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Evaluation of a Functional Time Series Model for Forecasting Inflation in Uganda

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Abstract: Inflation is a major economic problem in emerging market economies and requires accurate models to avoid high volatility and long periods of inflation. This paper is aimed at evaluating a Functional Time Series (FTS) model as compared to other models in forecasting inflation in Uganda. The monthly Time Series (TS) data for the general Consumer Price Index (CPI) was used during the period of Jul-2005 to Jun-2020. Box-Jenkins' Auto Regressive Integrated Moving Average (ARIMA) and Seasonal Autoregressive Integrated Moving Average (SARIMA) methodologies are explored to evaluate the FTS method of forecasting the general CPI where their accuracies are compared and validated using Mean Squared Error (MSE), Root Mean Square Error (RMSE), Akaike Information Criterion (AIC), and Bayesian Information Criterion (BIC) criteria. Existing inflation models in Uganda are outdated by structural changes in the economy igniting the need for a novel accurate model for forecasting inflation. FTS technique is overall considered accurate and particularly used to model high-frequency data such as Uganda general CPI data modeled as a functional observation after smoothing by kernel smoothing methods compared to traditional methods. Business operations and consumers normally base their decisions on modeled and forecasted inflation with their decisions affected by inflation uncertainties that hinder their motivations to invest and save in a given country as they try to avoid inflation-related risks. Findings therefore show FTS having great accuracies and recommended the method for forecasting Uganda inflation. This opens a new framework for extending the Box and Jenkin's methodology to the functional setting.
Keywords: ARIMA, CPI, SARIMA, AIC, BIC, Functional Time Series, RMSE, MSE.

1 Introduction

Determining a proper model for forecasting purposes that aid the stability of prices in an economy can be quite a challenging exercise. This is due to the inevitable structural changes in the economies that make inflation a major economic problem and a matter of growing concerns for most emerging economies. The study, therefore, considers the comparison and evaluation of an FTS model for forecasting inflation in Uganda. Its novelty has caused a diverse range of methods believed to merge theoretical developments with some practical implementations particularly in areas related to the dimensionality of reduction models. It explores the application of models that develop the FTS using a functional basis and applications. FTS techniques can be used to forecast inflation in Uganda whose economy has experienced volatile and fluctuating prices for two decades now.

Uganda's yearly inflation rates have been high in certain years although the economy never reached hyperinflationary levels. Bank of Uganda that is the regulatory body is tasked to control core inflation to at most 5% over the medium-term adopted a contractionary policy that saw the inflation declining to about 3.6%. A 2.30% decline was further anticipated towards the end of the last quarter of 2020. The central bank has always applied both univariate and multivariate macroeconomic models for policymaking and forecasting purposes with these models partially depending upon the generated data for the inflation variable [1].

Univariate or multivariate type of models establish procedures for specification and estimation linear TS models. Structural changes in the Uganda economy make almost all the existing models outdated and therefore vital to consider the FTS model whose effectiveness could be compared with other univariate and multivariate models. [2] emphasized on how the

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technique allows for nonparametric smoothing to be incorporated into the modeling procedure to obtain smoothed principal components. Its validity is investigated in the context of inflation in Uganda. The paper considers the FTS model alongside other models to assess the forecasting abilities of competing models. Considerable emphasis is on the forecasting accuracies for the high-frequency data that can produce the greatest out-of-sample forecasts. This is applied to the general CPI data and different forecast results are explored to improve the forecasts of each model.

2 Literature Review

Inflation is currently a serious economic problem slowing economic developments around the world. It is considered to be a guiding tool for major governmental programs although different economists and researchers have not agreed on a unified definition and measure. There is however a universal consensus to design tools/models to accurately forecast inflation. It is typically an increase in the price levels based on goods that are commonly consumed by the public and grouped under different baskets of commodities. CPI is the current price of goods and services for the same prices in the previous period is considered the main measure of inflation rates. [3] and [4] acknowledged the negative impacts of inflation and agreed on the use of CPI as a measure although [5] criticized the technique of at times being tedious.

[6] stressed on inflation to be affecting economic decisions because of its volatility and attracts the need to forecast future rates. Business operations and consumers normally base their decisions on modeled and forecasted inflation. Their decisions are affected by inflation uncertainties that hinder their motivations to invest and save in a given country as they try to avoid inflation-related risks. This calls for up-to-date models to fairly forecast inflation and give both the investors and governments the absolute confidence they need for future planning. Uganda's past inflation is grouped by [7] into three main episodes including a high inflation period in the 1980s, a moderately stable period during the 1995-2007 period, and a very volatile thou moderate inflation during the past few years.

