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Conversed Fractional Goal Programming: A Mathematical Programming Approach

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Abstract: In this article, the Fuzzy technique is used to identify the optimal distribution of sample sizes in Conversed Tertiary-objective fractional goal programming. LINGO software provides the answer to formulated bi-goal fractional programming. For practical resolution, if the value of the problem is non - integer then the solution of the problem is obtained through Branch and Bound method.

Keywords: Optimal solution, Stratification, Fuzzy programming, LINGO and Fractional goal Programming.

1 Introduction

Statistics is widely used in all fields' of scientific inspection and one of the important sections of statistics is stratified sampling Design. Reducing the heterogeneity of the population units is appropriate to increase the estimate's precision. It is accomplished using stratification technique, where total population comprising of N items is first distributed among L sub-units $N_1, N_2, \ldots, N_L$ of sizes one-to-one are termed as strata and together make the entire population i.e. $\sum_{i=1}^{L} N_i = N$. The strata/blocks are within homogeneous & between they are heterogeneous. Once the blocks created, a sample from each stratum is taken independently. In every block if the procedure of SRS (Simple Random Sampling) is taken, the entire practice is termed as stratified random sampling. N_i should be known in stratified design, the consideration of allocation of sample sizes is most important in every block/stratum and problem of optimally choosing the sample sizes either to minimize cost subject to variance or vice versa is known as the optimal allocation problem. In stratified sampling designs the problem of optimal allocation conferred by numerous authors (see, for example, [1-19] etc).

It is found that a number of optimization problems from different sector of sciences like engineering, agriculture, health, planning, etc and economics have a need of optimizing the ratio between two tasks physical &/or economic. Multi-task problems, which cares about decision-making issues, especially in the public sector where there are several conflicting purposes, was developed as a result of the optimization programming technique having more than one objective function. The multi-objective methodology is a popular alternative that can be applied together in public & private segments. Our daily routines are full of decision-making situations with several objectives that are incommensurable.

In Refs. [20, 21] the obtained integer optimal allocation in formulated Linear /Non -linear Plus fraction programming problem and in different situations. In mathematical modeling an expanding economy through a generalized fractional programming approach used [22]. Various algorithms found in ([23,24,25,26]) used to solve linear fractional programming problems. [27] presented two alternative algorithms—the fuzzy weighting average algorithm and the fuzzy critical point algorithm—to determine the ideal location for the facility centre.

This article discusses a fuzzy programming technique in which [28] converts a tertiary goal function into a bi-goal and solution of the programming problem is achieved by using Mathematical programming software [29]. For practical

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scenario, in rare cases, changing non-integer sample sizes to the adjacent integral value may yield an impossible and suboptimal outcome if the acquired result is not an integer. In that scenario, the Branch and Bound approach would be used instead of non-integer values to the neighbouring integer value.

2 Problem Formulations

Suppose size *N* of finite population is distributed among *M* strata of different sizes $N, N_2, \dots N_M$ in such a way that $\sum_{i=1}^{M} N_i = N$. It is also assumed that the drawing of the sample sizes n_i from each stratum is made independently.

Let Y = studying characteristic and

$\bar{y}$ = impartial estimation of the population's average ($\bar{Y}$).

Let $\bar{y}_i$ = impartial estimation of the stratum's average $\bar{Y}_i$

s.t $\bar{y}_i = \frac{1}{n_i}\sum_{h=1}^{n_i} y_{ih} \ \forall \left(i=1,2,3,...L\right)$ and $\bar{y} = \frac{1}{N}\sum_{i=1}^{M} N_i\,\bar{y}_i = \sum_{i=1}^{M} W_i\,\bar{y}_i$ is an impartial estimation of population mean $\bar{Y}$. This estimate's accuracy is restrained by variance of the sample-based estimation of the population's attributes.

$V(\bar{y}) = \sum_{i=1}^{M} a_i x_i$, where, $\sum_{i=1}^{M} W_i^2 S_i^2 = a_i$, $W_i = \frac{N_i}{N}$ and $x_i = \frac{1}{n_i} - \frac{1}{N_i}$.

Subsequently $\sum_{i=1}^{M} \frac{a_i}{N_i}$ keeps going and it is adequate to min. $V(\bar{y}) = \sum_{i=1}^{M} \frac{a_i}{n_i}$.

Furthermore, the CV is

$$CV = \left[\sum_{i=1}^{M} \frac{a_i}{n_i}\right]^{1/2} \Big/ \bar{Y}$$

The optimum allocation problem comprises of determining the sample sizes in such a way which minimizes the overall variance tested by the cost function.

