\alpha+3))}+\frac{n}{(1+\theta)^2}-\sum_{i=1}^{n}\frac{((1+\theta)^3+3\theta(1+\theta)^2)(3\alpha^2+6x\alpha+12\alpha+2x^2+9)}{(\theta^4(1+\theta)^3+(\alpha+x+1)(\alpha+x+2)(\alpha+x+3))}$$

The following equations can be solved maximum for likelihood estimates $\widehat{\zeta}$ *and* $\widehat{\eta}$ *of* ζ *and* η *of PWPD*

$$\begin{bmatrix}\frac{\delta^2\log L}{\delta\zeta^2} & \frac{\delta^2\log L}{\delta\zeta\delta\eta}\\ \frac{\delta^2\log L}{\delta\zeta\delta\eta} & \frac{\delta^2\log L}{\delta\eta^2}\end{bmatrix}_{\substack{\widehat{\zeta}=\zeta_0\\ \widehat{\eta}=\eta_0}}\begin{bmatrix}\widehat{\zeta}-\zeta_0\\ \widehat{\eta}-\eta_0\end{bmatrix}=\begin{bmatrix}\frac{\delta\log L}{\delta\zeta}\\ \frac{\delta\log L}{\delta\eta}\end{bmatrix}_{\substack{\widehat{\zeta}=\zeta_0\\ \widehat{\eta}=\eta_0}}$$

Estimates of $\zeta$and η can be obtained by solving the above system of matrices by using R software 3.5.2 ([12]).

8 Application of PWPD

Table 1: Number of Haemocytometer yeast cell counts per square studied by Gosset (1908)

Number of yeast cells per square	0	1	2	3	4	5	6
Observed Frequencies	213	128	37	18	3	1	0

From table 2 it is observed that the Poisson Weighted Pranav Distribution have the lesser Akaike information criterion (AIC) and Bayesian information criterion (BIC) values as compared to other competing models. Hence we can conclude that our model fits better as compared to other competing models. We have compute expected frequencies ofPoisson

Table 2: Model comparison criterion for fitted models to a dataset representing Number of Haemocytometer yeast cell counts per square.

Criterion	PD	PLD	PAD	PPD	PWPD
-LogL	449.5	452.6	454.65	458.65	446.5
AIC	901	907.2	911.3	919.3	897
BIC	900.94	907.14	911.24	919.24	896.9

Distribution (PD), Poisson Lindley Distribution (PLD), Poisson Akash Distribution (PAD), Poisson Pranav Distribution (PPD), and Poisson Weighted Pranav Distribution (PWPD) with the help of R software (3.5.2) to the data set given in the table 1. The expected counts, Parameter estimates and chi-square p-value are given in table 3. It is observed from table 3

Table 3: Fitted proposed distribution and other competing models to a dataset representing Number of Haemocytometer yeast cell counts per square.

No. of yeast cells per square	Observed Frequencies	PD	PLD	PAD	PPD	PWPD
0	213	202.65	234	236.8	242.7	214.5
1	128	137.8	99.4	95.6	90.35	122.8
2	37	46.85	40.5	39.9	37.45	44.8
3	18	10.6	16	16.6	16.5	13.3
4	3	1.8	6.2	6.7	7.35	3.5
5	1	0.2	2.4	2.7	3.75	9
6	0	0.1	1.5	1.7	1.9	2.1
Total		400	400	400	400	400
ML estimates (S.E)		$\theta = 0.68(0.04)$	$\theta = 1.95(0.12)$	$\theta = 2.26(0.48)$	$\theta = 2.264(0.08)$	$\zeta = 6.04(2.35)$ $\eta = 2.88(1.69)$
Chi-square		10.10	11.12	14.68	21.23	2.52
d.f.		2	2	2	2	1
P-value		0.006	0.003	0.000	0.000	0.11

that p-value for PWPD is > 0.05 whereas P-Value is < 0.05 for PD, PLD, PAD, PPD. Hence it employs that PWPD fits the data statistically well whereas PD, PLD, PAD, PPD does not fit at all.

Table 4: Number of micronuclei after exposure after exposure at dose 4 Gy of γ irradiation studied by Piug and Valero (2006)

Z	0	1	2	3	4	5	6	7
Observed Frequencies	1974	1674	869	342	102	26	13	2

The table 5 shows that the weighted Poisson model has the lesser AIC and BIC values than other competing models. This suggests that our model fits better in terms of its performance. The following table shows the expected frequencies