Inflation rates in Uganda and other under developed countries are so much affected by the political atmosphere. Uganda's inflation in particular is observed to increase every after five years mainly towards presidential elections with the rates on food and fuel prices always skyrocketing immediately after the general elections. The inflation rate in March 2011 rose from 11.2% to 30.5%, almost tripling the March figure during October of the same years. [8] pointed out some outdated models used by [9], [10], and [11] that emphasized inflation rates to have been affected by domestic money and foreign factors during the stabilization period of the economy. Inflationary determinants in Uganda comprise of amongst others world food and energy prices that are directly transmitted to the country through import prices or prices of internationally traded goods. Inflation in Uganda and other developing countries therefore suddenly rises due to both foreign and domestics factors. Foreign factors include rises in food and energy prices in the international market. With the importation of a considerable amount of food, any increase in foreign or world food prices directly pushes up domestic food prices. Food is a vital proportion in every basket of an average household in Uganda and imported inflation generally affects domestics prices. Likewise, world oil price fluctuations have instantaneous effects on domestic prices. Coffee is a major contributor to Uganda's Gross Domestic Product (GDP) generating about 56% of the revenue hence the international price of coffee is an important determinant of Uganda inflation as a result of its effects on the terms of trade and balance payments deficits. [12] elaborated on Kabundi using monthly data in a single equation equilibrium correction framework for log CPI following similar work by [13] highlighting Kenya and Ethiopia inflations. It highlights the role of foreign and domestic factors influencing the food and oil prices in the inflation process.

There has however been no single model used in all countries to forecast inflation at the same time since each economy has its major predictors of inflation. Different and several models with specific predictors can therefore be developed to best forecast inflation in a given economy. The application of these models to inflationary dynamics must consider model efficiencies and accuracies. Such considerations arise due to some models being more suitable for short term forecasts while others working better on medium to long term forecasting. The most common and recent theoretical models for Uganda inflation are drawn from the univariate Box Jenkins (ARIMA) and SARIMA models respectively in the series, the famous Philips curve or Vector autoregression (VAR) models applied to understand the influence among multiple time series. However, other researchers have used univariate models like autoregressive conditional heteroscedasticity (ARCH) models and cointegration for the multivariate responses with a series of predictors including unemployment and monetary variables used.

[14] based on Stock and Watson forecasting United States of America (USA) inflation in a modified bivariate Philips curve and their results later discarded unemployment as a less important predictor for USA inflation compared to capacity utilization and trade sales. Later studies and inflation forecasts in the USA have considered asset price being significantly successful in predicting US inflation. [15] emphasized the theoretical autoregressive (AR) model generally outperforming the bivariate Phillips curve. The analysis is supported by [16] that found the AR model mostly robust across different forecasting horizons. [17] and [18] conducted a related study for European countries identified price and labor variables as useful predictors for most countries. This is an indication that Uganda and every other economy has its major predictors of inflation, signifying development of different models that best forecast the inflation.

[19] used the monthly data of 2001 to 2011 to model inflation in Tanzania that found rainfall having a significant effect on the overall CPI of Tanzania and food prices. They were particularly driven by imported inflation on food and energy prices that provided a forecast for their long-run solutions. Money shocks mainly had peripheral long-run effects on overall CPI with no substantial impacts on food and energy prices. However, asymmetric adjustment to volatilities in food and energy prices provided some evidence of imported inflation to domestic prices. High international food prices in regards to domestic food prices render heavy spikes in domestic prices compared to relatively low foreign prices. [20] explored Box-Jenkins's methodology and used the ARIMA model while considering seasonality effects to suitably model Liberia's monthly inflation rates of 2006. The residuals did not find evidence of ARCH and serial correlation using effects Ljung-Box test. Most studies that have used SARIMA and other univariate models confirm SARIMA to enjoy vital forecasting advantages compared to other TS models. [21] assumed the SARIMA model as being better than the Holt-Winters exponential smoothing approach while forecasting transshipment in German. ARCH and generalized autoregressive conditionally heteroscedastic (GARCH) models however were suitable tools for uncertainty/volatility of inflation rates. Uganda has insufficiently considered GARCH or Multivariate GARCH models in forecasting the inflation rate, unlike other countries. [22] applied ARIMA and GARCH models to forecast Kenya's inflation using 2000 to 2014 monthly data. The ordinary least square (OLS) estimation method was used and concluded ARIMA to have provided better estimates than the GARCH model. The combination of the two models out-performed ARIMA and GARCH individually. [23] utilized a diminutive out-of-sample data for forecasting a short-term volatility feature of the Egyptian inflation rate and found GARCH as a better model for the purpose.

Evaluation of various models for forecasting accuracy is reputable for modeling, decision, and monetary policymaking. An accurate forecast curtails welfare losses arising from inaccurate forecasts due to over forecasting that unreasonably contracts the economy while under forecasted inflation ruins the general welfare of society by causing macroeconomic instability. Many evaluation criteria for the forecast are used including the mean forecast error, mean absolute forecast error, the mean squared forecast error, the mean percent forecast error, the mean squared percent error, and the Theil coefficients. The most popular ones being mean forecast error (ME) and RMSE that minimizes quadratic loss function for the forecasts. When RMSE is performed on pure econometric models, the results of forecast evaluation normally conform to the accuracy to improve steadily with better economic structure.