The cost function is given as

$$C = c^o + \sum_{i=1}^{M} c_i n_i \tag{i}$$

where, c_i charge per sample in i^{th} block, c^o is overhead charges and C is the survey's entire available budget. Let $C - c^o = C^*$. Now, optimal allocation of different sample sizes is determined from the below formulated tertiary objective allocation problem:

$$\left.\begin{array}{l} \text{Min } c^{**} = \sum_{i=1}^{M} c_i n_i \\ \text{Min } n^{*} = \sum_{i=1}^{M} n_i \\ \text{Min } (CV) = \left[\sum_{i=1}^{M} \frac{a_i}{n_i} \right]^{1/2} \Big/ \bar{Y} \end{array}\right\} \quad \text{(ii)}$$

Sub. to

$$V(\bar{y}) = \sum_{i=1}^{M} \frac{a_i}{n_i} \le v^{*}$$

$$2 \le n_i \le N_i \qquad n_i \text{ ,integer } i = 1,2,3,\ldots,L$$

Where v^{*} is prefaced variance of the population mean estimate?

3 Fuzzy Method for Conversed Fractional Goal Programming

The outlined proposed algorithm for (MONLPP) is under:

I : Identify the highest value of $K_p(n)$ say it $K_p(n^{*})$.

II. Perform division to each individual objective by $K_p(n^{*})$ and for left over $K_{p-1}(n)$.

III: we obtain fractional programming $\zeta_t(n)$.

IV: Construct the membership function for pth objective.

V: Use Taylor's first order theorem to transform the membership function.

VI: Frame the Fuzzy mathematical goal programming problem.

VII: Determine n^{*} later evaluating converted mathematical programme in Step 6.

VIII: Terminate.

4 Numerical Example

The below mention information has been barrow from [10],in which population comprises 64 units, the stratum weights & variance of a population that has been distributed among 3 divisions with studying one feature is given as under in tabular form 3.1.

Table 3.1: Information about strata with one feature.

i	N_i	$W_i = \frac{N_i}{N}$	S_i^2	$\bar{Y}_i^2$	a_i
1	16	0.2500	540.0625	62.9375	33.7539
2	20	0.3125	14.6737	27.6000	1.4330
3	28	0.4375	7.2540	14.0714	1.3885

It is Assumed the available budget denoted by C =100 items with c^{O} and c^{O} = 30 items (above cost). The total given budget aimed at the field study is C^{*} = 70 items. It is further assumed that the charge t of dimension C_i in different units are $c_1 = 4, c_2 = 1.5\, and\;\; c_3 = 1$ aimed at the below cost relationship $C = c^{O} + \sum_{i=1}^{M} c_i n_i$

Now, In next step the parameters were replaced by values given table 3.1 , the NLPP (1.1) can be framed as under (when t=3):

$$\left.\begin{aligned}
&Min\;\; K_1(n) = 4n_1 + 1.5n_2 + n_3\\
&Min\;\; K_2(n) = n_1 + n_2 + n_3\\
&Min\; K_3(n) = \left[\frac{33.7539}{n_1} + \frac{1.4330}{n_2} + \frac{1.3885}{n_3}\right]^{1/2}\\
&Subject\;\; to\\
&\frac{33.7539}{n_1} + \frac{1.4330}{n_2} + \frac{1.3885}{n_3} \le 2.90\\
&2 \le n_1 \le 16,\;\; 2 \le n_2 \le 20,\;\; 2 \le n_1 \le 28
\end{aligned}\right\} \quad \text{(iii)}$$

Resolving (iii) using LINGO software and by applying step 1, we get $F_1(n) = 70,\; F_2(n) = 23.15, and\;\; F_3(n) = 0.036$.

Applying step 2 and 3 we get developed (FGPP) and is written under:

$$\left.\begin{aligned}
&Min\;\; \xi_1(n)\\
&Min\;\; \xi_2(n)\\
&subject to\\
&\frac{33.7539}{n_1} + \frac{1.4330}{n_2} + \frac{1.3885}{n_3} \le 2.90\\
&2 \le n_1 \le 16,\;\; 2 \le n_2 \le 20,\;\; 2 \le n_1 \le 28
\end{aligned}\right\} \quad \text{(iv)}$$

Therefore, the converted programme (iv) has dual objectives attained by solving (iii) for each objective i, e O(1) = $F_1^{**} = 0.3170$ and O(2) = $F_2^{**} = 0.0002997$.

In the direction of n and to fulfill the subsequent fuzzy objectives and vague aspirations are 0.3170 and 0.0002997 s.t. $\xi_1(n)\;\le 0.3170 and\; \xi_2(n)\;\le\; and\, 0.0002997$.Therefore 0.56 & 0.00067 are tolerance limits to the above mentioned objectives respectively.