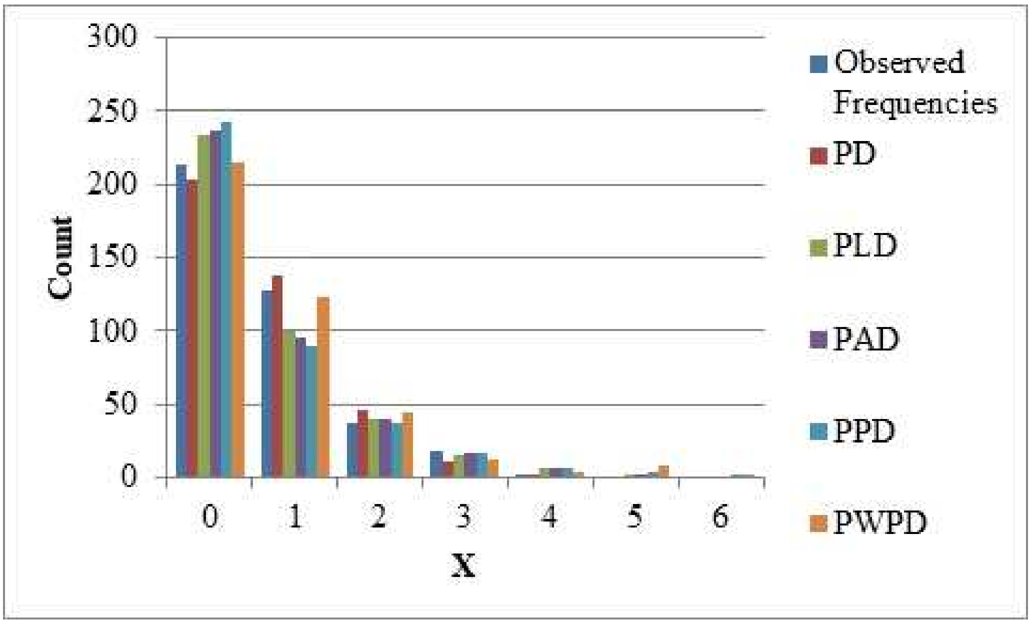

Fig. 2: Graphical overview of fitted models to a dataset representing number of yeast cells per square

Table 5: Model comparison criterion for fitted models to a dataset representing Number of micronuclei after exposure after exposure at dose 4 Gy of γ rradiation

Criterion	PD	PLD	PAD	PPD	PWPD
-LogL	6767.9	6918.35	6947.5	7038.9	6735.9
AIC	13537.8	13838.7	13897	14079.8	13475.8
BIC	13537.9	13838.8	13897	14079.9	13477.9

of various distributions, such as the Poisson Lindley, PLD, PAK, and PWD, using the R software. Table 4 shows the data set used for the computation. Table 6 shows the chi-square p-value and the phenomenological estimates.

It is observed from table 6 that p-value for PWPD is ¿0.05 whereas P-Value is ¡ 0.05 for PD, PLD, PAD, PPD . Hence it employs that PWPD fits the data statistically well whereas PD, PLD, PAD, PPD does not fit at all.

9 Conclusion

A new probability distribution is presented using the compounding technique. Its statistical properties are studied and applied to the data collected by count.

Conflicts of Interests

The authors declare that they have no conflicts of interests

Table 6: Fitted proposed distribution and other competing models to a dataset representing representing Number of micronuclei after exposure after exposure at dose 4 Gy of *gamma* irradiation

No. Of micro nuclei	Observed Frequencies	PD	PLD	PAD	PPD	PWPD
0	1974	1861	2396.8	2409.95	2483.6	1965.8
1	1674	1839.9	1300.3	1256.3	1175.4	1697.2
2	869	932.1	668.8	665.4	625.7	856.9
3	342	314.8	332.1	344	343.6	331
4	102	79.7	160.9	171.9	185.3	108.2
5	26	16.1	76.5	86.15	99.5	31.6
6	13	2.7	35.8	40	155.3	8.5
7	2	1.6	30.8	28.3	33.6	2.8
Total	5002	5002	400	400	400	400
ML estimatesS.E.		$\theta = 1.01(0.014)$	$\theta = 1.38(0.022)$	$\theta = 1.73(0.021)$	$\theta = 1.87(0.016)$	$\zeta = 6.74(0.78)$ $\eta = 5.35(0.83)$
Chi-square		62.21	337.08	392	563.8	3.44
d.f.		4	5	5	5	4
P-value		0.00	0.00	0.00	0.00	0.48

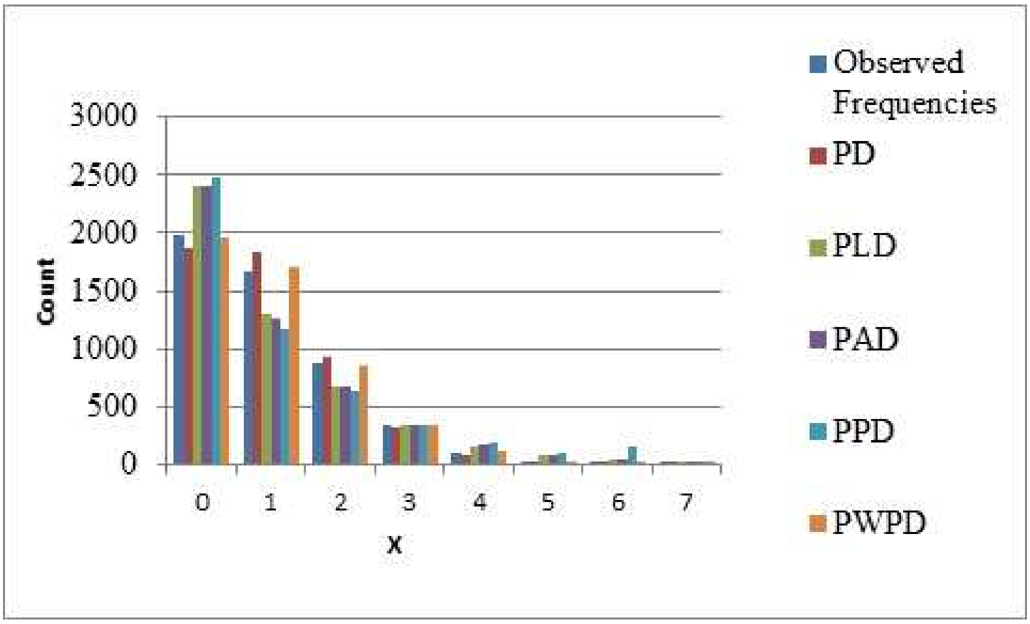

Fig. 3: Graphical overview of fitted models to a dataset representing number of micronuclei

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