With the structural changes of the Uganda economy, the FTS model can be relied while assessing time series forecasting abilities. It provides a certain set of tools for observations considered to the curves instead of separate data points. Comparison and evaluation of the methodologies pioneer the work that provides a platform for modern time series analysis and solving many bottlenecks in forecasting related to the frequency domain. Recent years have seen applications of numerous fields in which FTS methods of forecasting provided better and accurate results. [24] elaborated on FTS applications in Statistics and many other fields. The approach consisting of smoothing, data reduction, and forecasting methods can be particularly applied in forecasting inflation using CPI disaggregated data and smoothing the data using kernel methods to enhance modeling. Advancement in technology has motivated the application of FTS with sample elements recorded sequentially over time. Recent studies have shown functional principal component analysis easily aiding FTS forecast. Forecasting performances of FTS that produce high forecast errors due to outlying observations are corrected through the applications of the robust methodology.

3 Methodologies

The paper explores forecasting Uganda inflation or the general CPI of the country using the FTS model and compares it to ARIMA and SARIMA models respectively to confirm its accuracy in forecasting high-frequency data. In forecasting inflation, Uganda's monthly inflation data based on the general CPI is applied. The analysis is based on the secondary data from the central bank of Uganda and the World Bank data source for the period Jul-2005 to Jun-2020, a period within which Uganda is considered to have registered a volatile but moderate rate of inflation. It is also a period when the central bank of Uganda vigorously explored the contractionary monetary policy that brought down the rate from about 30% in 2011 to 3.6% in 2019. Auto ARIMA function in Python 3.7 provided the main statistical software for the identification and estimation of the model.

3.1 Functional Time Series (FTS)

FTS model is an application from Functional Data Analysis (FDA) in forecasting a high dimensional TS data. It uses the FDA as a tool for analyzing functional variables, where each observation is a continuous function and explores samples of data where each observation arises as a curve or function for forecasting purposes using different methodologies. The method is particularly applied in forecasting inflation disaggregated general CPI data and modeled as a functional observation after smoothing. Recent studies indicate using the functional principal component helps in performing analysis and the FTS model. The forecasting performance of functional time series models is affected by outlying observations although robust forecasting technique can be applied to correct the remedy. The smoothed curves are then decomposed using the Basis Function (BF) model as

$$X_i\,(t_j) = \mu(x) + \sum_{k=1}^{K} \beta_{j,k} \emptyset_k(x) + e_j(x) \ldots \quad (1)$$

Where
$\mu(x)$ is the mean across years of the functional observation,
$\emptyset_k(x)$ is a set of orthogonal BFs purely independent of time
$\beta_{j,k}$ is a univariate TS,
$e_j(x)$ is the model error term assumed to be serially uncorrelated
$i.e \;\; e_x \sim N(0, v(x))$
The coefficients $\{\beta_{j,k}\}$ are each forecasted, and then multiplied by the BFs $\{\emptyset_k(x)\}$ to compute forecasts of incidence curves. The exponential smoothing state-space models is used to forecast the TS coefficients $\{\beta_{j,k}\}$.
The parameters can then be estimated using the maximum likelihood score method. It uses a function of the Pearson residuals for which large values indicate the observations deviate from the underlying model. This, therefore, makes it possible to check whether the estimators are affected by a set of observations that are inconsistent with the model. The method also provides robust estimates with weighted residuals used to obtain point or interval forecasts.
A lot of methods have been developed for forecasting FTS models including parametric, non-parametric and dimensionality reduction approaches. FTS has created a high diversity of methods that combine both theoretical and practical implementations where the functional operators are used to model the dependency of integral operators. Dimensionality reduction expands the FTS into a functional basis and applies standard multivariate methods to the reduced coordinates. Once the model is fitted, the accuracy of the model is therefore checked by diagnostics including mean square error, root mean square error, AIC, and BIC criteria respectively.

3.2 Autoregressive Integrated Moving Average (ARIMA) Model

The Box-Jenkins (ARIMA) model that was popularized by [25] is in theory considered the utmost common model for forecasting TS. It models the serial dependence in a TS, where the AR-terms model the interdependency in Y and the MA-terms describe how the dependent variable depends on previous error terms, mainly applied to the stationary data series However, the model can be applied in some cases on data that show evidence of non-stationarity property when data is stationarized by transformations such as differencing and logging techniques [26]
Cases of non-stationarity in the data can be stationarized by transformations such as differencing and logging so that the mean or any discrepancy, and the autocorrelation function is constant over time.
The three parameters that represent the model comprise of the order of an Auto-Regressive (AR) component p, the differencing order d, and the order of a Moving Average (MA) component q. ARIMA model uses historical information of the data and decompose that data to make the data a stationary for easy forecast. The AR(p) is expressed as:

$$CPI_t = \phi + \theta_1 CPI_{t-1} + \theta_2 CPI_{t-2} + \ldots + \theta_p CPI_{t-p} + \varepsilon_t \ldots \quad (2)$$