Thu we describe the membership functions as:

$$\mu_1(n) = \begin{cases} 1 & if \quad \zeta_1(n) \le\; 0.3170 \\ \dfrac{0.56 - \zeta_1(n)}{0.56 - 0.3170} & if \quad 0.3170 \le \zeta_1(n) \le 0.56 \\ 0 & if \quad \zeta_1(n) \ge 0.56 \end{cases}$$

$$\mu_2(n) = \begin{cases} 1 & if \quad \zeta_2(n) \le 0.0002997 \\ \dfrac{0.00067 - \zeta_2(n)}{0.00067 - 0.0002997} & if \quad 0.0002997 \le \zeta_2(n) \le 64. \\ 0 & if \quad \zeta_2(n) \ge 0.00067 \end{cases}$$

Thus, first order series about points by Taylor n_1^* = (16, 3.85, 3.32) and n_2^* = (16, 20, 28) for $\mu_1(n)\ and\ \mu_2(n)$

$$\mu_1(n) = \mu_1(n_1^*) = 0.0150n_1 - 0.0.02965n_2 - 0.0385n_3 + 0.9965$$

$$\mu_2(n) = \mu_1(n_2^*) = 0.06n_1 + 0.013n_2 - 0.0090n_3 + 0.633$$

Consequently, FGP can be defined as

$$\left.\begin{array}{c} \text{Min } \mu_t(n) \\ \text{S. t} \\ V(\bar{y}) = \sum_{i=1}^{M} \frac{a_i}{n_i} \le V^* \\ 2 \le n_i \le N_i \qquad n_i\text{, integer } i = 1,2,3,\ldots,L \end{array}\right\} \quad \text{(v)}$$

The above-described relationship with 1 a highest value, and the following is the aspiration level of the above-described relationship with unity:

$\mu_t(n) + \delta_t^+ = 1$, Where δ_t^+ is the over deviational variable.

FGP is described as

$$\left.\begin{array}{l} \text{Min } \delta_t^+ \\ \text{S.t} \\ V(\bar{y}) = \sum_{i=1}^{M} \frac{a_i}{n_i} \le V^* \\ \mu_t(n) + \delta_t^+ = 1 \\ 2 \le n_i \le N_i \qquad n_i\text{, integer } i = 1,2,3,\ldots, L \end{array}\right\} \quad \text{(vi)}$$

Applying (vi)& the described mathematical model (iv) is given below:

Min $\delta_1^+ + \delta_2^+$

S. t

$$\left.\begin{array}{l} 0.0150n_1 - 0.0.02965n_2 - 0.0385n_3 + 0.9965 + \delta_1^+ = 1 \\ 0.06n_1 + 0.013n_2 + 0.0090n_3 - 0.633 + \delta_2^+ = 1 \\ \frac{33.7539}{n_1} + \frac{1.4330}{n_2} + \frac{1.3885}{n_3} \leq 2.90 \\ 2 \leq n_1 \leq 16,\ 2 \leq n_2 \leq 20,\ \ 2 \leq n_1 \leq 28 \end{array}\right\} \quad \text{(vii)}$$

After evaluating (vii) using LINGO s, we have

$n_1 = 16, n_2 = 4.19$, $and\ n_3 = 3$. Thus optimal allocation is $n_1 = 16, n_2 = 5, and\ n_3 = 3$.

Lingo Software's Branch and Bound approach produced the following results:

```
 Local optimal solution found.
Objective value:                              0.4482500
Objective bound:                              0.4482500
Infeasibilities:                               0.000000
Extended solver steps:                                8
Total solver iterations:                            190

Model Class:                                      MINLP

Total variables:                   5
Nonlinear variables:               3
Integer variables:                 3

Total constraints:                 4
Nonlinear constraints:             1

Total nonzeros:                   13
Nonlinear nonzeros:                3


                         Variable           Value        Reduced Cost
                               D1       0.2725000E-01        0.000000
                               D2       0.4210000            0.000000
                               N1       16.00000         -0.8500000E-01
                               N2        5.000000          0.1665000E-01
                               N3        3.000000          0.2950000E-01

                              Row    Slack or Surplus      Dual Price
                                1       0.4482500           -1.000000
                                2        0.000000           -1.000000
                                3        0.000000           -1.000000
                                4         0.3969792E-01        0.000000
```

5 Conclusions

It is determined that a fuzzy technique is used in developed tertiary objective stratified sampling design, which is translated into bi-objective, to determine the best distribution of sample sizes. Turning non-integer values to neighbouring integral values can occasionally produce an impractical result in real-world settings. Branch and Bound Method has been applied to determine integer optimum sample size allocation.

Conflicts of Interest: The authors affirm that the publishing of this paper does not involve any conflicts of interest.

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