Where the lag order is p and *t* = (1, 2, …, *n*)
Similarly, the q order moving average process, MA(q), can be expressed as:

$$CPI_t = \phi + \varepsilon_t - \lambda_1 \varepsilon_{t-1} - \lambda_2 \varepsilon_{t-2} - \ldots - \lambda_q \varepsilon_{t-q} \ldots \quad (3)$$

ARMA (p, q) model is expressed by joining the AR(p) and MA(q) models as

$$CPI_t = \phi + \theta_1 CPI_{t-1} + \theta_2 CPI_{t-2} + \cdots + \theta_p CPI_{t-p} + \varepsilon_t - \lambda_1 \varepsilon_{t-1} - \cdots - \lambda_q \varepsilon_{t-q} \ldots \quad (4)$$

Stationarity of real-life data is normally uncommon and a first- differenced inflation series is obtained as

$$CPI_t = (\nabla CPI_t) = CPI_t - CPI_{t-1} = \nabla CPI_t - \nabla CPI_{t-1} \ldots \quad (5)$$

Differencing process is usually used to make time series data stationary. The first order differencing process of time series y_t is defined as $\Delta CPI_t = CPI_t - CPI_{t-1}$ for example, if CPI_t is non-stationary series, we will take a first-difference of CPI_t so that the ARIMA (p, 1, q) model is:

$$CPI_t = \phi + \theta_1 CPI_{t-1} + \theta_2 CPI_{t-2} + \ldots + \theta_p CPI_{t-p} + \varepsilon_t - \lambda_1 \varepsilon_{t-1} - \ldots - \lambda_q \varepsilon_{t-q} \ldots \quad (6)$$

Where ε_t is a zero mean and constant variance white noise series denoted as WN (0,σ^2).
The lag operator is defined as $\mathrm{L}^{\mathrm{k}} CPI_t = CPI_{t-k}$ so $\Delta^{\mathrm{d}} CPI_t = (1 - \mathrm{L})^{\mathrm{d}} CPI_t$

CPI_t is the differenced inflation series of order 1 and ϕ, θ *and* λ are the parameters to be estimated [27]. Their estimations are conducted using least-squares estimation or Maximum Likelihood Estimation (MLE) that explore making the sum of squared errors of the fitted models absolutely small while the final model selection done using AIC or BIC.
Forecasts of an *ARIMA*(*p, p,q*) are obtained and the h-multistep(s) ahead forecast can be computed recursively as

$$\widehat{CPI_t}(\mathrm{h}) = \mathrm{E}[CPI_{t+h}/CPI_t] = \emptyset_0 + \sum_{i=1}^{p} \emptyset_i \widehat{CPI_t}(\mathrm{h}-\mathrm{i}) - \sum_{i=1}^{q} \theta_i \hat{\varepsilon}_t(\mathrm{h}-\mathrm{i}) \quad (7)$$

Where the error associated to this forecast is $\hat{\varepsilon}_t(h) = CPI_{t+h} - \widehat{CPI_t}(\mathrm{h})$ and the h-step ahead forecast error variance FEV(h) for $\{CPI_t\}$ can be obtained after expressing the *ARMA* (*p, q*) model weighted sum of disturbances-ε_t

$$CPI_t = \varepsilon_t + \pi_1\varepsilon_{t-1} + \cdots + \pi_k\varepsilon_{t-k}$$

3.3 Seasonal Autoregressive Integrated Moving Average (SARIMA) Model

The model has been extended from the ARIMA model and does not only explore regular difference or stationarity in the series as ARIMA does but captures both stationary and seasonal behavior in the TS data. Stationarity must be attained as a prerequisite for fitting the SARIMA model. Information contained in past observations and errors of the series is used during the process of building and selecting an appropriate model. SARIMA model is therefore proposed and used if the series contains both regular and irregular patterns. The model is therefore a byproduct from the Box-Jenkins (ARIMA) model containing seasonal and non-seasonal factors alongside the corresponding lags form of the model defined by

$$\theta(\mathrm{L})\vartheta(\mathrm{L}^S)\varepsilon_t = \emptyset(\mathrm{L})\varphi(\mathrm{L}^S)(1-\mathrm{L})^d(1-\mathrm{L}^S)^D y_t \quad (8)$$

Functions with the seasonal polynomial of order *P* and *Q* are represented as
$\varphi(\mathrm{L}^S) = 1 - \emptyset_1\mathrm{L}^S - \emptyset_2\mathrm{L}^{2S} - \cdots - \emptyset_{p-1}\mathrm{L}^{(p-1)S} - \emptyset_p\mathrm{L}^{PS}$
$\vartheta(\mathrm{L}^S) = 1 - \vartheta_1\mathrm{L}^S - \vartheta\mathrm{L}^{2S} - \cdots - \vartheta_{p-1}\mathrm{L}^{(p-1)S} - \theta_p\mathrm{L}^{PS}$
respectively, where
$\{y_t\}$: observable time series
$\{\varepsilon_t\}$: white noise series
p, d, q: Nonseasonal orders of AR part, differencing and MA part in the model respectively.
P, D,Q: Seasonal order AR, differencing and MA part respectively
L: lag operator $\mathrm{L}^k y_t = y_{t-k}$
S: seasonal order for the data.
During the process of SARIMA model selection, many tentative SARIMA models can be identified and the corresponding AIC, and BIC values for each of the models are determined. This will aid in determining an appropriate SARIMA model that best fits the series. The model with the smallest values of the above criterion is therefore chosen as the best model to work with. An Augmented dickey fuller test is performed to ascertain the stationarity in the seasonally differenced series. Otherwise, the order of the model is indicated by the number of significant spikes in ACF and PACF respectively.
The parameters of the appropriate SARIMA model identified are then estimated by the maximum likelihood estimation method and forecasted thereon. [28] demonstrated a one- step and two steps ahead forecast for the selected SARIMA model as

$$y_t - y_{t-1} = \emptyset(y_{t-12} - y_{t-13}) - \varepsilon_t - \theta\varepsilon_{t-1} - \vartheta\varepsilon_{t-12} + \theta\vartheta\varepsilon_{t-13} \quad (9)$$

The above equation gives the corresponding one-step-ahead forecast while below are the corresponding two steps ahead forecast equations
$\hat{y}_{t+1} = y_t + \emptyset(y_{t-11} - y_{t-12}) - \theta\varepsilon_t - \vartheta\varepsilon_{t-11} + \theta\vartheta\varepsilon_{t-12}$
$\hat{y}_{t+2} = y_{t+1} + \emptyset(y_{t-10} - y_{t-11}) - \vartheta\varepsilon_{t-10} + \theta\vartheta\varepsilon_{t-11}$

4 Data

Uganda government and World Bank monthly data observations on the general CPI series from Jul 2005 to Jun 2020 is used in this section to aid the model buildings and thereafter used for comparison and evaluation. Data on domestic variables were obtained from the Bank of Uganda while external variables were obtained from the World Bank group websites and transformations only done to allow the series to become stationary. The paper explores the use of R, auto ARIMA in Python 3.7 and SARIMA functions respectively to build the models under considerations.

4.1 Empirical Results and Discussion

This paper considers monthly data on the general CPI, exchange rate variable from July-2005 to June-2020. This is a period within which Uganda is considered to have registered a volatile but moderate rate of inflation. It is also a period when the central bank of Uganda vigorously explored the contractionary monetary policy that brought down the rate from about 30% in 2011 to 3.6% in 2019. ARIMA and SARIMA models being the most famous statistical models used in numerous studies to forecast inflation in several countries aided the evaluation and comparison to functional time series model. FTS is a new and emerging field of statistics to model high-frequency data. The monthly data series are modeled and a comparison made to assess the superiority of the models for forecasting inflation.

4.2 Descriptive Statistics

The descriptive statistics in Table.1 below highlights all the variables not to be normally distributed according to the Jarque-Bera normality test. The general CPI of Ugandan exhibits a high standard deviation, a clear indication of the volatility of prices over the past years in Uganda.

Table1: Descriptive Statistics of the Monthly General CPI of Uganda.

Statistics	Observations	Min	Median	Mean	Max	Standard deviation	Skewness	Kurtosis
Value	180	4.21360	4.89522	4.80324	5.21819	1.31620	-0.45912	1.81247

The general CPI plot on Fig.1 depicts how Uganda has experienced large swings in inflation way before 2000. The early 2000s saw rising general CPI levels with slow economic growth and declines (depreciation) in the value of the national currency. 2011 was mostly affected by the political atmosphere that happens every five years. Food and fuel prices skyrocketed immediately after the general elections. Coffee prices have also been fluctuating throughout the year and significantly affecting inflation or the general CPI of Uganda.

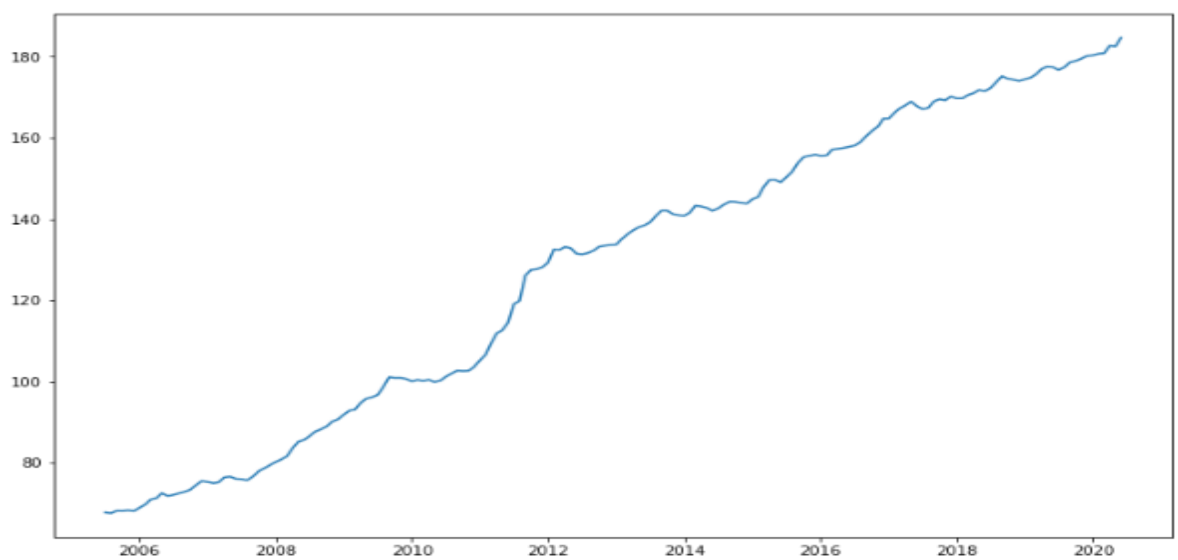

Fig.1: Time Series Plot of Log General CPI of Uganda.

5 Modeling General CPI

In modeling the general CPI of Uganda, ARIMA and SARIMA models are explored and compared with the FTS model to determine the forecast superiority of the models for high- frequency inflation data.

5. 1 Autoregressive Integrated Moving Average (ARIMA) Model

The auto ARIMA function in python 3.7 has aided the development and identification of the best ARIMA and SARIMA models for forecasting the general CPI of Uganda. It automates the tuning process of the model and sets the model parameters accordingly and providing a better solution without the need to traverse through data transformations, ACF, and PACF plots respectively.

Table.2: ARIMA Model

Coefficients	Coef	S. E	z-statistics	P-Value
Constant	0.6661	0.101	6.606	0.00001
AR 1	0.2858	0.076	3.743	0.00001
Model Results				
	ARIMA (1,1,0)			
Observations	180.00000			
S. E	29.91000			
Log-Likelihood	-210.67800			
AIC	460.99000			
BIC	486.68900			
MSE	8.30886			
RMSE	2.88251			

Auto ARIMA function identifies ARIMA (1,1,0) based on log-likelihood, the AIC and BIC criteria as the most appropriate and desirable ARIMA mode. The model and results in Table.2 are therefore explored in forecasting the general CPI of Uga nda. It suggests a non-seasonal MA (1) and AR (1) signature indicating the stationarity of the general CPI obtained during the first order differencing. Auto ARIMA confirmed the model parameters of ARIMA (1,1,0) having the smallest errors.

5.2 Forecasting the General CPI Using ARIMA Model

ARIMA (1,1,0) is then used to forecast the general CPI of Uganda and comparison of the forecasting ability and accuracy t o aid better decision making by the policymakers.

Table 3: ARIMA Forecasted General CPI Series.

Year	Actual CPI	ARIMA Predicted CPI	% CPI Variance $\left(\frac{\boldsymbol{Actual - Predicted}}{\boldsymbol{Actual}} \boldsymbol{x100}\right)$
Jun-19	177.39	179.36	(1.11)
Jul-19	176.66	180.02	(1.90)
Aug-19	177.36	180.69	(1.88)
Sep-19	178.56	181.35	(1.56)
Oct-19	178.90	182.02	(1.74)
Nov-19	179.47	182.69	(1.24)
Dec-19	180.15	183.35	(1.78)

Jan-20	180.29	184.02	(2.07)
Feb-20	180.67	184.68	(2.22)
Mar-20	180.78	185.35	(2.53)
Apr-20	182.65	186.02	(1.85)
May-20	182.47	186.68	(2.31)
Jun-20	184.64	189.69	(2.74)

5.3 Seasonal Autoregressive Integrated Moving Average (SARIMA) Model

In the identification of an appropriate SARIMA model, auto ARIMA function has automatically developed SARIMA (2,1,2) (1,0,[1],4) as the best model that corresponds to lower values of MSE, RMSE, AIC and BIC presented in Table.4. The package estimates the parameters of SARIMA (2,1,2) (1,0,[1],4) confirming all the coefficients of the model are significantly different from zero hence the model is white noise.

Table 4: SARIMA Model of General CPI.

	Coef	**S. E**	**z-statistics**	**P-Value**
Intercept	0.7644	0.422	1.811	0.07000
AR 1	0.9536	0.040	24.114	<0.00001
AR 2	-0.9483	0.036	-26.692	<0.00001
MA 1	-0.8226	0.036	-23.137	<0.00001
MA 2	0.9430	0.048	19.573	<0.00001
AR.S4	-0.1765	0.593	-0.298	0.76600
MA.S4	0.3456	0.557	0.620	0.53500
SIGMA2	0.6956	0.063	11.068	<0.00001
Model Results				
Model	SARIMA (2,1,2) (1,0,[1],4)			
Observations (n)	180.00000			
S. E	17.109491			
Log-Likelihood	-222.49500			
AIC	427.356000			
BIC	436.56300			
MSE	4.498502			
RMSE	2.120967			

5.4 Forecasting the General CPI Using SARIMA Model

The selected SARIMA (2,1,2) (1,0,[1],4) model is explored to conduct forecasts for the general CPI of Uganda. The forecast is constructed based on the estimated parameter using the period Nov 2018 to Jun 2020 and the predicted general CPI is compared with the observed values as shown in Table.5. It can be noticed that most of the predicted general CPI values are close to the actual values. The SARIMA (2,1,2) (1,0,[1],4) model, therefore, predicts a continued upward trend observed in the general CPI series of Uganda with the accuracy of the model evaluated based on the AIC, BIC, MSE, and RMSE indicated in Table. 4.

Table.5: SARIMA Forecasted CPI Series.

Year	Actual CPI	SARIMA Predicted CPI	% CPI Variance $\left(\frac{Actual - Predicted}{Actual} x100\right)$
Jun-19	177.39	176.74	0.37
Jul-19	176.66	177.03	(0.21)
Aug-19	177.36	177.28	0.05
Sep-19	178.56	177.53	0.58
Oct-19	178.90	177.83	0.60
Nov-19	179.47	178.08	0.77
Dec-19	180.15	178.32	1.02
Jan-20	180.29	178.55	0.97
Feb-20	180.67	178.80	1.04
Mar-20	180.78	179.04	0.96
Apr-20	182.65	179.25	1.86
May-20	182.47	179.47	1.64
Jun-20	184.64	180.69	2.14

5.5 FTS Model

In modeling the Uganda general CPI using FTS model, functional data analysis technique and principal component decomposition is used to estimate the basic functions. Basis provides a set of independent vectors that simultaneously model many data points. The estimated basis functions and corresponding parameters are shown on Table 7.

6 Model Comparisons and Evaluation

Comparison and evaluation of the model prediction accuracy highly depend on the model with the least error that is chosen as the best model for forecasting. Table.7 indicates FTS to have the smallest errors as compared to ARIMA and SARIMA. This is indicated by the overall MSE, RMSE, AIC and BIC prediction and forecast accuracies identifies FTS model as an ideal model and technique for in modeling the high frequency general CPI of the Uganda economy. The same depicts better forecasting ability for high frequency and confirms the superiority of FTS technique or model having great accuracies compared to the traditional methods and a highly recommended the method for forecasting Uganda inflation. The forecasted value using FTSA on Table.8 has less difference from the actual value than forecasted values using ARIMA and SARIMA models respectively.

Table 6: FTSA Estimated Parameters μ, φ and β of General CPI of Uganda.

μ	φ	β
0.005213	0.510739	0.107435
0.003732	0.010324	-0.036438
0.006948	-0.301792	-0.002634
0.0010482	0.370165	-0.004792
0.0025317	0.412653	0.003267
0.0042739	0.247842	-0.004369
0.017314	0.215273	-0.005902
0.010628	0.391604	0.003751
0.007136	0.157287	-0.006854
0.0058631	0.115983	0.009523
0.006215	0.072359	
0.0019342	-0.178325	
0.0046276	-0.265492	

Table 7: Forecast Accuracy Comparison of the General CPI of Uganda.

Method	MSE	RMSE	AIC	BIC
ARIMA	8.30886	2.88251	460.99000	486.68900
SARIMA	4.49850	2.12097	427.35600	436.56300
FTSM	2.17400	0.13900	253.49200	212.61405

Table 8: Comparison Based on June-2020 Forecast.

Actual	ARIMA	SARIMA	FTSA
184.64	187.35	180.69	**183.74**

7 Conclusions

Uganda's general inflation rates are verified by the result to be having seasonality components by the SARIMA model outperforming the ARIMA model based on the MSE and RMSE forecast accuracy respectively. The results also validate the general CPI, world coffee prices, world oil/fuel prices, and the exchange rates as the predominant determinants of inflation in the Ugandan economy.

FTS model has however outperformed ARIMA and SARIMA models respectively as manifested by MSE, RMSE, AIC, and BIC outmatching the two models in modeling the general CPI of Uganda. FTSA forecasted value have small variations compared to the two models. This is a case for the novelty and superiority of the FTS model for accurately modeling and forecasting high- frequency data such as Uganda general CPI. FTS approach, therefore, lays a foundation for future studies in modern time series analysis and forecasting especially highly dimensional and frequency data like the daily or weekly electricity demands as well as the current COVID-19 data in Uganda. It will resolve some complications in forecasting related to frequency domain compared to using traditional methods.

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References

[1] Anguyo, F.L., Gupta, R. and Kotzé, K., 2020. Inflation dynamics in Uganda: a quantile regression approach. Macroeconomics and Finance in Emerging Market Economies., 13(2), 161-187, 2020.

[2] Croux, C. and Joossens, K., 2008. Robust estimation of the vector autoregressive model by a least trimmed squares procedure. In COMPSTAT 2008, pp. 489-501. Physica-Verlag HD, 2008.

[3] Habimana, Norbert, Anthony Wanjoya, and Anthony Waititu."Modeling and forecasting consumer price index (Case of Rwanda)." American Journal of Theoritical and Applied Statistics., 5, 101-107, 2016.

[4] Lidiema, C., 2017. Modelling and Forecasting Inflation Rate in Kenya Using SARIMA and Holt-Winters Triple Exponential Smoothing. American Journal of Theoretical and Applied Statistics., 6(3), 161-169, 2017.

[5] Begg, D., Vernasca, G., Fischer, S. & Dornbusch, R. 2014. Economics. London: McGraw-Hill Education, 1197.

[6] Devereux, Banerjee, R., M.B. and Lombardo, G., 2016. Self-oriented monetary policy, global financial markets and excess volatility of international capital flows. Journal of International Money and Finance., 68, 275-297, 2016.

[7] Bariyo, N. 2011. Uganda Hikes Rates to Tackle Pre-Election Inflation. Surge. The Wall Street Journal, 2011.

[8] Aron, J., Muellbauer, J. & Sebudde, R. 2015. Inflation forecasting models for Uganda: is mobile money relevant? Centre for the Study of African Economies, University of Oxford, 2015.

[9] Kasekende, L. A. 1990. The Relationship between Monetary Aggregates and Inflation. Bank of Uganda Research Abstracts, 1,1183-98, 1990.

[10] Barungi, B. M. 1997. Exchange Rate Policy and Inflation: The Case of Uganda, 1997.

[11] Abuka, C. & Wandera, A. 2001. Determinants of price level dynamics in Uganda: Some stylized facts and empirical evidence, 26,Economic Research Centre, 2001.

[12] Opolot, J. and Mpagi, A., 2017. Inflation dynamics in Uganda: The role of disequilibria in the money and trade goods markets. Journal of Statistical and Econometric Methods., 6(1), 45-74, 2017.

[13] Durevall, D. and Sjö, B., 2012. The dynamics of inflation in Ethiopia and Kenya. African Development Bank Group,2012.

[14] Salisu, A.A. and Isah, K.O., 2018. Predicting US inflation: Evidence from a new approach. Economic Modelling., 71,134-158, 2018.

[15] Atkeson, A. & Ohanian, L. E. 2001. Are Phillips curves useful for forecasting inflation? Federal Reserve bank of Minneapolis quarterly review., 25, 2-11, 2001.

[16] Cecchetti, S. G., Chu, R. S. & Steindel, C. 2000. The unreliability of inflation indicators. Current issues in economics and finance., 6, 1-6, 2000.

[17] Banerjee, A., Marcellino, M. & Masten, I. 2005. Leading Indicators for Euro-area Inflation and GDP Growth. Oxford Bulletin of Economics and Statistics, 67, 785-813.

[18] Arratibel, O., Kamps, C. & Leiner-killinger, N. 2009. Inflation forecasting in the new EU member states., 2009.

[19] Adam, C., Kwimbere, D., Mbowe, W. and O'connell, S., 2012. Food prices and inflation in Tanzania,2012.

[20] Fannoh, R., Orwa, G. & Mung'atu, J. 2014. Modeling the inflation rates in Liberia Sarima approach. International Journal of Science and Research., 3, 1360-1367, 2014.

[21] Schulze, P.M. and Prinz, A., 2009. Forecasting container transshipment in Germany. Applied Economics., 41(22), 2809-2815, 2009.

[22] Uwilingiyimana, C., Munga'tu, J. & Harerimana, J. D. D. 2015. Forecasting Inflation in Kenya Using Arima-Garch Models. International Journal of Management and Commerce Innovations., 3, 15-27, 2015.

[23] Abdelaal, M. M., Saad, M. A. A., Morsy, H. A.-T. M. & Farouq, M. W. 2012. Forecasting the Volatility of Egyptian Inflation Using Garch Models. International Journal of Mathematics & Computation™., 14, 1-13, 2012.

[24] Ullah, S. and Finch, C.F., 2013. Applications of functional data analysis: A systematic review. BMC medical research methodology., 13(1), 43, 2013.

[25] Box, G.E.P. and Jenkins, G.M., 1970. Forecasting and Control. Time Series Analysis,3,75, 1970.

[26] Montgomery, D.C., Jennings, C.L. and Kulahci, M., 2015. Introduction to time series analysis and forecasting. John Wiley & Sons, 2015.

[27] Shumnay, R.H. and Stoffer, D.S. (2011).Time Series Analysis and its Application, Springer Science + Business Media, 2011.

[28] Cryer, J. & Chan, K. 2008. Springer Texts in Statistics. Time Series Analysis with Applications to R,2.New York:Springer, 2008.